THE DYNAMO

THE DYNAMO

ITS THEORY, DESIGN, AND MANUFACTURE

BY

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PREFACE TO VOLUME II.

In the opening chapter of the present volume, which completes the study of continuous-current dynamos, a detailed analysis of the effect of armature reaction on the flux-curve under load is given both for non-commutating-pole and commutating-pole machines. In the former, it has always appeared to the writer that the ordinarily accepted division of the armature ampere-turns into a "back" and a "cross" group rests on a somewhat insecure foundation, or at least that its logical presuppositions and its limitations as a truthful representation of the physical facts have not been sufficiently recognized. The revival and almost universal use of commutating poles has largely robbed this question of practical importance, yet it is the writer's belief that the non-commutating-pole machine fully analysed still affords the best approach to a correct understanding of exactly how a commutating pole works: its function is simple, but how it performs it and the inter-relations of the main and reversing fluxes are by no means so simple. The non-commutating-pole machine is still therefore given an important place in the chapter on sparking.

The subject of the eddy loss in solid and in laminated pole-pieces has again been reviewed in the light of further experimental results obtained since the last edition.

Two designs of dynamos have been worked out in full, in order to illustrate the application of the numerous formulæ which have been given in preceding chapters: as explained in the text, many of the calculations would not need to be carried through in such detail, the practised designer depending on his judgment and experience. Yet perhaps it may be permitted to express some regret that only too often, under the exigencies of business, insufficient time is allowed in the dynamo-designing office for careful design and tabulation of data for future reference. In the end such care is economical in time and money.

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C. C. H.

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LIST OF CHIEF SYMBOLS ADDITIONAL TO THOSE OF VOL. I.

AT_b . . . = demagnetizing armature AT on half magnetic circuit (eq. 142).

AT_{cx} . . = AT expended over armature core from plane of maximum flux or neutral line up to point x (Fig. 325, Chap. xix, § 8).

AT_{cx1}, AT_{cx2} = AT expended over armature core as above, on the leading and trailing sides respectively.

AT_{cc}' . . = AT_{cx} when point x is at distance c' from trailing pole-edge.

AT_{c1}, AT_{c2} . = AT expended over total length of path in armature core from plane of maximum flux up to plane of bifurcation, on the leading and trailing sides respectively.

AT_{gr} . . = AT expended over air-gap of commutating pole.

AT_P . . . = magnetic potential in AT of main pole-face.

AT_P' . . = magnetic potential of main pole-face as due to field AT (including comm. poles) acting alone.

AT_{PC}' . . = magnetic potential of commutating pole-face as due to field and comm. pole AT acting alone.

AT_{PC}'' . . = magnetic potential of commutating pole-face as due to armature AT acting alone.

AT_r . . . = ampere-turns of excitation on a commutating pole.

a . . . = height of rectangular packet of coil end-connexions (Fig. 391).

a, a' . . . = magnetic potential of main pole-face due respectively to field (including comm. poles) AT and to armature AT acting alone (Chap. xix, §§ 7, 11).

a'' . . . = $4\pi \times$ equiv. permeance per cm. length for wedge and slot-opening (Chap. xx, § 24).

a_{cp} . . . = cross-sectional area of commutating pole.

a_{gr} . . . = effective air-gap area of commutating-pole reversing field (Chap. xix, § 14; and xx, § 41).

B_g', B_g''. . = flux-density in air-gap at leading and trailing pole-corner respectively.

B_{gr} . . . = flux-density in air-gap under commutating pole.

B_{mr} . . . = flux-density in pole-core of commutating pole.

B_s . . . = flux-density at brush position from symmetrical main field excitation (Chap. xx, § 32).

B_{se} . . . = correct flux-density from symmetrical main field excitation for linear commutation (eq. 192).

B_q . = flux-density of armature cross flux.
B_r . = reversing flux-density from commutating pole.
B_{rc} . = correct reversing flux-density for linear commutation (eq. 193–195).
b . = width of rectangular packet of coil end-connexions (Fig. 391).
= $4\pi \times$ equiv. permeance per cm. along surface of core (Chap. xx, § 24).
= pitch of sectors at surface of commutator (Chap. xx, § 3).
b_1 = peripheral width of contact of brush (Chap. xx, § 3).
b_m = peripheral width of mica insulating strip in commutator.
b, b' = magnetic potential of yoke opposite commutating pole as due respectively to field and comm. pole AT and to armature AT acting alone (Chap. xix, § 11).

C = number of commutator sectors or coils.
C_k = number of sections simultaneously short-circuited at a brush (eq. 191).
c . = number of sectors per slot.
= half interpolar gap measured on armature circumference in non-commutating-pole machine or half distance between a main and a commutating pole.
c' = distance measured on armature circumference from trailing pole-edge to tangent point of main field component flux.
c'' = distance of bifurcation point of armature component flux ahead of IS.
c''' = distance of tangent point of resultant flux ahead of IS.
c_λ . = distance of diameter of commutation ahead of IS.

D_c = diameter of commutator.
d = distance by which zero of armature component cross flux falls behind centre of pole (Chap. xix, § 6).
d_r . = diameter of collecting ring in homopolar dynamo (Chap. xxii, § 21).
ΔE . = E.M.F. set up between extreme edges of brush (Chap. xx, § 31).
ΔE_1 . = ditto with brushes at no-load position, without commutating poles.

ΔE_2 . . . = ditto with brushes at half-load position without commutating poles.

E_b . . . = drop of potential over one set of brushes (Chap. xx, § 10).

F . . . = coefficient for eddy-current loss in armature (Chap. xxi, § 18).

F_u . . . = area of contact of one brush-set, $= l_b b_1$ (Chap. xx, § 5).

F_u' . . . = area of contact of one brush-set with leading sector of short-circuited section.

F_u'' . . . = area of contact of one brush-set with trailing sector of short-circuited section.

$f'(B)$ . . . = at per cm. at density B in iron.

$f(t)$ . . . = E.M.F. induced in short-circuited section by rotation in main or commutating-pole field.

$f(t)_c$ . . . = correct E.M.F. in short-circuited section for linear commutation.

$f(t)_z$ . . . = difference, $f(t) - f(t)_c$, causing i_z.

G . . . = specific torque coefficient (Chap. xxii, § 2) = (watts per rev. per min.)$/D^2L$.

H . . . = heat capacity of a body, *i.e.* joules which will raise its temperature 1°.

= hysteresis coefficient of armature = H_w/N (Chap. xxi, § 23).

h. . . . = $\frac{1}{2}(AT_{c2} - AT_{c1})$ = magnetic potential of armature core on neutral plane (Chap. xix, § 8).

$h - AT_{cx1}$. = magnetic potential in AT of armature core opposite commutating pole (Chap. xix, § 13).

h_2 . . . = height of wedge (Fig. 357).

h_3 . . . = height of lip of slot-opening (Fig. 357).

I_0 . . . = current in third wire of three-wire system (Chap. xxiii, § 7).

i . . . = instantaneous current in short-circuited section.

i_c . . . = correct instantaneous current in short-circuited section for linear commutation.

i_z . . . = additional instantaneous current in short-circuited section $= i - i_c$.

i_1 . . . = instantaneous current in leading commutator connector of short-circuited section.

i_2 . . . = instantaneous current in trailing commutator connector of short-circuited section.

j . . . . = joules required to raise 1 cub. inch 1° C. (Chap. xxi, § 11).
j (with suffix) = number of coil-sides simultaneously short-circuited and acting on the same permeance (Chap. xx, § 24).

k' . . . . = permeance factor which, when multiplied by h_w/w_s, gives for a barrel-winding in two layers part of the slot-inductance of short-circuited section per cm. length (Chap. xx, § 24).
K_r . . . . = extension coefficient for air-gap length of commutating pole.

L_r . . . = axial length of commutating-pole face.
l . . . = net axial length of armature core after deduction of air-ducts.
l' . . . = length of one complete end-connexion of coil.
l_{gr} . . . = length of air-gap under commutating pole.
l_b . . . = axial length of one set of brushes along commutator.
l_{cp} . . . = radial length of commutating pole.
$\mathscr{L}_{sz}$ . . . = apparent self-inductance of short-circuited section at final moment.
Λ . . . = $4\pi \times$ equivalent permeance, in calculations of inductance.
λ_e . . . = angle of lead of brushes in electrical degrees $= p\lambda$.
λ . . . = angle of lead of brushes in mechanical degrees.
= $4\pi \times$ equivalent permeance per cm. length, in calculations of inductance.
λ_1, λ_2 . . = ditto for the two sides of a drum coil respectively.
λ' . . . . = ditto for one complete end-connexion.

$\mathscr{M}$ . . . = mutual inductance.

$\mathscr{P}_{lr}$ . . . = leakage permeance of commutating pole.

R . . . = resistance of short-circuited section and commutator connectors $= r + 2r_c b/b_1$ (Chap. xx, § 7).
R_k . . . = specific resistance in ohms per unit area of brush contact on commutator.
R_k' . . . = specific resistance in ohms per unit area of brush contact with leading sector.
R_k'' . . . = specific resistance in ohms per unit area of brush contact with trailing sector.

$\mathcal{R}_{o}$ . . . = magnetic reluctance of half length of path in armature core.
$\mathcal{R}_{op}$ . . . = magnetic reluctance of commutating pole.
$\mathcal{R}_{gr}$ . . . = magnetic reluctance of commutating-pole air-gap.
$\mathcal{R}_{t}$ . . . = magnetic reluctance of tooth.
$\mathcal{R}_{t}'$, $\mathcal{R}_{t}''$. = magnetic reluctance of teeth per sq. cm. of path at leading and trailing pole-corner respectively (Chap. xix, § 18).
$\mathcal{R}_{lr}$ . . . = leakage reluctance of commutating pole.
$\mathcal{R}_{y}$ . . . = magnetic reluctance of length of path in double section of yoke or of half length of path in single section of yoke.
$\mathcal{R}_{y}'$ . . . = ditto of half length of path in single section of yoke on leading side (Fig. 328; and Chap. xix, §§ 10 and 11).
$\mathcal{R}_{y}''$ . . . = ditto of half length of path in single section of yoke on trailing side (Fig. 328; and Chap. xix, §§ 10 and 11).
r . . . = resistance of one section of armature winding (Chap. xx, § 5).
r_{c} . . . = resistance of one commutator connector (Chap. xx, § 5).
r_{ph} . . . = resistance of one leg of static balancer (Chap. xxiii, § 7).
r_{s} . . . = distance between centres of two adjacent coil-sides in same slot and layer (Chap. xx, §§ 36 and 45).

s_{u} . . . = normal current-density at brush-contact (Chap. xx, § 6) $= 2J/F_{u}$.
s_{u}' . . . = normal current-density at brush-contact with leading sector $= i_1/F_{u}'$.
s_{u}'' . . . = normal current-density at brush-contact with trailing sector $= i_2/F_{u}''$.

T . . . = time of short-circuit of armature section in secs. (eq. 190).
t_1 . . . = tooth-pitch.
t_r . = rise of temperature.

v . = peripheral speed of armature in cm. per sec.
v_c . = peripheral speed of commutator.
= potential of third wire (Chap. xxiii, § 7).
v_c' . = peripheral speed of commutator in ft. per min. (Chap. xxii, § 21).
v_r' . = peripheral speed of collecting ring in ft. per min.

w . . . = number of conductors in a coil-side of an armature coil (Chap. xx, § 24) = $Z/2C$.

w_c . . . = peripheral width of commutating-pole face (Chap. xix, § 11).

X_g . . . = ampere-turns of excitation over double air-gap = $2AT_g$.

X_t . . . = ampere-turns of excitation over teeth = $2AT_t$.

Φ_{mr} . . . = total flux (useful and leakage) in commutating pole.

ϕ_r . . . = useful flux of commutating pole.

ϕ_{lr} . . . = leakage flux of commutating pole.

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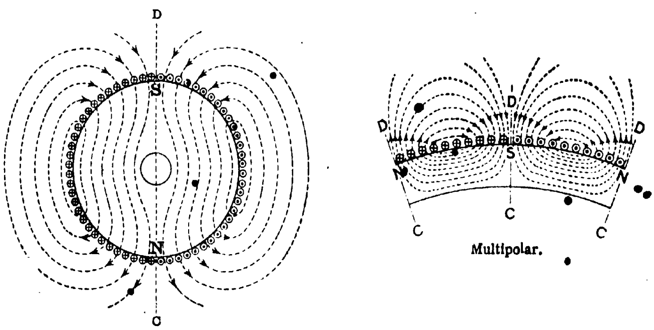
CHAPTER XIX

ARMATURE REACTION AND THE FLUX-DENSITY CURVE UNDER LOAD IN CONTINUOUS-CURRENT DYNAMOS

§ 1. Diameter of commutation and brush-position.—In the continuous-current closed-circuit dynamo, whatever the nature of the armature winding, it is divided by the brushes into as many current-sheets as there are poles. Thus in the case of a drum-wound



Bipolar.

Multipolar.

FIG. 314.—Magnetic field of continuous-current armature without field-magnet.

armature for a 2-pole machine or as part of a multipolar machine, two sheets of current as shown by the dotted and crossed circles of Fig. 314 flow along the external surface from end to end, and the direction of the currents in the active conductors changes at the diameter of commutation (DC) corresponding to the position of the brushes—unless the armature is chord-wound, when adjacent current-sheets partially overlap. It may here be recalled that the relative position of the line of the brushes on the commutator and the line passing through the coils undergoing commutation on the armature core may vary according to the way in which they are connected to the commutator sectors, but in the toothed drum machine the connexion is usually made at the centre of the end-connectors, so that the brush position is shifted

90 electrical degrees or $90/p$ mechanical degrees away from the true line of commutation. In every case, then, the line or *diameter of commutation* (a term extended to multipolars by analogy from the 2-pole dynamo) must be understood to refer to the actual position of the coils undergoing commutation rather than to the position of the brushes corresponding thereto.

§ 2. The ampere-turns of the armature.—In Fig. 315 (*a*) a few of the ampere-conductors of Fig. 314 are joined by end-connectors

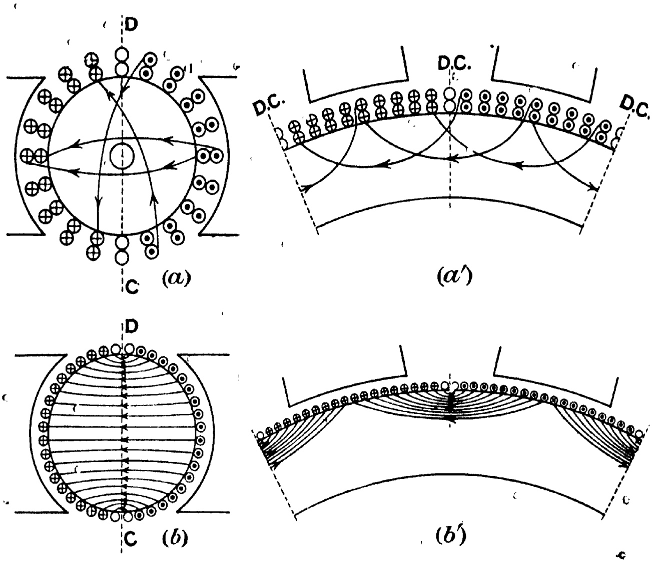


FIG. 315.—The armature conductors coupled up into loops by end-connexions (*b*, *b'*) replacing the real end-connexions (*a*, *a'*).

as they might be in the actual 2-pole machine. So far as their magnetic effect is concerned, it will be seen that in each pair, one on either side of the diameter of commutation, the components of the current parallel to the diameter of commutation, being in opposite directions, practically cancel out, and only the components at right angles to the diameter of commutation, being uniformly in the same direction across the core, are left to reinforce one another. It will therefore not conflict with the magnetic effect of the end-connexions if we ignore their actual paths in considering the M.M.F. of the current-carrying armature as a whole, and imagine each active conductor connected across to a corresponding active conductor on the opposite side of the diameter of commutation.

in planes at right angles thereto, as shown in Fig. 315 (*b*). A number of loops are thus obtained, each carrying a current equal to half the total armature current, of which the M.M.F. has the general direction parallel to the diameter of commutation. If acting by itself, with the armature in air, this M.M.F. system would yield lines of flux through the two halves of the core on either side of the diameter of commutation, issuing from the lower surface and circling round to enter in at the upper surface (Fig. 314).

In the multipolar case (Fig. 315 (*a'*)), the crossing of the end-connexions in opposite directions in the two layers of a barrel-wound armature, appears to introduce great complexity, but this vanishes when the end-connexions are developed on the flat and the directions of the currents are examined (Fig. 316). Let the

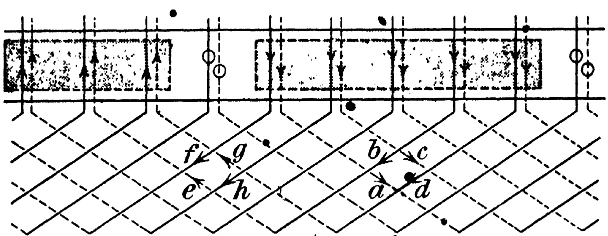


FIG. 316.—Currents in end-connexions of barrel-wound armature.

currents be resolved into axial and peripheral components. Then considering any one mesh such as *a b c d* opposite a pole, the peripheral components practically neutralize one another, so far as their magnetic effect is concerned (except for the space between the layers) and only the axial components need be considered. But in a mesh such as *e f g h* in the plane of commutation, it is the axial components which neutralize one another, and only the peripheral components which agree in their magnetic effect. Applying the same analysis to other meshes the whole system of end-connexions resolves itself into pairs of triangular current-sheets, one opposite the commutation plane in which the current flows circumferentially, the two triangles being bounded by the end-connexions of diametric loops in the middle of their commutation period (Fig. 395). When the strips of current are joined up, a series of loops is reached,[1] which justifies us in replacing the actual end-connexions by the imaginary system of Fig. 315 (*b'*). Thus the real armature winding may be assumed to be replaced

[1] See "A Note on the Distribution of Flux around a Continuous-current Armature," B. Hague and F. F. H. Schröder, *Electr.*, Vol. 75, p. 959 (1915).

in a two-pole machine by a single coil having for its axis the diameter of commutation, and in a multipolar machine by as many coils as there are poles, each of curved pancake shape (Fig. 315 *b* and *b'*).

The general effect of the $2p$ current-sheets given by these coils, when considered apart from any other M.M.F.'s, may in all cases be stated to be the formation of N. and S. polar surfaces about the opposite ends of the diameter of commutation and stretching along the entire length of the core. The one polarity shades off into the other at right angles to the diameter of commutation about the centre of the current-sheets, *i.e.* about the centre of the

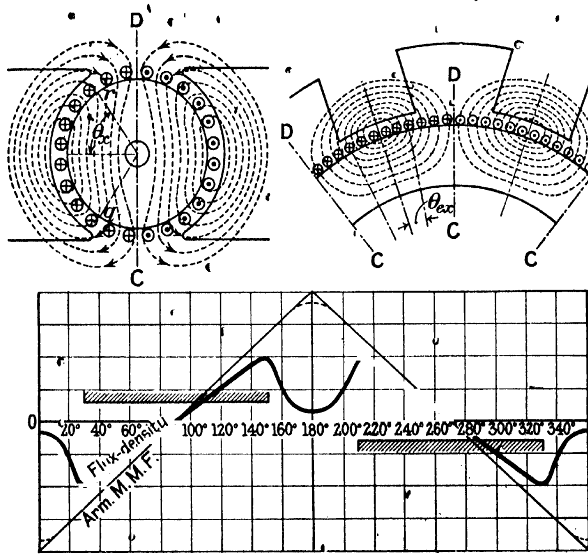


FIG. 317.—Magnetic field of armature with unexcited field-magnet.

equivalent coil-side of the 2-pole armature (Fig. 315 *b*) or the point where the equivalent coils of the multipolar armature divide (Fig. 315 *b'*).

§ 3. The 'field' of the armature alone.—When the armature is surrounded by air, the flux due to the armature ampere-turns will be comparatively weak. But when the armature, either bipolar or multipolar, is placed within the embrace of the iron pole-pieces of the field-magnet, the length of the air-path under the poles is so greatly reduced that the same armature M.M.F. will yield a strong flux, passing across the poles and traversing the two short air-gaps (Fig. 317). As the armature rotates, the

conductors pass successively from one current-sheet into the next, but the sheets themselves remain fixed in space,[1] and so also the flux to which they give rise remains stationary and unchanging.

The shape of the spacial curve of flux-density from the armature M.M.F. alone is, in the absence of any commutating poles, indicated at the foot of Fig. 317, from which it will be seen that the density rises to a maximum at the pole-tips and diminishes in the interpolar gap owing to the greater length of the air-path thereat, although

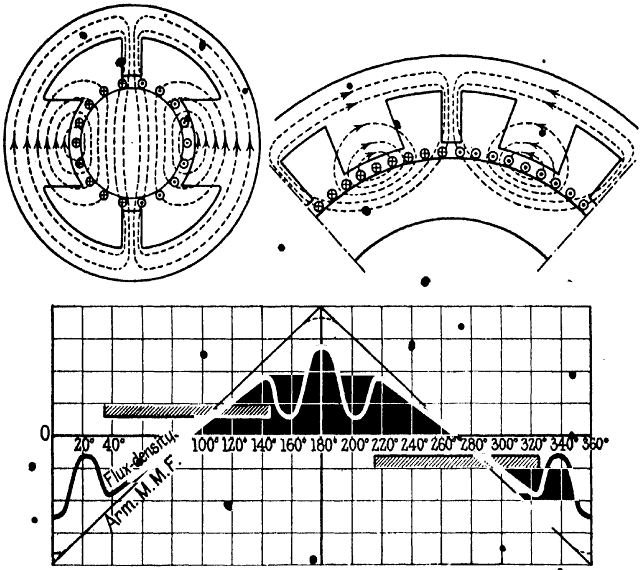


FIG. 318.—Magnetic field of armature with field-magnet and commutating poles unexcited.

this region is really the centre of the polar surface formed by the armature. When commutating poles are present, the curve will have the shape shown in Fig. 318, and the armature flux will be especially strong under the commutating poles.

It is here for the present assumed that the diameter of commutation coincides with the interpolar line of symmetry or with the centre of a commutating pole.

If Z be the total number of conductors distributed round the

[1] Except for minor commutation effects, and slot and tooth effects which are here neglected.

armature, and the current flowing in each be J, the ampere-conductors forming one current-sheet[1] are—

$$\frac{JZ}{2p} \text{ or } ac\,.\,Y$$

where Y is the pole-pitch and ac is the number of ampere-conductors per unit length in the circumference. Assuming the conductors to be uniformly distributed or evenly spaced in small groups, for any angle θ_{ex} in electrical degrees (Fig. 317), or distance x measured on the armature surface when θ_{ex} and x fall within one current-sheet, the ampere-conductors embraced are—

$$\frac{JZ}{2p} \times \frac{\theta_{ex}}{180^\circ} \text{ or } ac\,.\,x$$

Between two points making equal angles with the centre of the current-sheet or at equal distances from it, *i.e.* reckoning θ_{ex} or x from a point at right angles to the diameter of commutation, the ampere-conductors embraced are—

$$\frac{JZ}{2p} \times \frac{2\theta_{ex}}{180^\circ} \text{ or } ac\,.\,2x$$

their M.M.F. is

$$1{\cdot}257\frac{JZ}{2p} \times \frac{2\theta_{ex}}{180^\circ} \text{ or } 1{\cdot}257\ ac \times 2x.$$

and the maximum value of the M.M.F. reached on the diameter of commutation is $1{\cdot}257\ JZ/2p$ or $1{\cdot}257\ ac\,.\,Y$.

Taking any closed path, as between $r\,q$ (Fig. 317), the flux which traverses it is equal to the M.M.F. of the armature ampere-turns within those points, divided by the reluctance of the double air-gap, of the teeth and of the iron pole and core through which the lines pass. Under the present assumption that the armature AT are symmetrically placed with respect to the poles, there is no M.M.F. and no flux opposite the centre of the pole-face; thence the density rises to a maximum at the pole-tips. Outside the limits of the polar

[1] The values here given imply that the full current occurs even in the short-circuited coils on either side of the central commutation plane. The maximum that will be reached if commutation proceeds properly is therefore slightly over-estimated, and is really as shown by the dotted line at the apex of the M.M.F. triangle (Fig. 317). If it be assumed that in each commutation zone two sectors are always short-circuited out of a total of 18 per pole, *i.e.* covering a zone of 10° (electrical) on either side of the diameter of commutation and that the short-circuit current follows a linear law, the ampere-conductors rise at a uniform rate of ac per cm. from the centre of the current-sheet up to $ac\dfrac{Y}{2} \times \dfrac{8}{9}$, and thence at a rate decreasing from ac to zero and averaging $\frac{1}{2}\,ac$. The maximum over a pole-pitch is then $ac\,.\,Y\left(\dfrac{8}{9} + \dfrac{1}{18}\right) = ac\,.\,Y \times \dfrac{17}{18}$.

arc the armature M.M.F. continues to increase, but for Kl_g must now be substituted in the non-commutating-pole machine the increased length of air-path, namely, $Kl_g + \xi x$, as explained in Chapter XVI, § 6 (*a*); the density therefore decreases and falls to a minimum (but not to zero) on the line of symmetry. In the commutating-pole machine, although it falls to a minimum in the gap between a main and a commutating pole, it rises again to a second maximum under the commutating pole owing to the short length of the air-gap there.

§ 4. The distortion of the resultant field.—Let the sheets of armature current shown in Figs. 317 and 318 be due to rotation of the armature as a dynamo, say, in a clockwise direction; then in order to produce armature currents in the directions shown, there must be a N. pole on the left and a S. pole on the right-hand side. Let that edge or corner of the pole under which an active conductor first enters after passing through an interpolar gap be termed the "leading" edge as opposed to the "trailing" edge from under which it emerges into an interpolar gap. If the general direction of the two symmetrical sets of lines, namely, those that would be due to the field-magnet system by itself, and those of either Fig. 317 or 318 that would be due to the armature AT alone, are now compared together, it will be found that the lines are in the same direction in the air-gap and teeth under the trailing half of each main pole, *e.g.* under each trailing pole-tip (*r*, Fig. 317), but that their directions are immediately opposed in the air-gap under the leading half of each pole-face, *e.g.* under each leading pole-tip (*q*, Fig. 317). The commutating poles being in the dynamo virtually extensions of the leading pole-tip brought to the centre of the gap between the main poles, the direction of the lines in the commutating poles and their air-gaps as due to the field-magnet system (including the excitation of the commutating poles) is always directly opposed to that of the lines due to the armature AT acting alone.

But when the dynamo is at work and supplying current, the M.M.F.'s of both the field ampere-turns and the armature ampere-turns are simultaneously present, and in fact there is only one resultant distribution of field due to the combined effect of the two acting together. In nature the total flux is always the maximum that is possible with the given number and distribution of all the ampere-turns that are present, due regard being paid to the reluctivity of the material of the circuit which, when of iron, will alter as the flux-density at different points is varied with different distributions. *I.e.* the total flux is linked with the maximum possible number of forward AT, or conversely passes through a minimum number of back AT. If the symmetrical distribution which holds when the armature is on open circuit were retained,

the lines in the armature would run in paths parallel with the armature AT of Fig. 315 b and b', and would not thread *through* them; such a distribution could not therefore express the magnetic effect of the armature turns. It follows that the resultant lines of the field-system no longer pass straight across the armature from pole to pole, but tend to become bent so as to pass through as many forward AT of the armature as possible; in other words, the field is distorted or twisted round in the direction of rotation. Thus in Fig. 319 it is shown how in a 2-pole machine if a line which is initially central enters the armature in the upper left quadrant and emerges from the lower right quadrant, it will pass through a certain number of forward armature ampere-turns, and the same will be also found to be true in the multipolar case (Fig. 319 b). But all the lines cannot flow along the paths indicated; the action of the additional forward AT through which the lines,

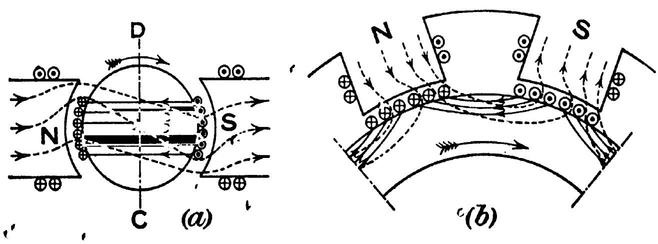


FIG. 319.—Resultant flux threading through armature ampere-turns.

when thus displaced and concentrated, pass is checked by the combined effect of two restraining causes, namely, the greater length of the path and the greater fall of magnetic potential that must result from the increased density of the flux in the trailing regions, even if the permeability of the material remained constant, coupled with the fact that in iron the permeability will in general decrease. The lines must therefore be more or less spread out, and the total flux is retained at its highest possible value and linked with as many forward AT as possible on the armature, so that the total energy stored in the resultant magnetic field, in relation to the exciting coils and the armature turns as a whole, is a maximum.

The general result is shown in Fig. 320. The distribution of the field or the density in the air-gap instead of being uniform over the greater part of the bored face of the main pole rises continuously from the leading to the trailing pole-tip. The distortion of the resultant field now corresponds to the inductance of the armature current-turns. If the armature current is reduced in strength, the lines will straighten themselves and in swinging back to a more symmetrical distribution will cut the armature conductors, giving

a forward E.M.F., tending to keep up the previous value of the current. In so doing, the stored energy of the armature current-turns, due to their reaction on the magnetic field, will reappear. The greater the armature current, the greater the displacement of the field as a whole and of the resultant neutral plane where the lines change their direction relatively to the armature surface.

§ 5. The combination of two or more systems of M.M.F.—When the lines of the resultant field are thus more or less distorted and concentrated, the length of path followed by any one line round its closed circuit and the reluctivity due to the flux-density at each portion of the path will exactly absorb the M.M.F. of the ampere-turns with which it is linked. But in the present case the varying lengths of path of the displaced lines and the actual number of armature ampere-turns to be traversed by any group of lines

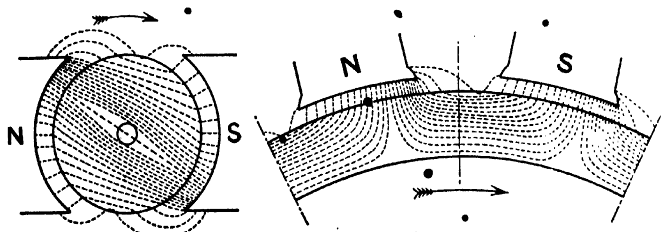


FIG. 320.—Distorted resultant field.

would have to be approximated by trial and error, so as to judge how closely the two checking causes that have been mentioned counterbalance any increased number of forward armature turns through which they pass. An immediate and direct determination of the resultant distribution would therefore be both tedious and uncertain. The process can, however, be simplified and shortened by considering each of the M.M.F. systems separately.

With two or more systems of M.M.F.'s on a common magnetic structure, and considering any closed path which follows exactly the actual path of the resultant lines, if only one system of M.M.F.'s is taken into account and differences of magnetic potential arising from the second or other systems of M.M.F. are left out of consideration, a true component flux or density which can be added as a scalar quantity to other such components is found, provided that the final reluctances of any iron parts are used in the calculation. This does not, however, meet the needs of the practical problem so long as the final path of the resultant flux is unknown. But next, if with a complete knowledge of the final iron reluctances in various directions, the entire field due to each system of M.M.F.'s be separately and independently determined, the composition of these fields vectorially with regard to their directions, sense and strength

will yield the true resultant field. For this actual field from point to point is simply the result of the vector potential due to the different systems of M.M.F. It might then be thought that, with the knowledge presupposed, the resultant flux might be determined more directly without the need for the use of imaginary component fluxes; these latter may diverge widely from the real path, and when two such components are in direct opposition, their superposition makes each appear somewhat unreal.[1] The justification for their retention lies, however, in their use as an intermediary means for readily discovering *in what proportions* to allot the expenditure of the M.M.F. of one system along various portions of the closed path followed by its component flux. Even in somewhat complicated cases, a correct apportionment of the component M.M.F. along the path can be made by using the analogy of an equivalent system of electric circuits.

But it must always be borne in mind that when the path of the component flux diverges from the actual resultant path, the reluctivity of any iron may differ widely from what would be its reluctivity to the component flux, if acting alone. Yet, lastly, in such cases when in parts the path of the component crosses the real path, it usually suffices to consider only those lengths over which they closely coincide; to the transversal lengths which complete the circuit may then be assigned zero reluctance or zero resistance in the equivalent electric system. The ground for the success of such an approximate treatment lies in the fact that in most practical cases the reluctance of the iron for the secondary component nearly at right angles to the true path usually has but a small effect in relation to the problem as a whole.

Adopting, therefore, the process of determining in the first instance component fluxes as due to the field-magnet and to the armature and afterwards superposing them, the cases of the non-commutating-pole machine and of the commutating-pole machine, although largely similar, are sufficiently distinct to warrant separate investigation. Although the latter is now the more usual type, a true understanding of its action magnetically is best gained by a study first of the non-commutating-pole machine.[2]

The Non-Commutating-Pole Machine

§ 6. The resultant field without angle of lead in the non-commutating-pole machine.—(*a*) *With circuit of constant reluctance.* First, let the iron parts of the circuit be assumed of constant permeability. A comparison of the cross circuits of Fig. 317 with the

[1] As in the case of the main and armature components through a commutating pole.

[2] For the plotting of the actual flux-distribution in both the non-commutating-pole and commutating-pole machine, see especially R. Richter, *Archiv für Elektrotechnik*, Vol. 11, p. 85.

main symmetrical paths of a 2-pole machine or of a multipolar as indicated (*e.g.* in Fig. 226) shows that the armature component flux passes transversely across the width of the pole and through the armature core, in each case more or less at right angles to the general direction of the symmetrical component field from the main excitation. The armature component is therefore termed the *cross flux*, and the ampere-turns which produce it are known as the *cross AT* of the armature. But in the air-gaps and teeth, as already stated, the two components have the same direction, although not necessarily the same sense. On the pole and core portions of the cross magnetic circuit there will be expended some small proportion of the armature M.M.F. embraced within any angle $2\theta_e$, where θ_e is the angle on either side of an axis at right angles to the diameter of commutation, *i.e.* reckoned from the centre of a current-sheet. But the proportion so expended will for the reason stated in the preceding section be very small, the transversal reluctance of the iron bearing only a small ratio to the total reluctance. Deducting then this small proportion of the armature M.M.F., the remainder is expended on the air-gaps and teeth, and the latter being for the present assumed as of constant reluctance, half of the remaining M.M.F. will be expended equally on each air-gap and on the teeth at similar points on either side of the centre of the pole. For any two points symmetrically situated in the trailing and leading halves of a pole-pitch, the armature cross component in the air-gap and teeth will thus be the same, and since in the trailing half it is in the same direction as the main component but in the opposite direction in the leading half, the resultant flux density in the one half is as much strengthened above the normal, as it is weakened in the other half. The resultant field is then merely twisted round in the direction of rotation, becoming much denser under the trailing pole-tip, and much weakened under the leading pole-tip, and gradually reverting to the initial value of the density at the centre of the pole. Thus the two systems of M.M.F. being in quadrature, and the iron being assumed of constant permeability, the total value of the flux is neither increased nor decreased but remains unaffected. But, as indicated in Fig. 319, the resultant flux in varying degree now threads through a certain proportion of the armature ampere-turns, and thereby the energy stored in the magnetic field is increased. *E.g.* taking any line in Fig. 319, the ampere-turns of the armature with which it is linked are proportional to the sum of the distances from the pole-centre at which it enters and at which it emerges from the armature surface.

But in the given case there remains the question whether owing to the distortion of the field there is any reduction of the E.M.F. of the armature even though the total flux may remain the same. The

brushes being assumed to be on the interpolar line of symmetry, all the active conductors within the angle by which the neutral line has been shifted ahead of the line of symmetry are positively harmful, so that on this account there would be a loss of E.M.F. But on the other hand, this back E.M.F. is balanced by an equal increase in the forward E.M.F. of the active conductors within the same angle behind the symmetrical line. There is in this case, therefore, no net loss of E.M.F. in spite of the displacement of the resulting field—a result which may also easily be arrived at by the following consideration. Since the E.M.F. of a drum armature as a whole is proportional to the algebraic sum of the resultant flux embraced within a loop situated under the brushes, if this resultant flux is reproduced by the superposition of two separate flux-distribution curves it is evident that the E.M.F. of the armature may equally well be calculated by considering the E.M.F. given separately by the two component flux-curves, each being summed up algebraically within the embrace of the short-circuited loop. The cross flux traversing the armature core in a direction parallel to the diameter of commutation, no portion of it is embraced by the loop in the position of short-circuit. It has therefore no net effect, and the initial symmetrical main field remaining unaltered, there is no reduction of the E.M.F. in spite of the fact that the diameter of commutation does not correspond with the position of the resultant neutral line.

If the reluctance of the transversal portions of the cross circuit be neglected as of small account, so that half of the armature M.M.F. may be credited to overcome the reluctance of air-gap and teeth on each side of the centre of the pole, the resultant density at any point distant θ_{ex} electrical degrees or x cm. from the centre of the current-sheet and under the pole is—

$$\frac{1{\cdot}257\left(AT_g + AT_t \pm \frac{JZ}{2p} \times \frac{\theta_{ex}}{180^\circ}\right)}{Kl_g + \mathcal{R}_t}$$

$$= \frac{1{\cdot}257\,(AT_g + AT_t \pm ac\,.\,x)}{Kl_g + \mathcal{R}_t} \quad . \quad . \quad . \quad (140)$$

(b) *With varying reluctance of iron.* In the presence of iron forming part of both the local cross and main magnetic circuits, it is possible in a smooth-core armature that under uniform distribution in the air-gap of about B = 6000 the iron of the pole-face might be working at its point of maximum permeability, so that a change in the resultant flux-density, whether an increase or a decrease, would equally cause an increased reluctance. Further, if the rise of reluctivity was equal for a similar variation above or below the uniform flux-density, both the main and the armature cross

flux would be slightly reduced, but the distribution of the latter would still remain symmetrical about the centre of the pole-face as in the previous case. Such a possibility would, however, rarely occur, and in general the rapid rise in the reluctance of the saturated iron in the trailing portion of the pole and under it, where the main and armature components agree in direction, cannot be balanced by an equal decrease in the reluctance of the less saturated iron under the leading portion of the pole. This is especially the case with the toothed armature in which the iron teeth become highly saturated near the trailing pole-tip.

The two portions of the armature cross flux in the air-gap under the poles then become unequal, yet the excess of the leading portion of the cross flux must in some way be enabled to complete its circuital path correctly. It is not the case that the actual ampere-conductors of the armature can be coupled up across the armature into two sets of ampere-turns of unequal magnitude, the greater to act on the path of higher reluctance; for the effect under one pole is repeated in the same order and is not spacially reversed under the next pole. The solution is, however, found in the redistribution of the resultant magnetic potentials; in relation to the armature ampere-turns as acting alone, the poles no longer remain at zero potential, but the potential of the N. pole of the main field rises as a whole above zero to some positive value $+ a'$, and the potential of the S. pole sinks as a whole to a similar negative value $- a'$ (Fig. 321). That is, in the resultant field $+ AT_p$ and $- AT_p$ of the polar faces exceed AT_p' and $- AT_p'$ (which would arise from the field excitation alone) by the numerical amount a'.

By this means two results are secured. The M.M.F. of the armature turns with which any line of the cross flux is linked is no longer expended in equal proportion over the two gaps and teeth, but over the leading gap at any distance y reckoned from the centre of the current-sheet (Fig. 321), there acts the difference of potential $+ ac.\ y - a'$, and over the trailing portion at any distance x there acts $a' - (- ac.\ x) = ac.x + a'$. No cross flux passes through the gap under one pole when the potential of the core facing it has the value $ac.d = a'$, and under the other pole when the potential of the core facing it has the value $- ac.d = - a'$. This implies that the point of zero armature flux moves *backwards* against the direction of rotation, and the distance d by which the zero point is shifted back, so that it falls behind the centre of the current-sheet or centre of the pole in our present case, is fixed by the relation $ac.\ d = a'$.

The inequality of the fluxes is thereby lessened owing to the greater proportion of the armature M.M.F. expended on the trailing side. But it is not necessary that the inequality should be more than reduced; for at the same time the difference of potential

$+a' - (-a') = 2a'$ acts between the poles and passes the excess of the flux of the leading side *via* the two paths of the yoke (in the reverse direction to the main flux) from one pole to the other, and the circuit for the excess flux is thereby completed. The above two effects thus proceed together, and owing to the comparatively low reluctance of the yoke a very small value of a' suffices to equalize a considerable divergence between the amounts of the cross fluxes.

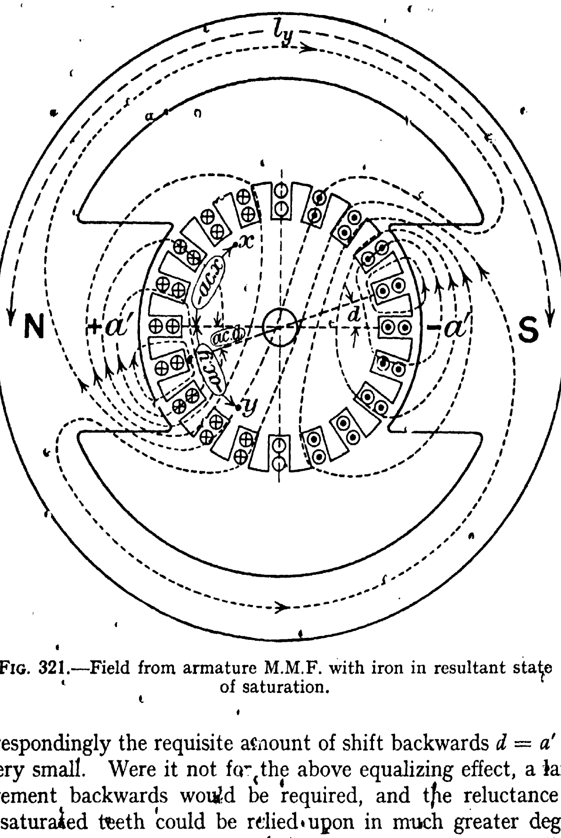


FIG. 321.—Field from armature M.M.F. with iron in resultant state of saturation.

Correspondingly the requisite amount of shift backwards $d = a' /ac$ is very small. Were it not for the above equalizing effect, a large movement backwards would be required, and the reluctance of the saturated teeth could be relied upon in much greater degree to limit displacement of the resultant field. Further, it will be noticed that it is due to the excess flux acting as a back flux opposed to the main field that the tooth saturation does actually cause a diminution in the total resultant flux below the value that would otherwise be given by the field excitation with teeth of constant reluctance.

If l_y = length of path in the yoke from pole to pole and a_y is

the double cross-section of the yoke (Fig. 321), the reluctance of the two paths in parallel is l_y/a_y, or if it is known that about 1500 AT will be expended over the yoke in a complete magnetic circuit. when, say, the useful resultant flux is 300,000 per cm. of armature core length, the resultant reluctance for the main and armature fluxes after eliminating the area of yoke required to carry the leakage flux is—

$$\mathcal{R}_y = \frac{1{\cdot}257 \times 1500}{300{,}000} = 0{\cdot}0063 \text{ per cm. of armature core}$$

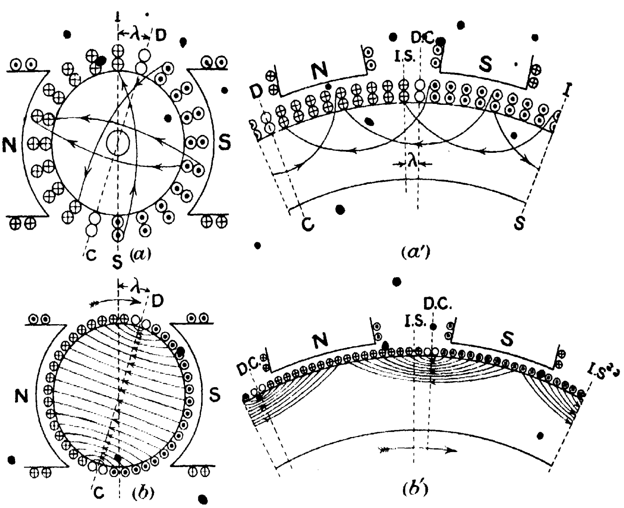


FIG. 322.—Armature ampere-turns with angle of lead.

The back flux which the yoke will carry to equalize the cross fluxes in the leading and trailing portions of the pole is then

$$\frac{1{\cdot}257 \times 2\,a'}{0{\cdot}0063} = 400\,a'$$

If $ac = 260$ per cm., since $a' = ac \,.\, d = 260 \times d$, this becomes $104{,}000 \times d$ per cm. of core-length, which shows how great is the effect of even a small amount of shift d by way of balancing the fluxes.

Thus with the diameter of commutation retained on the interpolar line of symmetry, the armature cross flux can affect the E.M.F., but only by reason of its altering the magnetic state of the iron and thereby influencing the main component field.

§ 7. The resultant field with a forward angle of lead.—(*a*) *With iron of constant reluctivity.* When the armature is carrying current

without any angle of forward lead being given to the brushes, the previous investigation will have shown that the neutral line is advanced ahead of the interpolar line of symmetry through some distance c'' and is by the same amount ahead of the diameter of commutation, so that the coils short-circuited by the brushes in the commutation zone are moving in a more or less strong field of the incorrect sign, *i.e.* in a field which so far from assisting to reverse the current is actively maintaining it in its original direction before commutation.

The effect of advancing the diameter of commutation forwards beyond the line of symmetry has therefore now to be considered. Let an angle of lead of λ mechanical degrees or λ_e electrical degrees $(= p\lambda)$ be given to the brushes, the corresponding distance through which the diameter of commutation is moved ahead of the interpolar line of symmetry being c_λ cm. measured on the surface of the armature. It will be seen from Fig. 322 that the armature ampere-turn system then becomes unsymmetrically placed with respect to the field poles, and that the two systems of M.M.F.'s are no longer strictly in quadrature. If the directions of the currents in the armature conductors within the belt of 2λ degrees, *i.e.* λ degrees on either side of the interpolar line of symmetry, be compared with the direction of the current in the field-magnet wires which are on the same side of the magnetic circuit, they will be found to be opposed.

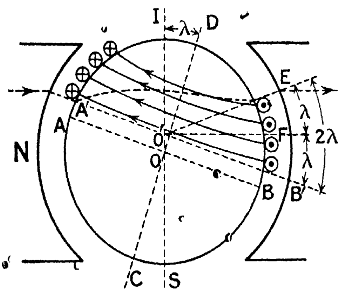


FIG. 323.

Now *if* the resultant field retained a truly symmetrical distribution about IS, as shown by a dotted line in Fig. 323, *i.e.* if each line entered and left the armature at the same distance on either side of the interpolar line of symmetry IS, an examination of Fig. 323 will show that all lines outside the interpolar zone of $2\lambda_e$ degrees will have threaded through a number of armature ampere-turns opposed to their direction, and proportional to $2\lambda_e$, for the angle $IO'A' = IOA = 90° - \lambda = IO'E$, so that $EO'F = \lambda$, and $EO'B' = 2\lambda$. Numerically, therefore, these *back ampere-turns* are $\frac{JZ}{2p} \times \frac{2\lambda_e}{180}$ or $ac \,.\, 2c_\lambda$. It would then be justifiable to imagine the armature conductors within $2\lambda_e$ to be coupled up directly to form a belt of "back ampere-turns" on each magnetic circuit, leaving

the remaining armature conductors to be coupled up into a belt of *cross ampere-turns* in strict quadrature with the field ampere-turns. The entire number of armature conductors is thus divided into two sets of ampere-turns as shown in Fig. 324, of which the former or the " back ampere-turns " embrace practically the entire magnetic circuit and being in direct opposition to the exciting ampere-turns, partially demagnetize the field, while the cross-magnetizing ampere-turns have the same effect as has been previously described in § 4, twisting the resultant field round in the direction of rotation, and weakening it at the leading pole-corners but strengthening it at the trailing pole-corners. If $\theta_e' = 90° - \lambda_e$, the maximum value

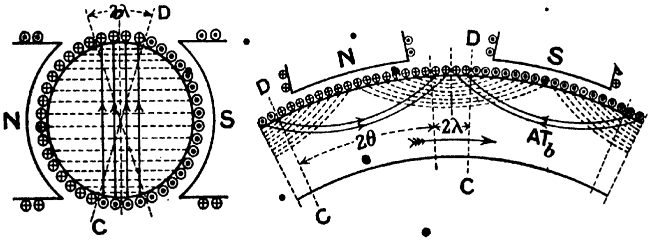


FIG. 324.—Back and cross ampere-turns of armature with angle of lead.

of the cross ampere-turns acting between opposite corners of the belt on a cross circuit would be—

$$\frac{JZ}{2p} \times \frac{2\theta_e'}{180°} \quad \text{or on the half circuit,} \quad \frac{JZ}{2p} \times \frac{\theta_e'}{180°} \qquad (141)$$

while the back ampere-turns are $\frac{JZ}{2p} \times \frac{2\lambda_e}{180°}$ or in relation to a half magnetic circuit are—

$$AT_b = \frac{JZ}{2p} \times \frac{\lambda_e}{180°} = ac \, . \, c_\lambda \qquad (142)$$

The relative proportions of the two sets will depend on the angle of lead, and together in relation to a magnetic circuit they make up the ampere-turns of a current-sheet $JZ/2p$.

Neglecting then any weak fringe entering or leaving the armature within the double angle $2\lambda_e$, the total flux if it remained symmetrical would be that due to $AT_p - AT_b$ acting on the single air-gap, teeth and core, and owing to the presence of the back ampere-turns its amount for the same field excitation would be less than if there were no lead given to the brushes.

But the cross effect of the cross ampere-turns has not been taken into account, and although the above division of the armature

ampere-turns is commonly adopted as a convenient basis for calculation, it does not truly represent the facts. Owing to the distorting effect of the cross ampere-turns the distance from the interpolar line of symmetry at which any resultant line enters and the distance at which it leaves the armature are rendered different. It will then be followed from Fig. 323 that in proportion to the degree to which the resultant lines are twisted round into a general direction at right angles to the diameter of commutation in the 2-pole machine or so that the distance from IS at which any line passes through the armature surface is less on the trailing than on the leading side, the number of the true back ampere-turns of Fig. 323 through which they pass is progressively reduced. Finally, if all the resultant flux was twisted round so that it traversed the core at right angles to the diameter of commutation or so that the lines entered and left the armature surface at the same distance from the diameter of commutation, there would again be no back ampere-turns. A closer analysis of the component fields on the true basis of the ampere-turns of Fig. 322 can therefore profitably be made, and first for the case of iron assumed as of constant reluctivity.

Reckoning the armature ampere-turns from the centre of the current-sheet 90 electrical degrees from the diameter of commutation, *i.e.* from a point $c\lambda$ cm. ahead of the centre of a pole in the direction of rotation, Fig. 322 shows that a greater proportion of the armature M.M.F. is brought to bear on the leading portion of the pole. Further, if the zero line of the armature flux remained at the centre of the current-sheet, the area of pole-face over which the larger part of the M.M.F. would act would be increased. Conversely, the trailing portion acted on by a smaller magnetic potential would be of decreased area. We thus meet with a second and new cause which would result in the leading portion of the cross flux being considerably in excess of the trailing portion even if the teeth were of constant reluctance. The same equalizing action that has been already described in § 6 (*b*) then comes into play; the poles diverge in magnetic potential, and there is a shift backwards of the point of zero armature flux, while some back flux flows through the yoke, reducing the resultant total field.

(*b*) *With iron teeth of varying reluctance.* When the effect of varying reluctance in the teeth is combined with that of an angle of lead, both of the causes described above and in § 6 (*b*) unite to reinforce one another, and the values of a', of d and of the back flux are correspondingly increased. It thus results that for the same field excitation the total resultant flux is further reduced by the angle of lead, but still remains distorted forwards. Actually, some portion of the resultant flux may be sufficiently distorted to pass through some *forward* armature ampere-turns, yet the bulk of the flux is not so much displaced as to follow up the diameter of

commutation; consequently on the whole there remains some back effect, although much less than is given by $2\,AT_b = ac\,.\,2c_\lambda$.

It has been stated above that in the design of the non-commutating-pole machine, it is customary to calculate the ampere-turns expended over air-gap, teeth and armature core for the required normal densities as given by a symmetrical distribution of the main field, and then to add the full value of the back ampere-turns $AT_b = ac\,.\,c_\lambda$ to obtain AT_p. The assumption that all the useful flux passes through such back ampere-turns contains little error, since only a fringe of weak density is concerned in the region covered by 2λ or $2\,c_\lambda$. But the true back effect is over-estimated, and the approximate accuracy obtained by the assumption is only due to the fact that the value of a' due to both causes—angle of lead and tooth reluctance—is ignored; the back effect of $2a'$ not being taken into account supplies a correction, and the result thus obtained may actually be either a small under-estimate or over-estimate. The view of the matter above presented does not therefore possess any great superiority in practical accuracy, but gives a more strictly correct account of what actually happens.

It remains to be added that with a constant armature current, if c''' be the distance of the neutral line (where the flux changes direction relatively to the armature surface) ahead of the interpolar line of symmetry, the first effect of shifting the brushes forwards is to increase the distance c''' as compared with its value, without any angle of lead, but progressively by less than the amount c_λ. A maximum value of c''' is thus reached when the diameter of commutation has overtaken the neutral line and $c_\lambda = c'''_{max}$. A further movement forwards of the brushes, so that $c_\lambda > c'''$, causes c''' itself to move slightly backwards, and thus finally the short-circuited coils are enabled to reach a sufficiently strong reversing field for commutation purposes.

§ 8. **The complete curve of flux-density over a pole-pitch.**—(*a*) *The magnetic potential of the armature core.* One of the two surfaces between which the flux density is to be found, viz., the cylindrical surface, level with the bottom of the teeth and slots, being at a magnetic potential varying from $+AT_c$ to $-AT_c$, the first step must be to determine what value of this varying potential of the armature core will correspond to any given point x on the surface.

When the field is displaced under the action of the armature ampere-turns on load, the state of saturation of the armature core at similar distances ahead of and behind the interpolar line of symmetry no longer remains the same, so that the centre point on the armature core between the pole-tips no longer falls on the plane of zero magnetic potential. Neither is the saturation of the core symmetrical on either side of the resultant neutral line or plane of maximum flux in the armature core, so that the plane of zero potential no longer coincides with the resultant neutral line.

In some radial plane ahead of the centre of a pole in the direction of rotation the resultant flux in the armature core bifurcates, and proceeding in either direction it grows to a maximum on the resultant neutral line where flux ceases to enter the armature core and begins to emerge from it. From the plane of bifurcation under say a N pole (Fig. 325), let AT be the

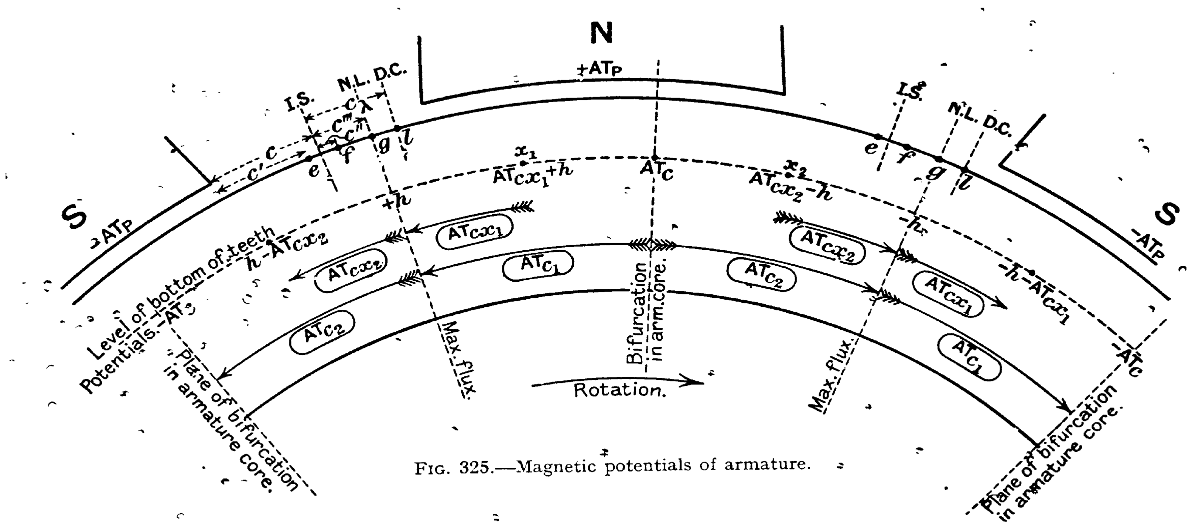


FIG. 325.—Magnetic potentials of armature.

I.S. = interpolar line of symmetry.
N.L. = neutral line or plane of maximum flux in armature core.
D.C. = diameter of commutation.

e at distance	c'	from trailing corner	marks	tangent point of main field component;
f ,,	c''	ahead of *I.S.*	,,	bifurcation point of armature component;
g ,,	c'''	,,	,,	tangent point of resultant field;
l ,,	c_λ	,,	,,	diameter of commutation.

total ampere-turns expended over the core below the teeth against the direction of the armature's rotation up to a neutral line or plane of maximum flux, *i.e.* in the leading portion relatively to the pole; and let AT_{c2} be the total ampere-turns expended over the core from the same plane of bifurcation, but following the direction of the rotation up to the next plane of maximum flux, *i.e.* in the trailing portion of the circuit. If then h stands for half the difference between these two values, *i.e.* for $\frac{AT_{c2}-AT_{c1}}{2}$, the magnetic potential in ampere-turns on the first neutral line must be $+h$ and on the second neutral line $-h$ (Fig. 325). The potential of the plane of bifurcation under the N. pole, whether it be reached from the first or the second neutral line, must be the same, viz., $AT_c = \frac{AT_{c1}+AT_{c2}}{2}$, and only thus can we have as the difference of magnetic potential between the bifurcation point under the N. pole and the resultant neutral lines on either side the required values of $AT_c - h = AT_{c1}$, and $AT_c - (-h) = AT_c + h = AT_{c2}$. Whether the value of h is itself positive or negative, *i.e.* whether AT_{c2} exceeds AT_{c1}, depends upon the circumstances of the case and especially on the degree of saturation in the core. In a non-commutating-pole machine, the resultant neutral plane of maximum flux is nearer to the leading than to the trailing pole-edge, and the flux is withdrawn more quickly as we proceed in the direction of rotation towards the leading pole-edge than in the reverse direction, although this no longer holds when the poles are reached. If the core is not highly saturated, the expenditure of ampere-turns AT_{c1} corresponding to the leading section is then less than AT_{c2}, although it may be very slightly. In a commutating-pole machine the opposite is the case; the neutral plane is again nearer to a commutating than to a main pole, but so little flux is withdrawn by the former that the longer path to the next main pole makes AT_{c1} exceed AT_{c2} appreciably, and the signs of h in Fig. 325 are then exactly reversed.

Hence, in a non-commutating-pole machine, if AT_{cx1} and AT_{cx2} are respectively the ampere-turns expended over the armature core from the neutral line up to any point x on the leading and trailing sides respectively, the resultant potential of the core at the point x will be $\pm (AT_{cx1} + h)$ on the leading side and $\pm (AT_{cx2} - h)$ on the other side (Fig. 325; *cp.* also the curve at the foot of Fig. 327 and column 8, Table XI).

(*b*) *The main field component.* In order to determine the radial component of the flux in air-gap and teeth, let $+AT_p'$ and $-AT_p'$ be the potentials of the pole-faces corresponding to the main field component, these values being slightly less than the resultant values $+AT_p$ and $-AT_p$ for the reason given in § 6 (*b*). They are to be considered in relation to the cylindrical surface, level with the bottom of the teeth, at its resultant potential varying unsymmetrically on either side of the resultant neutral line where it is $+h$. The point of balance between the tendency for the main component flux to pass into the core from, say, a trailing N. pole-tip and the tendency to pass out of the core into the leading S. pole-tip thus no longer falls on the interpolar line of symmetry. The distance c' of this point from the trailing pole corner, measured on the armature surface, must fall short of the resultant neutral line or be on its trailing side, since the armature ampere-turns displacing the resultant field are at present left out of consideration. It will depend on the potential $AT_{cc'} - h$ of the armature core surface below the teeth and underneath the point c', for this must be deducted from AT_p' on the trailing side and added to AT_p' on the leading side, as reducing or increasing the difference of magnetic potential acting on the air-gap and teeth. If $\Re_{tc}'$ be the equivalent reluctance of the teeth per sq. cm. of path in the air at point c', the position of that point is given by the relation

$$\frac{1{\cdot}257\, m\{AT_p' - (AT_{cc'} - h)\}}{\xi c' + Kl_g + \Re_{tc'}} = \frac{1{\cdot}257\, m'\{AT_p' + (AT_{cc'} - h)\}}{\xi(2c - c') + Kl_g + \Re_{tc'}} \quad . \quad (143)$$

whence (if the very small difference between m and m' is neglected)

$$c' = c - (c + Kl_g/\xi + \Re_{tc'}/\xi)\frac{AT_{cc'} - h}{AT_p'} \quad . \quad . \quad (144)$$

The point c', being the *tangent point* at which the component field flux just touches the cylindrical surface below the teeth on its way from a N. to a S. pole but does not enter it, is thus found to have moved against the angle of lead, *i.e.* to fall short of the midway point c. The numerical value of the flux from either the N. or S. pole if each were acting alone is exactly the same as if the saturation of the core was symmetrical about the centre line and $\Re_{tc'}$ was the reluctance per sq. cm. of path in the teeth, viz.—

$$\frac{1{\cdot}257\,AT_{\mathrm{P}}'}{\xi c + Kl_g + \Re_{tc'}}$$

but its position is shifted away from the centre.

In the interpolar gap not far from its centre, $\Re_{tc'}$ is entirely negligible as compared with the reluctance of the air-path, and the *at* per cm. length in the armature core is nearly constant, since but little flux is there entering or leaving the core. From a knowledge of the maximum flux density in the core as due to the flux which each pole is to give, an estimate can be made of the maximum *at* per cm. length in the core; let this, when reduced in the proportion $\dfrac{\text{mean diameter below teeth}}{\text{armature diameter}}$, be denoted by at'. The neutral line in the armature core being set at about $\frac{3}{8}c_\lambda$, where c_λ is the distance to which the diameter of commutation is assumed to be advanced ahead of the interpolar centre, $AT_{cc'}$ can be approximately assumed as $at'\,(c - c' + 0{\cdot}375\,c_\lambda)$. Substituting this value in the above equation, the value for c' can be determined from the data of design, as

$$c' = c - (c + Kl_g/\xi)\,\frac{0{\cdot}375\,c_\lambda\,.\,at' - h}{AT_{\mathrm{P}}' - at'\,(c + Kl_g/\xi)}\,.$$

In either expression the last fraction is so small that the divergence of c' from c only possesses a theoretic interest, and in practice hardly needs to be closely determined.

The negative flux to the leading S. pole must now be made to grow at the rate $\dfrac{x'}{c'}$ for any distance x' from the trailing N. pole-tip up to c', and the positive flux from the trailing N. pole ahead of c' must be made to grow at the rate $\dfrac{y'}{2c - c'}$ from $y' = 0$ up to $y' = 2c - c'$, where y' is the distance from the *leading* pole-edge. But when the point c' is left, we are justified, as in Chapter XVIII, § 3, in taking into account the actual potential of the core at the level of the bottom of the teeth, provided that the actual reluctance of the teeth, $\Re_{tx}$ per sq. cm. of air-gap area, is inserted in the denominators. Thus when we pass from a N. to a S. pole in the direction of rotation (Fig. 325), in place of $AT_{cc'} - h$ in (144), the potential of the core is $AT_{cx2} - h$ behind the resultant neutral line, and $-h - AT_{cx1}$ ahead of it.

If x' be the distance of any point from the *trailing* corner of a N. pole, the component density at the armature surface due to the main field considered separately will then be from $x' = 0$ to $x' = c'$

$$1{\cdot}257\left[\frac{m\{AT_{\mathrm{P}}' - (AT_{cx2} - h)\}}{\xi x' + Kl_g + \Re_{tx}} - \frac{m'\{AT_{\mathrm{P}}' + (AT_{cx2} - h)\}}{\xi(2c - c') + Kl_g + \Re_{tx}} \times \frac{x'}{c'}\right]$$

If y' = the distance of any point from the *leading* corner of the N. pole, the main component is from $y' = 0$ to $y' = c - c'''$

$$1{\cdot}257\left[\frac{m\{AT_{\mathrm{P}}' - (AT_{cx1} + h)}{\xi y' + Kl_g + \Re_{tx}} - \frac{m'\{AT_{\mathrm{P}}' + (AT_{cx1} + h)\}}{\xi c' + Kl_g + \Re_{tx}} \times \frac{y'}{2c - c'}\right]$$

and from $y' = c - c'''$ to $y' = 2c - c'$

$$1{\cdot}257\left[\frac{m\{AT_{\mathrm{P}}' + (AT_{cx2} - h)\}}{\xi y' + Kl_g + \Re_{tx}} - \frac{m'\{AT_{\mathrm{P}}' - (AT_{cx2} - h)\}}{\xi c' + Kl_g + \Re_{tx}} \times \frac{y'}{2c - c'}\right]$$

while under the pole it will be on the leading side

$$1{\cdot}257\,\frac{AT_p' - (AT_{cx1} + h)}{Kl_g + \Re_{tx}} \text{ or } 1{\cdot}257\,\frac{AT_p' - (AT_{cx2} - h)}{Kl_g + \Re_{tx}} \text{ on the trailing side,}$$

the change occurring when $AT_{cx1} + h = AT_{cx2} - h$. (*Cp.* columns 15–17, Table XI.)

(*c*) *The armature component field.* So far as the armature component flux is concerned, the core below the teeth may for simplicity be treated as throughout at zero potential, the further progress of the resultant flux after reaching the cylindrical surface below the teeth being cared for in the item AT_{cx}. The latter as calculated should, strictly speaking, be related to AT_p, and not to AT_p', from which it has been deducted; but the calculation of separate values of AT_{cx} for the main and armature components respectively would be unnecessarily tedious. Neglecting, then, any expenditure of armature ampere-turns over the core, when armature component flux enters the teeth at any point distant x cm. on the trailing side from the centre of the current sheet and passes through some of the ampere-turns, its potential is raised from a negative value by the M.M.F. $ac \, . \, x$ so that it becomes zero at the level of the bottom of the teeth, and as it enters the teeth and passes through some of the ampere-turns to leave the surface at point y on the leading side, its potential is raised from zero by the M.M.F. $ac \, . \, y$ to a positive value. The surface of one half of the two-pole armature is thus in relation to the armature flux at potential $-$ ($ac \, . \, x$ $-$ loss over teeth) and of the other half at potential $+$ ($ac \, . \, y$ $-$ loss over teeth).

Now, if the *bifurcation point of the armature flux* indifferently into either a N. or a S. pole be supposed actually to follow up and coincide with the diameter of commutation, the difference of potential acting on the trailing and leading sides respectively would be $ac \, . \, (Y/2) + a'$ and $ac \, . \, (Y/2) - a'$, which may also be expressed as $ac \, . \, (Y/2 \pm d)$. From the relation

$$\frac{m \, . \, ac(Y/2 + d)}{\xi(c + c_\lambda) + Kl_g} = \frac{m' \, . \, ac(Y/2 - d)}{\xi(c - c_\lambda) + Kl_g}$$

we have

$$d = Y/2 \, \frac{(m' + m)c_\lambda + (m' - m)\,(c + Kl_g/\xi)}{(m' + m)\,(c + Kl_g/\xi) + (m' - m)c_\lambda}.$$

This would give a very large value for d or $a' = ac \, . \, d$, which acting on the yoke would far more than balance any difference in the cross fluxes. The bifurcation point c'' therefore falls considerably behind c_λ, although in advance of c. The magnetic system of the armature flux tends in fact to maintain the configuration which it would have if the armature M.M.F. were disposed symmetrically in relation to the poles. In spite of the fact that the rise and fall of the potential of the poles above and below zero, which increases the proportion of the M.M.F. expended over the trailing side, also tends to increase the trailing fringe, this is more than offset by the alteration in the two denominators as c'' is increased.

Since it has been shown that the bifurcation point c'' of the armature flux must fall behind c_λ, but must be in advance of the interpolar line of symmetry, the relation which determines it is

$$\frac{m \, . \, ac\{Y/2 - (c_\lambda - c'') + d\}}{\xi(c + c'') + Kl_g} = \frac{m' \, . \, ac\{Y/2 - (c_\lambda - c'') - d\}}{\xi(c - c'') + Kl_g} \qquad (145)$$

whence

$$(c'')^2 + c''\{Y/2 - c_\lambda + \frac{m' - m}{m' + m}(c + Kl_g/\xi - d)\} =$$
$$(c + Kl_g/\xi)\ \{d - \frac{m' - m}{m' + m}\,(Y/2 - c_\lambda)\} \qquad (146)$$

c'' always has some small value, but in practice it is so small that, as in the case of c', it only possesses a theoretic interest. Under the action of the armature ampere-turns, as determining the final state of saturation in the

teeth, while the tangent point of the main component field falls back behind the interpolar line of symmetry very slightly, the bifurcation point of the armature component field only advances very slightly forwards.

A preliminary and approximate calculation of the value to be assigned to d may be made, by first assuming that there is no shift and taking $ac(\beta Y/2 + c_\lambda)$ as acting on the leading corner and $ac(\beta Y/2 - c_\lambda)$ on the trailing corner. The density from the former, $-B_{cg}'$, is practically determined solely by the length of the air-gap, the tooth reluctance for the resultant low density thereat being negligible, and is therefore closely $= 1{\cdot}257 \dfrac{ac(\beta Y/2 + c_\lambda)}{Kl_g}$. The resultant density at the trailing corner will on the given hypothesis be

$$B_g'' = 1{\cdot}257 \frac{AT_p' + ac(\beta Y/2 - c_\lambda) - AT_{cx2} + h)}{Kl_g + \Re_{tx}}.$$

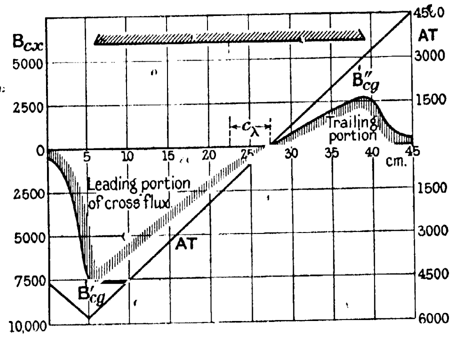


FIG. 326.—Preliminary estimate of cross fluxes without any shift d.

By trial corresponding values of $\Re_{tx}$ and B_g'' are found, and the armature component is then $B_{cg}'' = B_g'' \dfrac{ac(\beta Y/2 - c_\lambda)}{AT_p' + ac(\beta Y/2 - c_\lambda) - AT_{cx2} + h}$. Plotting then $-B_{cg}'$ and B_{cg}'' for the two corners and joining them by a line passing through a point c_λ ahead of the centre of a pole, after the manner shown in Fig. 326, the important part of the curve is determined, and the remainder can be approximately completed. The total of each of the two portions of the cross flux can thence be estimated, and their difference found. The value to be taken for d will then be slightly less than the value which will pass the whole of the difference as back flux through the yoke.

Reckoning, then, x and y from a point at right angles to the diameter of commutation, i.e. distant c_λ cm. ahead of the centre of the N. pole, and bearing in mind that the N. pole-face is itself at potential $+ a' = d \,.\, ac$ in relation to the armature flux, the component density at the armature surface due to the armature ampere-turns is

$$-1{\cdot}257 \frac{ac(y - d)}{Kl_g + \Re_{tx}} \text{ on the leading side from } y = 0 \text{ to } y = \beta Y/2 + c_\lambda,$$

$$\text{and } 1{\cdot}257 \frac{ac(x + d)}{Kl_g + \Re_{tx}} \text{ on the trailing side from } x = 0 \text{ to } x = \beta Y/2 - c_\lambda.$$

Continuing on the trailing side, if x' be the distance of any point from the trailing pole corner, the flux-density is from $x' = 0$ to $x' = c + \delta''$

$$1{\cdot}257 \frac{m \,.\, ac(\beta Y/2 - c_\lambda + d + x')}{\xi x' + Kl_g + \Re_{tx}}.$$

On the leading side of the N. pole, if y' be the distance of any point from the leading pole corner, the flux-density from $y'=0$ to $y'=c-c_\lambda$, *i.e.* up to the diameter of commutation, is

$$-1{\cdot}257\,\frac{m\,.\,ac(\beta Y/2+c_\lambda-d+y')}{\xi y'+Kl_g+\Re_{tx}}$$

and from $y'=c-c_\lambda$ to $y'=c-c''$, *i.e.* over the region between the diameter of commutation and the point of bifurcation of the armature flux into either pole indifferently,

$$-1{\cdot}257\,\frac{m\,.\,ac(Y/2+c-c_\lambda-d-y')}{\xi y'+Kl_g+\Re_{tx}}.$$

(*Cp.* column 18, Table XI.)

(*d*) *The combination of the main and armature component fields over a complete pole-pitch.* In order to map out the spacial distribution of the resultant flux over a complete pole-pitch (the minor ripples in the actual curve at any moment due to the slots being supposed to be smoothed out, but due account being taken of the tooth reluctance), an approximate preliminary estimate must in the first place be made of the magnetic potential of the core below the teeth.

Assuming a certain curve of the resultant flux density, tilted towards the trailing pole corner by the action of the armature ampere-turns, the gradual concentration or withdrawal of flux in the armature core leads to certain densities for which the *at* required per cm. length of path at a number of points can be found. Plotting these values and integrating the curve on either side of the neutral plane of maximum flux (*cp.* Fig. 302), the resultant expenditure of ampere-turns in the core in the leading and trailing sides respectively is AT_{c1} and AT_{c2}. Half of their difference is h, and adding this to the one and subtracting it from the other, a curve of the magnetic potential of the core can be plotted as at the foot of Fig. 327 or as tabulated in column 8 of Table XI.

A preliminary estimate of the back movement d of the zero of the armature component, so that it precedes the centre of the pole by the amount $c_\lambda-d$, must further be made, and both assumptions must finally be checked with the result until agreement is obtained. Column 10 is then given as $a'=ac\,.\,d$, and column 11 can be calculated, leaving the differences of column 12. The sum or difference of the two columns 9 and 12 is written down as column 13, and its values multiplied by $\dfrac{1{\cdot}257}{Kl_g+\Re_{tx}}$ under a pole or by $\dfrac{1{\cdot}257\,m}{\xi x'+Kl_g+\Re_{tx}}$ within the fringe will give the required resultant flux density (column 19). Within the fringe there must, however, first be deducted the back effect from the pole of opposite sign (column 15).

By reference to a curve such as Fig. 303 for the reluctance of the teeth per sq. cm. of air-gap for varying densities in the air-gap, corresponding values for $\Re_{tx}$ and resultant B_g are quickly found. The resultant flux must in any case be determined first, but thereafter in the case of the two component fluxes, the smaller should be determined first to secure accuracy.

Grouping the analytical expressions previously given and starting from c' which is behind the interpolar line of symmetry, the flux-density is from $y'=2c-c'$ to $y'=c-c''$.

$$1{\cdot}257\left[\frac{m\{AT_p'+(AT_{cx2}-h)\}}{\xi y'+Kl_g+\Re_{tx}}-\frac{m\,ac(\beta Y/2-c_\lambda+d+2c-y')}{\xi(2c-y')+Kl_g+\Re_{tx}}\right.$$
$$\left.-\frac{m'\{AT_p'-(AT_{cx2}-h)}{\xi c'+Kl_g+\Re_{tx}}\times\frac{y'}{2c-c'}\right]$$

from $y'=c-c''$ to $y'=c-c'''$

$$1{\cdot}257\left[\frac{m\{AT_p'+(AT_{cx2}-h)-ac(Y/2+c-c_\lambda-d-y')\}}{\xi y'+Kl_g+\Re_{tx}}\right.$$
$$\left.-\frac{m'\{AT_p'-(AT_{cx2}-h)\}}{\xi c'+Kl_g+\Re_{tx}}\times\frac{y'}{2c-c'}\right]$$

from $y' = c - c'''$ to $y' = c - c_\lambda$

$$1{\cdot}257\left[\frac{m\{AT_p' - (AT_{cx1} + h) - ac(Y/2 + c - c_\lambda - d - y')\}}{\xi y' + Kl_g + \Re_{tx}} - \frac{m'\{AT_p' + (AT_{cx1} + h)\}}{\xi c' + Kl_g + \Re_{tx}} \times \frac{y'}{2c - c'}\right]$$

and from $y' = c - c_\lambda$ to $y' = 0$

$$1{\cdot}257\left[\frac{m\{AT_p' - (AT_{cx1} + h) - ac(\beta Y/2 + c_\lambda - d + y')}{\xi y' + Kl_g + \Re_{tx}} - \frac{m'\{AT_p' + (AT_{cx1} + h)\}}{\xi c' + Kl_g + \Re_{tx}} \times \frac{y'}{2c - c'}\right]$$

Thence under the pole

$$1{\cdot}257\,\frac{AT_p' - (AT_{cx1} + h) - ac(y - d)}{Kl_g + \Re_{tx}} \text{ and } 1{\cdot}257\,\frac{AT_p' - (AT_{cx2} - h) + ac(d + x)}{Kl_g + \Re_{tx}}$$

and from the trailing pole corner from $x' = 0$ to $x' = c'$

$$1{\cdot}257\left[\frac{m\{AT_p' - (AT_{cx2} - h) + ac(\beta Y/2 - c_\lambda + d + x')\}}{\xi x' + Kl_g + \Re_{tx}} - \frac{m'\{AT_p' + (AT_{cx2} - h)\}}{\xi(2c - c') + Kl_g + \Re_{tx}} \times \frac{x'}{c'}\right]$$

It will be seen that owing to c' and c'' not coinciding, there are in the complete interpolar zone five regions. The apparent reckoning of AT_c and h twice over throughout the interpolar gap arises from the approximate and empirical expression employed to take account of the counter effect of an adjacent pole, as if the field flux therein really contained two components not following the same path and not therefore having one and the same denominator; the method confessedly does not yield a true physical expression for the magnetic potential function throughout the interpolar gap.

In the above it has been assumed that the angle of lead has been advanced beyond the resultant neutral line in order to provide a reversing field for commutation purposes. When there is no angle of lead, the armature ampere-turns acting on the resultant neutral line are $ac\,(Y/2 - c''')$; when lead is given to the brushes and c_λ rises from zero, this becomes $ac\{Y/2 - (c''' - c_\lambda)\}$ and reaches a maximum when $c_\lambda = c'''$. But at the same time d is increasing and the counter effect from the opposite main pole decreases, so that c_λ is enabled to overtake c'''. After this point has been reached, if the angle of lead is further increased, the armature ampere-turns acting on the resultant neutral line become $ac\{Y/2 - (c_\lambda - c''')\}$; c''' becomes reduced and the divergence of c_λ and c''' progressively reduces the armature effect, so that the short-circuited coils find themselves in a reversing field of increasing strength. On the neutral line, the loss of potential from reluctance of the teeth and core is eliminated, but the half difference h is still present, so that the magnetic potential of the armature surface where the resultant flux changes direction as regards entering or leaving the surface is $-h$. Hence, provided that c_λ is greater than c''', as it should be for commuting the current, the distance c''' for any assumed value of c_λ is given by the relation

$$\frac{m[(AT_p' - h) - ac\{Y/2 - d - (c_\lambda - c''')\}]}{\xi(c - c''') + Kl_g} - \frac{m'(AT_p' + h)}{\xi c' + Kl_g} \times \frac{c - c'''}{2c - c'} = 0 \quad . \quad (147)$$

a quadratic equation which can be solved for c''' when the known quantities are inserted with an approximate value for m.

§ 9. Example of a non-commutating-pole machine.—For the 8-pole machine assumed in Chapter XVIII, § 2, let the diameter of commutation under full

load be advanced through a distance of 5 cm., *i.e.* as near to the leading pole-tip as would be safe, and ac being 260, the back ampere-turns AT_b on the half-magnetic circuit would be $AT_b = 5 \times 260 = 1300$. Thus for $B_g = 8100$ and a flux of about 280,000 per cm. length of core, the magnetic potential of a pole-face without allowance for distortion, but reckoning AT_b at its full value as ordinarily estimated, is approximately

$$\begin{aligned} AT_p &= AT_g + AT_t + AT_c + AT_b \\ &= 5720 + 440 + 375 + 1300 = 7835. \end{aligned}$$

Proceeding now in closer detail, the potential curve AT_c for the armature core must first be calculated, even if only provisionally from an assumed flux-distribution curve, after the manner described in Chapter XVIII, § 2. For simplicity, the final results (from the curve shown at the foot of Fig. 327) are here at once inserted, and the values of AT_{c1} and AT_{c2} are found to be 376·5 and 381·5, the mean being 379 and $h = 2·5$ only. $\xi = 0·9 \times \pi/2 \times 1·295 = 1·83$, and c being 6 cm., $\xi c + Kl_g = 10·98 + 0·882 = 11·862$, and $c + Kl_g/\xi = 6·482$. Using the curve to obtain the values of $AT_{cx} \pm h$ at different points, the three distances c', c'', and c''' are as follows—

(i) By equation (144) $c' = 6 - (6·482)\dfrac{125}{7835} = 6 - 0·1 = 5·9$ from trailing pole corner, and at this spot by equation (143) the two fluxes from N. and S. pole respectively are—

$$\frac{1·257(7835 - 125)}{1·83 \times 5·9 + 0·882} = \frac{1·257(7835 + 125)}{1·83 \times 6·1 + 0·882}$$

$$\frac{1·257 \times 7710}{11·68} = \frac{1·257 \times 7960}{12·045}$$

$$\frac{7710}{9·3} : \frac{7960}{9·6}$$

$$830 : 830$$

(ii) The bifurcation point of the armature flux (equation 146) is given closely by

$$(c'')^2 + c''\,(22·5 - 5) = 6·482 \times 0·5,$$

since m and m' are but little different; whence $c'' = 0·17$ cm. ahead of the interpolar line of symmetry, and at this point by equation (145)

$$\frac{1·257 \times 260\,\{22·5 - 4·83 + 0·5\}}{1·83 \times 6·17 + 0·882} = \frac{1·257 \times 260\,\{22·5 - 4·83 - 0·5\}}{1·83 \times 5·83 + 0·882}$$

$$\frac{1·257 \times 4724}{12·182} \quad \frac{1·257 \times 4464}{11·532}$$

$$487 \quad 487$$

(iii) Lastly, for the position of the resultant neutral line (equation 147)

$$\frac{1·05\,\{(7835 - 2·5) - 260\,(22·5 - 0·5 - 5 + c''')\}}{1·83 \times (6 - c''') + 0·882} - \frac{7837·5}{11·68} \times \frac{6 - c'''}{12 - 5·9} = 0$$

whence $c''' = 2·44$.

The complete results are tabulated in Table XI and plotted in Fig. 327. In the first line the denominator by which −4654 in column 12 must be divided to give −500 in column 18 is 9·3 as given in the last line of the Table. In the second line for the armature bifurcation point c'', and onwards, the same denominator from column 6 can be used as the divisor for both the armature and the main field differences of potential acting on gap and teeth. The total flux of a pole is obtained from Fig. 327 as 274,000 per cm. of armature core length, being less than the preliminary estimate which assumed the back ampere-turns due to the lead of c_λ cm. to embrace all the flux, but in which no allowance was made for increased tooth saturation under the trailing pole corner. The true $AT_p = AT_p' + a' = 7835 + 130 = 7965$ as compared with the original estimate of 7835 for the greater flux.

Table XI.

	1	2	3	4	5	6	7	8	9	10
	Distance in cm.	Fl_g	$\xi y'$	$\Re_{tx}$	Reluctance of gap and teeth per sq. cm.	$\frac{\text{Col. 5}}{1{\cdot}257}$	AT_p'	Potential of core.	Diff. acting on gap and teeth.	a'
I.S. → −	0·1	0·882	11·15	0·0003	12·032	9·6	+ 7835	− 125	7960	− 130
+	0·17	„	10·65	0·0002	11·532	9·17	„	− 120	7955	+ 130
	1	„	9·15	0·0001	10·032	8·0	„	− 70	7905	„
	2·44	„	6·51	0·0000	7·392	5·87	„	2·5	7832·5	„
	3	„	5·49	0·0001	6·372	5·07	„	40	7795	„
	4	„	3·66	0·0004	4·542	3·61	„	105	7730	„
	5	„	1·83	0·001	2·713	2·16	„	160	7675	„
Leading edge	6	„	—	0·002	0·884	0·703	„	217	7618	„
	6·89	„	—	0·003	0·885	0·704	„	250	7585	„
	8·5	„	—	0·005	0·887	0·706	„	268	7567	„
	13·5	„	—	0·008	0·89	0·707	„	348	7487	„
	18·5	„	—	0·03	0·912	0·725	„	370	7465	„
N. pole	23·5	„	—	0·09	0·972	0·773	„	377	7458	„
	27	„	—	0·152	1·034	0·824	„	378	7457	„
	28·5	„	—	0·183	1·065	0·849	„	377	7458	„
	33·5	„	—	0·278	1·16	0·923	„	375	7460	„
	38·11	„	—	0·39	1·272	1·012	„	357	7478	„
Trailing edge	39	„	$\xi x'$	0·265	1·147	0·912	„	345	7490	„
	40	„	1·83	0·008	2·72	2·16	„	330	7505	„
	41	„	3·66	0·0025	4·544	3·615	„	303	7532	„
	42	„	5·49	0·0015	6·373	5·07	„	270	7565	„
	43	„	7·32	0·0005	8·202	6·52	„	230	7605	„
	44·9	„	10·8	0·0003	11·682	9·3	„	125	7710	„

Non-commutating-pole Machine.

11	12	13	14	15	16	17	18	19
Potential of core.	Diff. acting on gap and teeth.	Sum of 9 + 12 acting on gap and teeth.	m	Deduction for effect of adjacent pole in interpolar gap.	$\frac{\text{Col. 9}}{\text{Col. 6}}$	Main component density.	Armature component density.	Resultant density, B_{gx}
4524	− 4654		1·00	$-\frac{7835-125}{9\cdot3}\times\frac{6\cdot1}{6\cdot1}=-830$	830	0	− 500	− 500
4594	− 4464		1·00	$-\frac{7835-120}{9\cdot3}\times\frac{5\cdot83}{6\cdot1}=-794$	870	76	− 487	− 411
4810	− 4680		1·02	$-\frac{7835-70}{9\cdot3}\times\frac{5}{6\cdot1}=-685$	1015	330	− 596	− 266
5202	− 5072		1·05	$-\frac{7835+2\cdot5}{9\cdot3}\times\frac{3\cdot56}{6\cdot1}=-492$	1400	908	− 908	0
5330	− 5200		1·075	$-\frac{7835+40}{9\cdot3}\times\frac{3}{6\cdot1}=-416$	1650	1234	− 1100	134
5590	− 5460		1·1	$-\frac{7835+105}{9\cdot3+0\cdot003}\times\frac{2}{6\cdot1}=-280$	2350	2070	− 1662	408
5850	− 5720		1·14	$-\frac{7835+160}{9\cdot3+0\cdot008}\times\frac{1}{6\cdot1}=-141$	4050	3909	− 3020	889
5590	− 5460	2158	0·84			9105	− 6525	2580
5359	− 5229	2356				10,760	− 7440	3320
4940	− 4810	2757				10,710	− 6810	3900
3640	− 3510	3977				10,565	− 4955	5610
2340	− 2210	5255				10,300	− 3045	7255
1040	− 910	6548				9630	− 1175	[illegible]455
130	0	7457				9060	0	9060
− 260	390	7848				8800	460	9260
− 1560	1690	9150				8080	1830	9910
− 2759	2889	10,367				7380	2860	10,240
− 2990	3120	10,610	0·84			6890	2870	9750
− 3250	3380		1·14	$-\frac{7835+330}{9\cdot6}\times\frac{1}{5\cdot9}=-144$	3965	3821	1785	5606
− 3510	3640		1·1	$-\frac{7835+303}{9\cdot6}\times\frac{2}{5\cdot9}=-287$	2290	2003	1115	3118
− 3770	3900		1·075	$-\frac{7835+270}{9\cdot6}\times\frac{3}{5\cdot9}=-430$	1602	1172	826	1998
− 4030	4160		1·04	$-\frac{7835+230}{9\cdot6}\times\frac{4}{5\cdot9}=-570$	1216	646	665	1311
− 4524	4654		1·00	$-\frac{7835+125}{9\cdot6}\times\frac{5\cdot9}{5\cdot9}=-830$	830	[illegible]	500	500

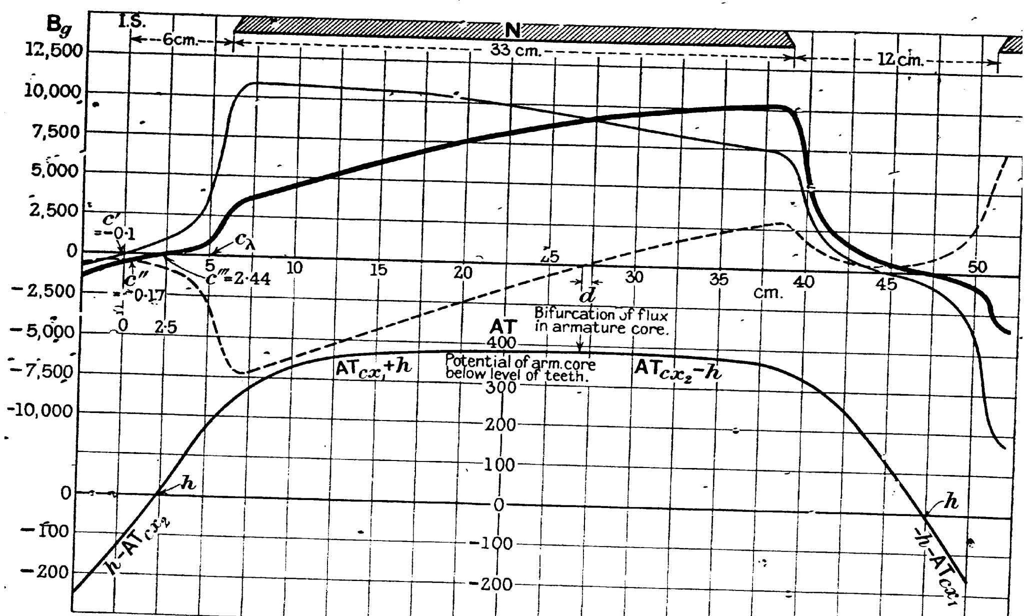


FIG. 327.—Main and armature component fluxes and resultant flux.

The centre of bifurcation of the resultant flux in the armature core having advanced 4·5 cm. ahead of the centre of the pole, as compared with $c_\lambda = 5$ cm., the lines from the bifurcation plane pass through a minimum of $ac \times 1$· back armature ampere-turns. The *difference* between the distance at which they enter and the distance at which they leave the armature, measured from the diameter of commutation, progressively increases until the back armature ampere-turns threaded through rise to a maximum of about $ac \times 6\frac{1}{2}$. The resultant flux therefore never passes through so many as $ac \times 2c_\lambda$ back ampere-turns, or $ac \times c_\lambda$ on the half-magnetic circuit, but is actually over-estimated on that assumption when tooth saturation is ignored.

The Commutating-Pole Machine

§ 10. The total resultant armature flux not greatly affected by commutating poles.—In a machine with commutating poles (as many as there are main poles) for the same reason as in § 8 (*a*), the magnetic potential of the points of bifurcation of the resultant flux in the armature core under a N. and S. main pole respectively must have the same numerical value; the expenditure over the reluctance of the teeth and main air-gap must also be the same under each pole, so that again the resultant potential of a N. and S. main pole-face above and below zero will be the same, viz., AT_p and $-AT_p$. The similarity continues up to the seat of the salient poles, but thence, owing to the unequal division of the resultant flux in the sections of the yoke (Chapter XVI, § 14), the expenditure of ampere-turns in the two directions becomes dissimilar and the point of zero potential in the yoke is not midway between two main poles, *i.e.* it is not opposite to the centre of a commutating pole, when the latter is excited.

When no current flows in the armature and the main poles are excited, the commutating poles, if not excited, face the planes of zero magnetic potential on the armature, and, the centre of the yoke opposite the centre of a commutating pole being also at zero potential from considerations of symmetry, the commutating poles remain throughout their length in the central plane at zero potential (Fig. 301). But now, as soon as they are excited and some flux passes up or down them, the ampere-turns expended over the two halves of the path in the yoke are unequal in amount, and this is especially the case owing to one path being more highly saturated and therefore of higher reluctivity than the other. In other words, the potentials at the roots of the commutating poles must rise above or fall below zero to some values $+b$ and $-b$ as indicated in Fig. 328.

If $2\mathfrak{R}_y$ is the reluctance of the full length of path in the single section of the yoke for equal division of the flux, and $\mathfrak{R}_y'$ and $\mathfrak{R}_y''$ are the reluctances of half that length in the single section on the leading and trailing side respectively, $\mathfrak{R}_y' + \mathfrak{R}_y''$ exceeds $2\mathfrak{R}_y$, and to that extent the original total flux is reduced. But, on the other hand, the additional path presented by the commutating

poles and the more rapid withdrawal of the flux from the armature core with consequent shortening of the path therein in the trailing section of reluctance $\mathcal{R}_c''$ tend to counterbalance the above

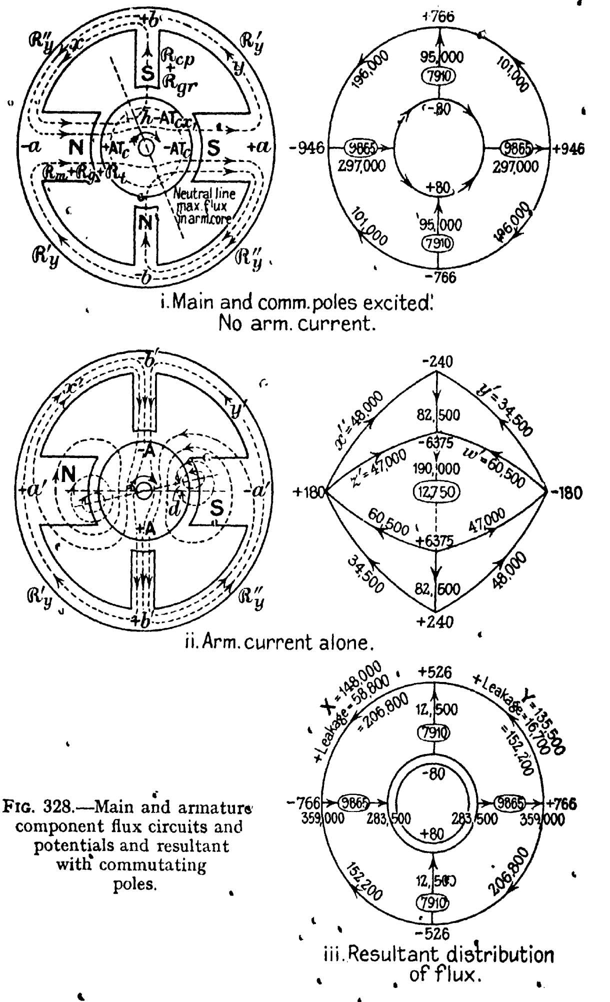


FIG. 328.—Main and armature component flux circuits and potentials and resultant with commutating poles.

mentioned change. If $2\mathcal{R}_c$ be the reluctance of the single section of the armature core in the absence of commutating poles, $\mathcal{R}_c' + \mathcal{R}_c''$ is less than $2\mathcal{R}_c$, and some ampere-turns are thereby saved. On the whole, the total useful armature flux (after deducting any

reversing flux behind the line of commutation) is little different from what it would be without the commutating poles, although differently distributed.

§ 11. (a) The main and commutating pole component field.— But the component field as due to the main and commutating-pole excitations alone and apart from the presence of the armature ampere-turns is of course greatly increased by the additional path afforded by the commutating poles, the final state of saturation of the iron being presupposed in making the comparison. The main and commutating-pole M.M.F.'s are best taken in conjunction, as giving rise to a single component field which is now to be considered.

From the fact that the flux is rapidly withdrawn as we proceed from a plane of maximum resultant flux in the armature core towards and under the trailing pole-edge, while it is only slowly withdrawn as we proceed under a commutating pole and the path to the leading edge of a main pole is of greater length, it follows that the AT_{c2} expended over the armature core under the trailing portion up to the line of bifurcation is appreciably less than AT_{c1} expended under the commutating pole and leading portion of the main pole. The case is therefore the reverse of that shown in Fig. 325 for the non-commutating-pole machine, and, when passing from a N. main pole to a S. commutating pole in the direction of rotation, h on the neutral line or plane of maximum flux is positive. The commutating pole being situated on the leading side of the neutral line, the magnetic potential of the core at the level of the bottom of the teeth beneath, say, a S. commutating pole is $h - AT_{cx1}$. The negative starting point from which the reversing flux proceeds up the commutating pole is not therefore at potential $- AT_{cx1}$, but at this amount less h; whence the potential is raised by the excitation AT_r on one commutating pole up to the positive value $+ b$ at its root, sufficient to pass the greater flux x over $\mathcal{R}_y''$ (Fig. 328). The ampere-turns acting on the commutating pole, gap, and teeth are, therefore, virtually increased beyond the amount AT_r to $AT_r + h$, the number h being as it were released from the main excitation as mentioned above. If, then, $\mathcal{R}_{cp}$ be the final reluctance of the commutating pole, the component flux passing up the commutating pole as due to the reversing and field excitations alone is

$$x - y = 1{\cdot}257 \frac{AT_r + h - AT_{cx1} - b}{\mathcal{R}_t + \mathcal{R}_{gr} + \mathcal{R}_{cp}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (148)$$

x and y being the main fluxes from the system of main and commutating M.M.F.'s through $\mathcal{R}_y''$ and $\mathcal{R}_y'$ respectively. In this and the following expressions containing $\mathcal{R}_{cp}$, a deduction from the pole section must be made in order to allow enough iron to carry the leakage flux, *i.e.* $\mathcal{R}_{cp}$ must be taken as its total reluctance raised in the proportion $\frac{\text{total flux}}{\text{useful flux}}$. Since the final density and saturation in the teeth under the commutating pole are very small, $\mathcal{R}_t$ may, in practice, be neglected. The quantity AT_{cx1} increases slightly as we pass under the commutating pole-face, but a mean value can be taken.

The potential a in ampere-turns at the junction of main pole and yoke is

$$AT_f - AT_c - \frac{(x + y)(\mathcal{R}_m + \mathcal{R}_g + \mathcal{R}_t)}{1{\cdot}257} = a \quad . \quad . \quad . \quad (149)$$

where AT_c will be somewhat less than $\frac{1}{2}(AT_{c1} + AT_{c2})$, and is an averaged value fairly representing the varying potential of the core under the main pole below the level of the teeth.

Also $\qquad 1{\cdot}257\,(a + b) = x \times \mathcal{R}_y''$

$\qquad\qquad 1{\cdot}257\,(a - b) = y \times \mathcal{R}_y'$

whence by addition and subtraction

$$a = \frac{x\Re_y'' + y\Re_y'}{1\cdot257 \times 2} \quad . \quad . \quad . \quad (150)$$

$$y = \frac{1\cdot257 \times 2b + x\Re_y''}{\Re_y'} \quad . \quad . \quad . \quad (151)$$

Equating the two values of a given by (149) and (150) with the value for y in terms of x and b inserted in each

$$x - y = 1\cdot257 \frac{b\left\{1 + \frac{4(\Re_m + \Re_g + \Re_t)}{\Re_y'' + \Re_y'}\right\} - (AT_f - AT_c)\frac{\Re_y'' - \Re_y'}{\Re_y'' + \Re_y'}}{\Re_m + \Re_g + \Re_t + \frac{\Re_y' \times \Re_y''}{\Re_y'' + \Re_y'}} \quad . \quad (152)$$

It will be noticed that the denominator is in a form representing the main pole and gap in series with the two sections of the yoke placed in parallel.

Lastly, equating the two expressions for $x - y$ as given by (148) and (152) with all quantities known or estimated except b, the value of b is found, and thence $x - y$ or x and y can be determined. Thus

$$x = 1\cdot257 \frac{(AT_f - AT_c + b)\Re_y' + 2b(\Re_m + \Re_g + \Re_t)}{(\Re_y'' + \Re_y')(\Re_m + \Re_g + \Re_t) + \Re_y'' \times \Re_y'} \quad . \quad . \quad (153)$$

and

$$y = 1\cdot257 \frac{(AT_f - AT_c - b)\Re_y'' - 2b(\Re_m + \Re_g + \Re_t)}{(\Re_y'' + \Re_y')(\Re_m + \Re_g + \Re_t) + \Re_y'' \times \Re_y'} \quad . \quad . \quad (154)$$

The value thus found, divided by $(w_c + K_1 . l_{gr})\,L$ where w_c is the width of the commutating pole-face and L its axial length, gives the density which when multiplied by 0·8 $K_r . l_{gr}$ gives the loss over the air-gap of the commutating pole for the present flux system and enables AT_{PC}', the potential of the commutating pole-face for the same system, to be found. The limits of the reversing field can now be calculated as follows, and with these can again be checked the effective width $(w_c + K_1 . l_{gr})$ already assumed in the calculation of the density and of $\Re_{gr}$.

In the interpolar gap between a main and a commutating pole of opposite polarity lies the tangent point c' at which the main flux just touches the armature on its way from the one to the other. The magnetic potentials of the main N. and commutating S. pole-faces, viz., $+AT_P'$ and $-AT_{PC}'$ above and below zero are not necessarily equal numerically as in the non-commutating-pole machine, nor are their air-gaps l_g and l_{gr} necessarily equal, so that the tangent point c' of the main component flux from the field excitation of main and commutating poles will seldom occur at the centre of the gap, although usually not far therefrom. The point c' must lie behind the final neutral line, since no armature ampere-turns are yet present. The potential of the core below the level of the teeth on the trailing side at point c' is, therefore, $+AT_{cc'} + h$, since h is positive.

The location of c' is then fixed by the relation

$$\frac{1\cdot257\, m(AT_P' - AT_{cc'} - h)}{\xi c' + Kl_g + \Re_{tc'}} = \frac{1\cdot257\, m'(AT_{PC}' + AT_{cc'} + h)}{\xi(2c - c') + K_r . l_{gr} + \Re_{tc'}} \quad . \quad . \quad (155)$$

whence neglecting $\Re_{tc'}$

$$c' = \frac{m(AT_P' - AT_{cc'} - h)(2c + K_r . l_{gr}/\xi) - m'(AT_{PC}' + AT_{cc'} + h)Kl_g/\xi}{m(AT_P' - AT_{cc'} - h) + m'(AT_{PC}' + AT_{cc'} + h)} \quad (156)$$

c is here the distance measured on the surface of the armature from a main pole-edge to the centre of the gap between a main and a commutating pole, and c' is the distance from the trailing corner of the main pole. When l_g

and l_{gr} are not far different, so that m and m' can be identified, the above expression reduces to—

$$c' = \frac{(AT_P' - AT_{cc'} - h)(2c + K_r \,.\, l_{gr}/\xi) - (AT_{PC}' + AT_{cc'} + h)\,(Kl_g/\xi)}{AT_P' + AT_{PC}'} \tag{157}$$

The corresponding point c' ahead of the resultant neutral line and of the commutating pole must now be interpreted as the point at which the main flux divides or passes indifferently into a commutating S. pole and a main S. pole in the next interpolar gap. It is given by

$$\frac{m(AT_{PC}' - AT_{cc'} + h)}{\xi c' + K_r \,.\, l_{gr} + \mathfrak{R}_{tc'}} = \frac{m'(AT_P' - AT_{cc'} + h)}{\xi(2c - c') + Kl_g + \mathfrak{R}_{tc'}} \tag{158}$$

where c' is reckoned from the commutating pole-edge. When with a less degree of accuracy $\mathfrak{R}_{tc'}$ is again neglected, approximately the second value of c' is

$$c' = \frac{m(AT_{PC}' - AT_{cc'} + h)\,(2c + Kl_g/\xi) - m'(AT_P' - AT_{cc'} + h)\,K_r \,.\, l_{gr}/\xi}{m(AT_{PC}' - AT_{cc'} + h) + m'(AT_P' - AT_{cc'} + h)} \tag{159}$$

§ 12. **(b) The armature component field.**—Next upon the component system of flux from main and reversing excitations thus found has to be superposed the component flux due to the armature turns when current flows therein. Under the main poles the action is the same as has already been described for a machine without commutating poles. But a new path is now presented by the commutating pole as shown in Fig. 328 (ii). Within the limits from a' through $-b'$ to $-A$, or from $-a'$ through b' to $+A$, the lines of the supposed flux follow the same path as the resultant flux from main and commutating-pole excitation. Upon these portions of the path must then be expended part of the ampere-turns which are linked with the supposed flux and which reach their maximum on the diameter of commutation, *i.e.* from $-A$ to $+A$, if commutation is assumed to take place exactly under the centre of the commutating pole. On this assumption the mean armature AT acting on one commutating pole are $ac(Y/2 - w_c/4)$.

The equivalent electric system is shown on the right-hand side of Fig. 328 (ii); here the passage of the armature flux through the armature core from $-A$ to $+A$ is shown dotted to indicate zero reluctance, yet within the length AA' there acts the total M.M.F. averaging $2ac(Y/2 - w_c/4)$. The reluctance of the teeth under the commutating pole being negligible, the total armature flux through the commutating poles and through the two portions of the yoke of unequal reluctance in series *in the absence of the main poles* would be

$$1{\cdot}257\,\frac{2ac(Y/2 - w_c/4)}{2\mathfrak{R}_{gr} + 2\mathfrak{R}_{cp} + \dfrac{\mathfrak{R}_y' + \mathfrak{R}_y''}{2}} = 1{\cdot}257\,\frac{ac(Y/2 - w_c/4)}{\mathfrak{R}_{gr} + \mathfrak{R}_{cp} + \dfrac{\mathfrak{R}_y' + \mathfrak{R}_y''}{4}} \tag{160}$$

But the points now marked as $+a'$ and $-a'$ would not then be at zero potential, but respectively above and below zero by an equal amount. Now, as in the non-commutating-pole machine, the cross flux in the trailing half of the main pole for a half of the armature M.M.F. would, owing to the higher saturation of the teeth, be much less than that in the leading half. For the same reasons, then, as are given in § 6 (*b*) for the non-commutating-pole machine, the potential of the N. pole will rise to some potential $+a'$ and that of the S. pole fall to $-a'$. If c'' be the distance from the corner of a main pole at which the armature flux bifurcates or passes indifferently to a main or commutating pole, the maximum differences of potential acting on the trailing and leading portions respectively of the air-gap and teeth under a main pole are $1{\cdot}257\,ac(\beta Y/2 + c'' + d)$ and $1{\cdot}257\,ac(\beta Y/2 + c'' - d)$ where $d = a'/ac$.

If the armature amperes are now imagined to be gradually increased from zero in the presence of tooth reluctance, two results are seen to follow. As

the potentials of the poles diverge from zero, a greater share of the magnetomotive force of the armature is available to cause flux over the trailing path of higher tooth saturation under the main poles. But as soon as the trailing cross flux ceases to balance the leading cross flux, the excess of the latter can pass from the point $+ a'$ through the $\mathfrak{R}_y''$ path and through the commutating pole. The potentials $+ a'$ and $- a'$ thus for each increase of the armature amperes continue to exceed the values that would be found from equation (160) for equal division; more than half of the commutating pole flux is caused to follow the $\mathfrak{R}_y''$ path in the yoke and less to follow the $\mathfrak{R}_y'$ path, and the difference exactly balances the difference between the leading and trailing portions of the cross flux under the main poles (Fig. 328 (ii)). As compared with the simple series system of equation (160) under which half of the flux through the commutating pole passes unchanged through $\mathfrak{R}_y'$ and $\mathfrak{R}_y''$ in series, the total flux through the commutating pole is always slightly reduced owing to a larger proportion having to pass through the more saturated portion of the yoke.

Thus, as contrasted with the non-commutating-pole machine, a shorter path is present to carry the excess cross flux which does not have to pass through the entire yoke. The commutating pole is itself called in to redress the balance of the cross fluxes under a main pole. Even though the air-gap be smaller and the tooth saturation higher in the commutating-pole machine, the amount of the backward shift d or the value of $a' = d \,.\, ac$ is but small, as in the non-commutating-pole machine.

A similar analysis to that of § 11 (*a*) can thus be made for the purpose of finding the potential a' instead of b. The potential b' (in ampere-turns) at the junction of commutating pole and yoke is for the armature system

$$b' = ac(Y/2 - w_c/4) - \frac{(x' + y')(\mathfrak{R}_{gr} + \mathfrak{R}_{cp})}{1{\cdot}257} \quad . \quad . \quad . \quad . \quad . \quad (161)$$

where x' and y' are the fluxes through $\mathfrak{R}_y''$ and $\mathfrak{R}_y'$ respectively and $x' + y'$ is the total armature flux through the commutating pole. Also

$$1{\cdot}257(b' + a') = x'\mathfrak{R}_y''$$
$$1{\cdot}257(b' - a') = y'\mathfrak{R}_y'$$

whence by addition or subtraction

$$b' = \frac{x'\mathfrak{R}_y'' + y'\mathfrak{R}_y'}{1{\cdot}257 \times 2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (162)$$

$$\text{and } y' = \frac{1{\cdot}257 \times 2a' \times x'\mathfrak{R}_y''}{\mathfrak{R}_y'}$$

Equating the two values of b' as given by (161) and (162) with the value for y' in terms of x' inserted in each case, we have—

$$x' = 1{\cdot}257\,\frac{ac(Y/2 - w_c/4)\left(1 - \dfrac{\mathfrak{R}_y'' - \mathfrak{R}_y'}{\mathfrak{R}_y'' + \mathfrak{R}_y'}\right) + a'\left\{1 + \dfrac{4(\mathfrak{R}_{gr} + \mathfrak{R}_{cp}) - (\mathfrak{R}_y'' - \mathfrak{R}_y')}{\mathfrak{R}_y'' + \mathfrak{R}_y'}\right\}}{2\left(\mathfrak{R}_{gr} + \mathfrak{R}_{cp} + \dfrac{\mathfrak{R}_y'' \times \mathfrak{R}_y'}{\mathfrak{R}_y'' + \mathfrak{R}_y'}\right)} \quad (163)$$

$$y' = 1{\cdot}257\,\frac{ac(Y/2 - w_c/4)\left(1 + \dfrac{\mathfrak{R}_y'' - \mathfrak{R}_y'}{\mathfrak{R}_y'' + \mathfrak{R}_y'}\right) - a'\left\{1 + \dfrac{4(\mathfrak{R}_{gr} + \mathfrak{R}_{cp}) + (\mathfrak{R}_y'' - \mathfrak{R}_y')}{\mathfrak{R}_y'' + \mathfrak{R}_y'}\right\}}{2\left(\mathfrak{R}_{gr} + \mathfrak{R}_{cp} + \dfrac{\mathfrak{R}_y'' \times \mathfrak{R}_y'}{\mathfrak{R}_y'' + \mathfrak{R}_y'}\right)} \quad (164)$$

Further (Fig. 328 ii)

$$w' = 1{\cdot}257\,\frac{ac(Y/2 - w_c/4) - a'}{\mathfrak{R}_g' + \mathfrak{R}_t'} \quad (165)$$

$$z' = 1{\cdot}257\,\frac{ac(Y/2 - w_c/4) + a'}{\mathfrak{R}_g'' + \mathfrak{R}_t''} \quad (166)$$

and, lastly, $x'+z'$ must be equal to $y'+w'$, so that

$$x'-y'=w'-z'.$$

Equating the two values obtained for $x'-y'$ from the above, we have

$$\frac{a'\left\{1+\dfrac{4(\mathcal{R}_{gr}+\mathcal{R}_{cp})}{\mathcal{R}_y''+\mathcal{R}_y'}\right\}-ac(Y/2-w_c/4)\dfrac{\mathcal{R}_y''-\mathcal{R}_y'}{\mathcal{R}_y''+\mathcal{R}_y'}}{\mathcal{R}_{gr}+\mathcal{R}_{cp}+\dfrac{\mathcal{R}_y''\times\mathcal{R}_y'}{\mathcal{R}_y''+\mathcal{R}_y'}}$$

$$=\frac{ac(Y/2-w_c/4)\,(\mathcal{R}_g''+\mathcal{R}_t''-\mathcal{R}_g'-\mathcal{R}_t')-a'(\mathcal{R}_g''+\mathcal{R}_t''+\mathcal{R}_g'+\mathcal{R}_t')}{(\mathcal{R}_g''+\mathcal{R}_t'')\,(\mathcal{R}_g'+\mathcal{R}_t')} \quad . \quad (167)$$

The values of w' and z' are calculated exactly as for a non-commutating pole machine upon the assumption of a certain small value of d (probably less than 1 cm.) or of $a'=d\,.\,ac$. The values, then, to be taken for $\mathcal{R}_g'+\mathcal{R}_t'$ and $\mathcal{R}_g''+\mathcal{R}_t''$ for insertion in (167) must be such that when acted upon by the full maximum M.M.F. of $1{\cdot}257\,ac\,.\,Y/2$ *plus* or *minus* d, the fluxes w' and z' are actually obtained.

Inserting the known or estimated quantities, the value to be assigned to a' or to $d'=a'/ac$ is found. If the value first found does not agree with the value assumed in the calculation for w' and z', the shift d must be adjusted until its value and that of $a'=ac\times d$ satisfies all conditions.

Then

$$x'+y'=1{\cdot}257\,\frac{ac(Y/2-w_c/4)-a'\dfrac{\mathcal{R}_y''-\mathcal{R}_y'}{\mathcal{R}_y''+\mathcal{R}_y'}}{\mathcal{R}_{gr}+\mathcal{R}_{cp}+\dfrac{\mathcal{R}_y''\times\mathcal{R}_y'}{\mathcal{R}_y''+\mathcal{R}_y'}} \quad . \quad . \quad . \quad . \quad (168)$$

The present denominator is, therefore, in a form representing the two sections of the yoke placed in parallel, but now in series with the commutating pole and gap.

The number of ampere-turns expended over the air-gap of the commutating pole by the armature flux component therein is $0{\cdot}8\dfrac{x'+y'}{(w_c+K_1l_{gr})L}\times K_r\,.\,l_{gr}$, and this number deducted from $ac(Y/2-w_c/4)$ leaves a remainder AT_{PC}'' which is the magnetic potential in AT of the whole of the commutating pole-face relatively to the armature surface, so far as the armature component flux is concerned.

The component flux density will rise to a maximum at the centre of the commutating pole with the given position for the diameter of commutation and will fall towards the edges. But the bifurcation point of the armature component flux into a main and commutating pole respectively is now given by the relation

$$\frac{ac\,(\beta Y/2\pm d+c'')}{\xi c''+Kl_g+\mathcal{R}_{tc''}}=\frac{ac(\beta Y/2+c'')-AT_{PC}''}{\xi(2c-c'')+K_rl_{gr}+\mathcal{R}_{tc''}} \quad . \quad . \quad (169)$$

$+\,d$ applying to the trailing edge of the main pole and $-\,d$ to the leading edge, c'' being in each case reckoned from the main pole-edge. It will be found that the bifurcation points c'' are again usually not far from the centre of the gap, so that there is not a great difference in the values of m.

Thence if $\mathcal{R}_{tc''}$ is neglected

$$(c'')^2+c''\left\{\beta Y/2\pm d/2-\frac{AT_{PC}''}{2ac}-\left(c-\frac{Kl_g-K_r\,.\,l_{gr}}{2\xi}\right)\right\}$$

$$=\beta Y/2\left(c-\frac{Kl_g-K_r\,.\,l_{gr}}{2\xi}\right)+\frac{AT_{PC}''}{ac}\,.\,\frac{Kl_g}{2\xi}\pm d\left(c+\frac{K_r\,.\,l_{gr}}{2\xi}\right) \quad . \quad . \quad (170)$$

With the value for c'' thus found may again be checked the value assumed for $\mathcal{R}_{gr}$ and for $w_c+K_1\,.\,l_{gr}$, but usually there will be no great difference as between the two cases of the main and armature components.

§ 13. (c) The resultant reversing field.—Grouping, therefore, our results it is evident that the actual magnetic potential at the root of the commutating pole must assume the intermediate value $b - b'$, and from the previous expressions this is equal to

$$-\frac{1}{1\cdot257}\left(\frac{X\mathcal{R}_v'' - Y\mathcal{R}_v'}{2}\right),$$ where $X = x - x'$ and $Y = y + y'$. This again is equal to $-\frac{1}{1\cdot257}\left\{\frac{f'(B_v'') - f'(B_v')}{2}\right\}\frac{l_v}{2}$ where $l_v/2$ is the half length of the path in the yoke between two main poles, *i.e.* the length of path between a commutating pole and a main pole. But in calculating the two values for $f'(B_v)$ or the *at* per cm. of path in the yoke, it must be remembered that if the full section of the yoke be used there must be added to X and Y in each case their proportionate share of the total leakage flux. Finally, therefore, the net resultant reversing flux is

$$\phi_r = x - y - (x' + y') = X - Y$$
$$= \frac{1\cdot257\,\{AT_r + h - AT_{cr1} - ac(Y/2 - w_c/4)\} - \{f'(B_v'') - f'(B_v')\}l_v/4}{\mathcal{R}_{gr} + \mathcal{R}_{cp}} \qquad (171)$$

It will be found that the difference in the saturation of the two sections of the yoke may very appreciably lower the reversing field that might otherwise be expected.

If $\mathcal{P}_l$ be the estimated leakage permeance in relation to the interpolar M.M.F. $1\cdot257\ AT_p \times 2$ of the main poles when the commutating poles are present but unexcited, the leakage permeance of the commutating pole in relation to its M.M.F. $1\cdot257\ AT_{pc}$ will be, as stated in Chapter XVI, § 14, about $1\frac{1}{2}\ \mathcal{P}_l$. Thence may be estimated ϕ_l and ϕ_{lr}, the leakage fluxes per main and commutating pole respectively. Of the latter only about $\frac{3}{8}$ths should be reckoned as passing effectively through the yoke path. Hence if Φ_a be the useful flux per pole $= \Phi_m - \phi_l$, the last term of the numerator of (171) becomes

$$\left\{f'\left(\frac{\frac{1}{2}\Phi_a + \frac{1}{2}\phi_l + \frac{3}{8}\phi_{lr} + \frac{1}{2}\phi_r}{\frac{1}{2}a_y}\right) - f'\left(\frac{\frac{1}{2}\Phi_a + \frac{1}{2}\phi_l - \frac{3}{8}\phi_{lr} - \frac{1}{2}\phi_e}{\frac{1}{2}a_y}\right)\right\}\frac{l_y}{4} \qquad (172)$$

where a_y is the double cross-section of the yoke.

The quantity $h - AT_{cr1}$ is of the order of $-100\ AT$ in normal cases, but if not assumed in the first instance, it can provisionally be approximated by plotting an assumed flux-density curve and thence calculating such a curve as that shown at the foot of Fig. 329, the gradual concentration and withdrawal of the flux in the

armature core being taken into account and the progressive expenditure of AT over the length of path therein being calculated in steps.

In calculating the width of fringe peripherally of the reversing field in order to determine the effective area and $\mathfrak{R}_{gr}$, it must be remembered that, strictly speaking, on one side half of the values of Fig. 253 should be used ; on the other side, between a main and commutating pole of the same polarity, half of the values of Fig. 254 is more appropriate. In each case the ratio c/l_{gr} may provisionally be taken as the abscissa, as will be shown later. Knowing the density at which it is intended to work the iron of the commutating pole, an estimate must be made of its reluctance upon an approximate assumption as to the total flux, useful and leakage, through it, and this reluctance must then be raised in the proportion total flux/useful flux to give $\mathfrak{R}_{cp}$.

It is then possible by giving increasing values to ϕ_r to determine quickly the point when equality of the two sides of equation (173) is reached.

$$\frac{\phi_r \times \mathfrak{R}_{gr}}{1{\cdot}257} = AT_r + (h\,AT_{cx1}) - ac(Y/2 - w_c/4) - f'\left(\frac{\phi_r + \phi_{lr}}{a_{cp}}\right) l_{cp}$$

$$- \left\{ f'\left(\frac{\Phi_a + \phi_l + \frac{3}{4}\phi_{lr} + \phi_r}{a_v}\right) - f'\left(\frac{\Phi_a + \phi_l - \frac{3}{4}\phi_{lr} - \phi_r}{a_y}\right) \right\} \frac{l_v}{4} \quad . \quad (173)$$

§ 14. **Example of a commutating-pole machine.**—In the example now to be considered, the pole-pitch is 45 cm. as in § 9, but with commutating poles added the pole-arc ratio β is reduced to 0·666. The polar arc is therefore 30 cm. and the width of the commutating pole-face is $w_c = 5$ cm., leaving gaps of 5 cm. between main and commutating poles.

$$Kl_g = 1{\cdot}1 \times 0{\cdot}635 = 0{\cdot}7 \text{ cm.}$$

$$K_r \,.\, l_{gr} = 1{\cdot}16 \times 0{\cdot}38 = 0{\cdot}44 \text{ cm.}$$

and the angle made by the pole shoe tips with the armature is such that in each case $\xi = 1{\cdot}7$.

ac is to be 300 ampere-conductors per cm. of periphery, so that

$ac \times Y/2 = 300 \times 22{\cdot}5 = 6750$

$ac(Y/2 - w_c/4) = 300 \ (22{\cdot}5 - 1{\cdot}25) = 6375$

If $B_{g\,max} = 9250$, $AT_g = 0{\cdot}8 \times 9250 \times 0{\cdot}7 = 5175$,

and from Fig. 302 $AT_t = 1290$.

The flux under the main pole would then be $9250 \times 30 = 277{,}500$ per cm. of axial length of armature core, *i.e.* $277{,}500\ L$, and for

convenience in what follows, all reluctances and fluxes are expressed in terms of L after reduction to their equivalents per cm. of armature core length. Without any commutating poles, the interpolar fringes between main poles would add on about 20,000 L, making the total flux 297,500 L. This amount is approximately to be retained as the maximum in the armature core, or 148,750 L in the single section, and for this a preliminary estimate gives $AT_c = 355$, so that $AT_g + AT_t + AT_c = AT_r$ would then be $5175 + 1290 + 355 = 6820$. But in this no allowance has been made for the loss of flux by reason of the difference in the increase and decrease of tooth reluctance under the trailing and leading portions of the main pole respectively. If this be 13,500 L, when $d = 0{\cdot}6$ cm. and $ac \times d = 180$, AT_r becomes $6820 + 180 = 7000$, and the actual flux from a main pole will then be $(297{,}500 - 13{,}500)\ L = 284{,}000\ L$. The loss is, however, more than made good by the flux from the commutating pole of the same polarity, so that we return to about 297,500 L for the total flux in the armature core, although all of it will not be available for the production of E.M.F. owing to the diameter of commutation being ahead of the neutral line or plane of maximum flux.

Finally $AT_r = 7910$

and $AT_f = 9865$

although according to the circumstances of the investigation these either remain to be calculated to meet the required values of Φ_a and ϕ_r, or are assumed known when the actual values of the fluxes are to be determined.

(*a*) *The resultant reversing field.* First, to calculate the net value of the reversing flux for the immediate purpose of determining the average density of resultant flux in which the short-circuited coils are moving.

The leakage flux is estimated at 75,000 L, so that $\Phi_m = \Phi_a + \phi_l = (284{,}000 + 75{,}000)\ L = 359{,}000\ L$, and at a normal density of 15,000 without unequal division of the fluxes in the two sections of the yoke in series, its double cross-section must be $\dfrac{359{,}000L}{15{,}000} = 24\ L$ sq. cm. The half length of path between two main poles or the length between a main and a commutating pole is $l_y/2 = 30$ cm.

The ratio c/l_{gr} being $2{\cdot}5/0{\cdot}38 = 6{\cdot}6$, for the effective width of the two fringes of the commutating pole will be taken $K_1 l_{gr} = 3 \times 0{\cdot}38 = 1{\cdot}14$, and the width w_c being 5 cm. the area of the commutating pole air-gap if the pole is given the same length as the armature core will be $6{\cdot}14\ L$. S_{gr} is then $0{\cdot}44/6{\cdot}14\ L = 0{\cdot}0717/L$. For $h - AT_{ex1}$ will be taken the figure -80 as finally established, and AT_r being assumed as 7910, the known quantities are $AT_r + (h - AT_{ex1}) - ac\ (Y/2 - w_c/4) = 7910 - 80 - 6375 = 1455$. Reckoning

ϕ_{lr} as $0{\cdot}75\,\phi_l = 56{,}200\,L$, then by equation (173) ϕ_r per cm. of armature core length $\times \frac{0{\cdot}717}{1{\cdot}257} =$

$$\phi_r \times 0{\cdot}57 = 1455 - 30{\cdot}3\,f'\left(\frac{\phi_r + 56{,}200}{5{\cdot}66}\right)$$

$$- 15\left\{f'\left(\frac{401{,}100 + \phi_r}{24}\right) - f'\left(\frac{316{,}900 - \phi_r}{24}\right)\right\}$$

Trial with different values of ϕ_r gives equality when

$\phi_r = 12{,}500$	$B_{cp} = 12{,}100$	$B_y'' = 17{,}240$	$B_y' = 12{,}650$
$at = f'(B)$	$7{\cdot}15$	$43{\cdot}1$	8
		difference	$35{\cdot}1$
$AT =$	$7{\cdot}15 \times 30{\cdot}3 = 217$		$35{\cdot}1 \times 15 = 526.$

Therefore air-gap AT of commutating pole

$$AT_{gr} = 12{,}500 \times 0{\cdot}57 = 712 = 1455 - 217 - 526,$$

and the average density of the resultant reversing flux $= 12{,}500/6{\cdot}14 = 2040$.

(*b*) *The distribution over a complete pole-pitch.* In order to map out the spacial distribution of the resultant flux over a complete pole-pitch (with tooth ripples smoothed out), a preliminary calculation must be made, as in Chapter XVIII, § 2, of AT_{c1} and AT_{c2} for an assumed resultant flux wave. Let it be supposed that on the leading and trailing sides respectively of the neutral plane of maximum flux in the armature core $AT_{c1} = 464$ and $AT_{c2} = 240$ (the values subsequently established are here at once inserted for simplicity). The mean value is therefore $AT_c = 362$, and $h = 122$. Plotting then such a curve as that shown at the foot of Fig. 329, which to an appropriate scale will also serve as a guide for other normal machines, and assuming AT_p' to remain at 6820, since the same value for B_g is approximately to be retained, the values of $AT_p' - (AT_{cx1} - h)$ or of $AT_p' - AT_{cx2} - h$ can be written down for small intervals as in Table XII (columns 7, 8, 9). Allowing some small value for d or $ac \times d$ (viz. 0·6 cm. and $0{\cdot}6 \times 300 = 180$), $ac(Y/2 - x + d)$ can also be written down (columns 10 and 11). Thence by the same procedure as in § 8 (*c*), the whole of the main polar area is covered up to points midway between main and commutating poles.

Some value of d has above been assumed, so that before going further this must be checked. Plotting the two component fields of Table XII (columns 17 and 18) for the polar arc up to the midway points between main and commutating poles, and summing up the two fluxes separately, the main flux is found to be 297,000 per cm. length of armature core, and the cross flux on leading and trailing sides respectively to be $w' = 60{,}500$ and $w'' = 47{,}000$ (Fig. 328 (ii)). The shift of the dividing line behind the centre of the pole being taken as $d = 0{\cdot}6$, the maximum ampere-turns acting on the two portions of the air-gap area are for our present purpose $6375 - 180 = 6195$ and $6375 + 180 = 6555$. Consequently the apparent reluctances are

$$\mathcal{R}_g' + \mathcal{R}_t' = \frac{1{\cdot}257 \times 6195}{60{,}500} = 0{\cdot}1285$$

and

$$\mathcal{R}_g'' + \mathcal{R}_t'' = \frac{1{\cdot}257 \times 6555}{47{,}000} = 0{\cdot}176$$

Table XII.

		1	2	3	4	5	6	7	8	9
		Cm. from centre of commutating pole.	$K_r l_{gr}$	$\xi x'$	$\Re_{tx}$	2+3+4 Reluctance of gap and teeth per sq. cm.	$\frac{\text{Col.5}}{1.257}$	AT_{PC}'	$-AT_{cx}+h$	7+8 AT from main field turns on gap and teeth.
N. commutating pole	Centre	0	0·44	–	0·001	0·441	0·35	548	80	5400
		1·25	0·44	---	0·0015	0·4415	0·351	5480	– 125	5355
	Trailing edge	2·5	0·44	- -	0·002	0·442	0·352	5480	– 170	5310
		3·5	0·44	1·7	0·001	2·141	1·71	5480	– 192	5288
		4·5	0·44	3·4	0·0005	3·8405	3·06	5480	– 220	5260
			Kl_g	$\xi y'$				AT_P'	$-AT_{cx}+h$	
		5·5	0·7	3·4	0·0005	4·1005	3·26	6820	– 250	6570
		6·5	0·7	1·7	0·001	2·401	1·91	6820	– 270	6550
N. main pole	Leading edge	7·5	0·7	---	0·004	0·704	0·56	6820	– 292	6528
		8·5	0·7	- -	0·005	0·705	0·561	6820	– 307	6513
		12·5	0·7	—	0·015	0·715	0·569	6820	– 343	6477
		16·5	0·7	---	0·06	0·76	0·605	6820	– 355	6465
		20·5	0·7	—	0·145	0·845	0·671	6820	– 360	6460
		21·9	0·7	- -	0·18	0·88	0·7	6820	– 361	6459
		24·5	0·7	- -	0·238	0·938	0·746	6820	– 362	6458
		28·5	0·7	-	0·34	1·04	0·829	6820	– 360	6460
		32·5	0·7	—	0·45	1·15	0·916	6820	– 354	6466
		36·5	0·7	- -	0·56	1·26	1·005	6820	– 320	6500
	Trailing edge	37·5	0·7	$\xi x'$	0·433	1·133	0·904	6820	– 293	6527
		38·5	0·7	1·7	0·011	2·411	1·92	6820	– 255	6565
		40·2	0·7	4·6	0·001	5·301	4·22	6820	– 170	6650
			$K_r l_{gr}$	$\xi y'$						
		41·5	0·44	1·7	0·0002	2·1402	1·705	5480	– 100	– 5580
S. commutating pole	Leading edge	42·5	0·44	---	0·0025	0·4425	0·352	– 5480	– 50	– 5530
		43·75	0·44	---	0·002	0·442	0·352	– 5480	+ 10	– 5470
	Centre	45	0·44	---	0·001	0·441	0·35	– 5480	+ 80	– 5400

Commutating-pole Machine.

10	11	12	13	14	15	16	17	18	19
$-ac(Y/2-x)+AT_{pc}''$		AT from armature on gap and teeth.	Sum of 9 + 12 acting on gap and teeth.	m	Deduction for effect of adjacent pole in interpolar gap.	Col. 9 − Col. 6	Main field density.	Armature field density.	Resultant B_{gr} or B_g
$-6750+1675$		-5075	325				15,400	14,473	927
$-6375+1675$		-4700	655				15,270	$-13,400$	1870
$-6000+1675$		-4325	985	0·84			12,700	$-10,350$	2350
$-5700+1675$		-4025	1263	1·07			3320	-2525	795
$-5400+1675$		-3725	1535	1·03			1772	-1255	517
$+ac(Y/2-x)+ac.d$									
$-5100+180$		-4920	1650	1·06			2136	-1600	536
$-4800+180$		-4620	1930	1·13			3870	-2730	1140
$-4500+180$		-4320	2208	0·84			9800	-6490	3310
$-4200+180$		-4020	2493				11,620	-7170	4450
$-3000+180$		-2820	3657				11,400	-4960	6440
$-1800+180$		-1620	4845				10,690	2680	8010
$-600+180$		-420	6040				9625	625	9000
$-180+180$		0	6459				9250	0	9250
$+600+180$		$+780$	7238				8660	1045	9705
$+1800+180$		$+1980$	8440				7800	2390	10,190
$+3000+180$		$+3180$	9646				7060	3480	10,540
$+4200+180$		$+4380$	10,880				6470	4370	10,840
$+4500+180$		$+4680$	11,207	0·84			6100	4360	10,460
$+4800+180$		$+4980$		1·13	$\dfrac{(5480+255)}{3{\cdot}1+0{\cdot}556+0{\cdot}008}\times\dfrac{1}{2{\cdot}7}=580$	3020	3340	2930	6270
$+5310+180$		$+5490$		1·04	-1650	1650	0	1350	1350
$+5700+1675$		$+4025$		1·07	$\dfrac{1{\cdot}04(6820-100)}{3{\cdot}65+0{\cdot}558}\times\dfrac{1}{2{\cdot}3}=725$	-3510	2785	$+2530$	-255
$+6000-1675$		$+4325$	-1205	0·84			$-13,180$	$+10,310$	-2870
$+6375-1675$		$+4700$	770				$-15,570$	$+13,380$	-2190
$+6750-1675$		$+5075$	-325				$-15,400$	$+14,473$	927

while $w' - z' = 60,500 - 47,000 = 13,500 = x' - y'$. The useful flux of a main pole being $297,000 - 13,500 = 283,500$, differing but little from the previous preliminary estimate of 284,000, the corrected value is here inserted, and from the previous calculations we have

$$\Re_v'' = \frac{1.257 \times 43.1 \times 30}{141,750 + 6250} = \frac{1.257 \times 1293}{148,000} = 0.011 \quad \text{sum} = 0.01322$$

$$\Re_v' = \frac{1.257 \times 8 \times 30}{141,750 - 6250} = \frac{1.257 \times 240}{135,500} = 0.00222 \quad \text{diff.} = 0.00878$$

$$\Re_{cp} = \frac{1.257 \times 217}{12,500} = 0.0218 \qquad \frac{\text{product}}{\text{sum}} = 0.00185 = \frac{\Re'' \Re_v'}{\Re'' + \Re_v'}$$

Hence

$$\Re_{gr} + \Re_{cp} = 0.0717 + 0.0218 = 0.0935$$

By equation (167),

$$\frac{a'\left\{1 + \frac{4 \times 0.0935}{0.01322}\right\} - 6375\left(\frac{0.00878}{0.01322}\right)}{0.0935 + 0.00185} = \frac{13,500}{1.257}$$

whence $a' = 180$ in agreement with the assumed $a' = d \cdot 300 = 0.6 \times 300$, so that we may proceed.

Returning to the main component field, the true reluctance of gap and teeth per cm. length of core, when the potential of the core below the teeth is reckoned as uniformly 345 instead of at its actual value which varies slightly, is

$$\Re_g + \Re_t = \frac{1.257(6820 - 345)}{297,000} = 0.0275$$

The calculated leakage flux per cm. being 75,000, and the resultant flux of a main pole being about 297,000, the AT required over it is reckoned to be 2100. The reluctance of the main pole to the useful flux per cm. length of armature core is therefore

$$\Re_m = \frac{1.257 \times 2100}{297,000} = 0.009$$

so that

$$\Re_m + \Re_g + \Re_t = 0.009 + 0.0275 = 0.0365.$$

Thence by equations (148) and (152)

$$\frac{7830 - b}{0.0935} = \frac{b\left\{1 + \frac{0.146}{0.01322}\right\} - (9865 - 345)\frac{0.00878}{0.01322}}{0.0365 + 0.00185}$$

$$b = 766.$$

By equations (153) and (154)

$$x = 196,000 \qquad x + y = 297,000$$

$$y = 101,000 \qquad x - y = 95,000 \text{ (Fig. 326 (i))}.$$

The potential of the commutating pole face AT_{pc}', so far as the main field is concerned, can now be calculated as

$$AT_{pc}' = \frac{95,000 \times 0.0717}{1.257} + AT_{cx1} - h$$

$$= 5400 + 80 = 5480.$$

By substituting the varying values $AT_{cx1} - h$, the main reversing flux density under the commutating pole is completed as shown in Table XII.

Similarly for the armature flux, from equation (168)

$$x' + y' = \frac{1{\cdot}257\left(6375 - 180\,\frac{0{\cdot}00868}{0{\cdot}01492}\right)}{0{\cdot}0935 + 0{\cdot}00185} = 82{,}500 \quad (\text{Fig. } 328\,(n))$$

and since $x' - y' = 13{,}500$,

$$x' = 48{,}000 \qquad y' \;\; 34{,}500$$

Thence

$$AT_{pc}{}'' \;\; 6375 \;\; \frac{82{,}500 \;\; 0{\cdot}0717}{1{\cdot}257}$$

$$= 6375 \;\; 4700 \;\; 1675$$

and the armature flux density under the commutating pole is completed from columns 10 and 11 of Table XII. In the gap between a main and commutating pole of opposite polarity, the back effect from the opposing pole is taken into account in the same way as in § 8 (*b*) and as shown in column 15.

A commutating and a main pole of the same sign really form one field, but the bifurcation of the flux enables a certain portion of it to be assigned separately to the commutating pole. The limits of this commutating pole field for the main and armature components respectively, as fixed by c' and c'', are not necessarily coincident owing to the different values of the potential differences acting on the air-gap in the two cases, and for neither component is the fringe on the two sides of the commutating pole edge precisely equal. In the case of the main field, the latter divergence is due to the fact that one value of c' gives the tangent point between a main and commutating pole of opposite sign, while the other value of c' gives the bifurcation point between a main and commutating pole of the same sign. Thus by equation (157) if $AT_{pc}{}'$ be estimated at 5400 + 80 5480, the former value of c' is

$$\frac{1{\cdot}04(6820 \;\; 170)\,(5 + 0{\cdot}259) \;\; 1{\cdot}00\,(5480 + 170) \;\; 0{\cdot}41}{1{\cdot}04 \;\; 6650 + 5650}$$

2·7 cm. from the trailing edge of a main pole, or 2·3 cm. from the *leading* edge of the commutating pole, and the component main fluxes thereat are

$$\frac{1{\cdot}257 \times 1{\cdot}04 \;\; 6650}{1{\cdot}7 \times 27 + 0{\cdot}7} \;\; \frac{8700}{5{\cdot}29} \;\; 1650 \;\; \frac{7160}{4{\cdot}35} \;\; \frac{1{\cdot}257 \;\; 5650}{1{\cdot}7 \;\; 2{\cdot}3 + 0{\cdot}44}$$

But the latter value of c' is by equation (159)

$$c' = \frac{1{\cdot}04(5480 \;\; 245)\,(5{\cdot}41) \;\; 0{\cdot}99\,(6820 \;\; 245) \times 0{\cdot}259}{1{\cdot}04 \times 5235 + 0{\cdot}99 \;\; 6575}$$

2·34 cm. from the *trailing* edge of the commutating pole.

In the case of the armature component field, the divergence between the extent of the fringe on the two sides of the commutating pole only arises from the different signs of a' $ac \times d$ in the N. and S. main poles respectively, and is equally small. By equation (169) if $AT_{pc}{}''$ 1675

c'' = 2 cm. from the *leading* edge of the commutating pole
and 2·09 cm. " *trailing* " " "

The fringe of the main field from the commutating pole is therefore the wider, and actually the resultant width of fringe has been over-estimated in the figure 6·14 for the effective area. But in general the positions of c' or c'' do not diverge greatly from midway between pole and pole, so that for simplicity this approximation may be adopted in the preliminary calculations.

§ 15. Shape of the flux-density curve under load with commutating poles.—The final spacial distribution is shown in Fig. 329. The dip in the resultant reversing flux density at the centre of the commutating pole is simply due to the armature ampere-turns

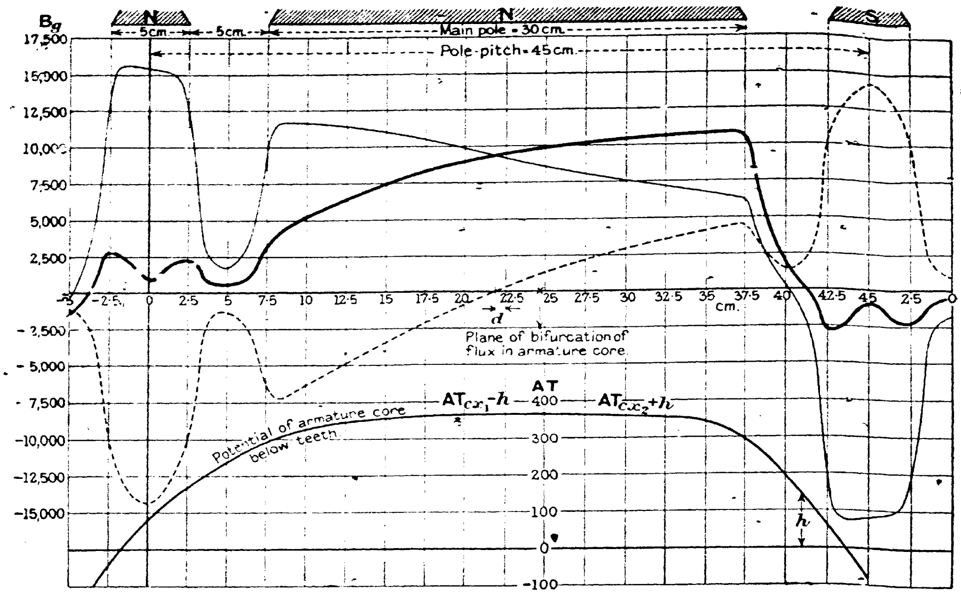


FIG. 329.—Main and armature component fluxes and resultant flux with commutating poles.

having been taken to their maximum of $300 \times 22{\cdot}5 = 6750$. Actually, if commutation proceeds properly, the apex over the zone of the short-circuited coils should be rounded off (*cp.* Fig. 318), and the average value of 6375 might in fact give the more accurate result. The resultant potential of the main pole-faces is 6820 + 180 = 7000, and the total AT on each of the main poles is 766 + 2100 + 7000 = 9866, where $766 = \frac{1}{2}(1293 + 240)$.

The useful flux of the two machines above investigated in §§ 9 and 14 is not greatly different, and the dimensions of the armatures and slots are the same. But while in the non-commutating-pole

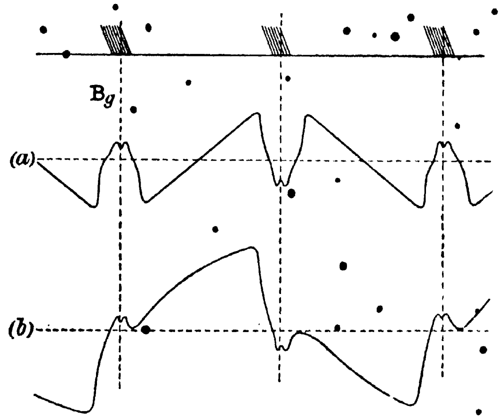


FIG. 330.—Flux-distribution curves with commutating poles. (After Rezelman.)

(*a*) Armature and commutating poles with full-load ampere turns.
(*b*) Ditto with main poles also excited.

machine the angle of lead has been advanced to the farthest limit of safety and a reversing density of 889 is secured with $ac = 260$, in the commutating-pole machine with a lesser main field excitation a reversing density of about 1000 at the lowest has been obtained with ac raised to 300.

The shape of the flux-density curve round the armature surface of an actual machine under load is shown in Fig. 330 (*a*) and (*b*); here in (*a*) the main field poles are unexcited, and the full-load ampere-turns of the commutating poles have been grouped with the full-load armature ampere-turns so as to reproduce the net reversing field and the armature cross-flux. In (*b*) the main poles are also fully excited. The depression in the centre of the reversing field was in this case due to the opening of a slot facing the commutating pole.

GENERAL

§ 16. Limiting value of ampere-conductors per unit length of circumference of armature.—One of the two important quantities that determine the necessary dimensions of a machine for a given output is the value of ac or the ampere-conductors per unit length of circumference of the armature. Although not a constant for machines of greatly different diameter, experience proves that there is a certain limiting value for each diameter beyond which it is inadvisable to push $ac = JZ/\pi D$. Fig. 331[1] shows that as the size of the armature is increased, the limiting value of ac approaches

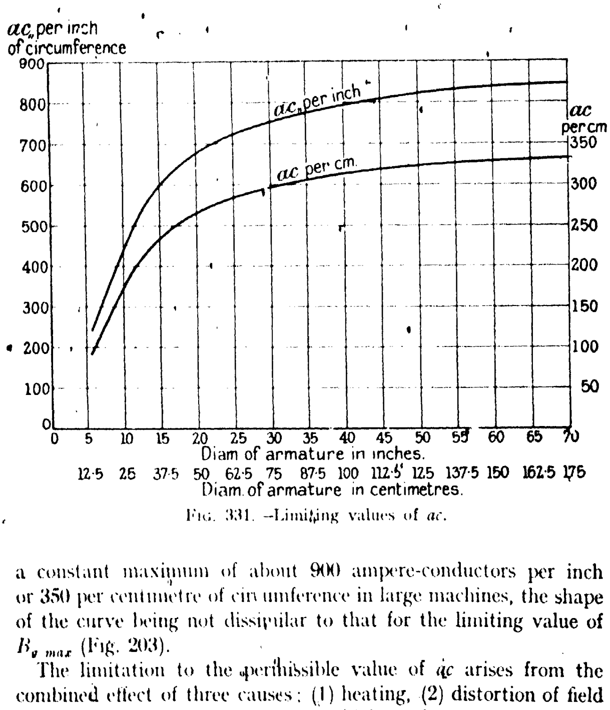


FIG. 331.—Limiting values of ac.

a constant maximum of about 900 ampere-conductors per inch or 350 per centimetre of circumference in large machines, the shape of the curve being not dissimilar to that for the limiting value of $B_{g\,max}$ (Fig. 203).

The limitation to the permissible value of ac arises from the combined effect of three causes: (1) heating, (2) distortion of field due to armature reaction causing too high a voltage to be generated between adjacent sectors (*cp.* Chap. X, § 13), and (3) sparking. With the first and third we are not here immediately concerned, but in regard to the first it may be pointed out that with larger diameters it is the possibility of using within certain limits a deeper

[1] *Cp.* an analogous curve giving the estimate of Dr. R. Pohl, *Journ. I.E.E.*, Vol. 40, p. 242.

slot (Fig. 202) without reducing too much the section at the root of the tooth that enables the volume of armature copper to be increased even more than in proportion to the increased diameter.

The sparking limit in the non-commutating-pole machine will be further discussed in the succeeding chapter, and is in fact closely connected with the reaction of the armature ampere-turns which appears as the second cause. In the commutating-pole machine the restrictive effect from sparking is largely removed, and the heating limit becomes the more important. But with or without commutating poles, even if the machine operates sparklessly, Figs. 327 and 329 will have shown that under the trailing pole-tip the flux density is greatly above the average. Hence, as a coil passes it, the E.M.F. which it generates is proportionately high, and the voltage between the sectors that form its ends will greatly exceed the average voltage, and may exceed the permissible limit, when the machine becomes liable to "flash over." With the increased number of ampere-conductors per unit length of armature circumference that commutating poles permit, the danger of this happening would be greater, were it not for the fact that the polar arc is usually less than in the non-commutating-pole machine

§ 17. "Flashing-over" at the commutator. When distortion of the field leads to the voltage between adjacent sectors becoming unduly high under the trailing half of the main poles, then under sudden changes of load the commutator is liable to "flash over"; arcs leap across from sector to sector until they span from a + to a − brush arm, or pass almost directly across between brush arms or to any bare metal parts in either case practically short-circuiting the armature and probably damaging both commutator and brush-gear. The magnetic leakage field blows the arcs outwards along the brush arms until they are ruptured with explosive violence. The cause is no doubt largely due, not so much to the actual distortion and the volts locally generated thereby under steady load, but to the sudden return to a more symmetrical distribution of the field when the armature load is suddenly reduced, or to distortion in the opposite direction when the load is reversed as in the operation of a large rolling-mill motor. The very rapid swing-back of the field then generates instantaneously a high voltage in an armature coil.

The exact combination of circumstances required to cause "flashing-over" can, however, hardly be said to be completely known. Although undoubtedly assisted by a high average or maximum voltage between sectors, the possible amount of the increased voltage at parts of the commutator due to the armature field varying under a sudden change of a heavy load, though often assigned as the sole cause, would not appear to be sufficient to start

the arc. The actual origin may perhaps be traced to heavy sparking at the brushes and the formation of small arcs between neighbouring sectors as they leave the brush tip, together with ionization of the air near the brush-tips as a contributory cause. Carbon particles embedded in the mica surface would assist the arcs to survive until the sectors arrive under the main field, where the arcs are maintained and magnified by the generated voltage. A chain of arcs might conceivably thus arise, each one as it forms partially short-circuiting an armature section and helping to maintain the disturbance of the field.[1] It is, however, more probable that the sequence of events is in many cases as follows: Sparking under heavy short-circuit first generates an appreciable quantity of conducting copper vapour, the circuit-breaker opens the external short-circuit, and then the above-mentioned momentary rise of the voltage induced per coil starts the arc through the vapour.

Apart from the use of a fan and air-blast along the commutator surface to drive the conducting vapour or air away, a method which has been employed by the Westinghouse Co., U.S.A., for the prevention of flashing-over in high-voltage continuous-current dynamos is to connect to the armature winding three slip-rings: a very high-speed switch, when tripped by a heavy overload, short-circuits these, and thereby the voltage at the brushes is reduced almost to zero so quickly that the original over-load has not time to cause any serious flashing, and there is no flashing-over.[2]

But to whatever extent field distortion is the originating or the assisting cause, the chance of "flashing over" is greatly minimized by the employment of a compensating field winding which annuls the armature distorting effect, as will be explained in § 19.

§ 18. Limiting number of ampere-conductors per pole.—The second cause limiting the value ac therefore falls within the present chapter as due to armature reaction causing distortion of the field to an amount dependent on the value of $ac \,.\, \beta Y$ or the ampere-conductors under the polar arc. Whether the distortion causes the local voltage between sectors to reach a dangerous amount will, of course, depend on the initial average value, which may be so low that no danger need be feared.

As a practical limit to the maximum volts per sector may be

[1] *Cp.* W. W. Firth, "Flashing-over in Commutator Machines," *Journ. I.E.E.*, Vol. 48, p. 858, and the discussion which followed this paper. The phenomenon has been mostly studied in connexion with rotary converters, which are peculiarly susceptible to it: *see* F. P. Whitaker, "Rotary Converters," *Journ. I.E.E.*, Vol. 60, p. 501, with the following discussion, pp. 512 and 835; also *Trans. Amer. I.E.E.*, Vol. 39, Part I, pp. 617 (J. J. Linebaugh) and 631 (M. W. Smith); *Journ. Amer. I.E.E.*, Vol. 41, p. 174 (E. B. Shand).

[2] *Journ. I.E.E.*, Vol., 60, p. 514; and *Trans. Amer. I.E.E.*, Vol. 39, Part I, p. 648.

given 40 volts,[1] and assuming the increase of volts under the trailing pole-edge to be 33 per cent., the average volts per sector are limited to a maximum of 30, or if the increase is 50 per cent., to a maximum of 20.[2] In the cases shown in Figs. 327 and 329, the increase of the volts at the trailing pole-tip will be not less than 60 per cent. of the average volts per sector. Yet above the normal value under a pole-face without distortion, the increase in the two cases is not more than 25 and 15 per cent. respectively.

Identifying $AT_r - AT_{cr2} + h$ with the normal $AT_g + AT_t$ calculated without distortion, the density at the trailing tip of a main pole is approximately when the shift d is neglected

$$B_g'' = 1\cdot257 \frac{\left\{AT_g + AT_t + \frac{JZ}{2p}\left(\frac{\varphi_e}{360} - \frac{\lambda_e}{180}\right)\right\}}{Kl_g + \mathfrak{R}_t''}$$

where ϕ_e is the polar angle in electrical degrees. Since $\phi_e = \beta \times 180°$,

$$B_g'' = 1\cdot257 \frac{\left\{AT_g + AT_t + \frac{JZ}{2p}\left(\frac{\beta}{2} - \frac{\lambda_e}{180}\right)\right\}}{Kl_g + \mathfrak{R}_t''}$$

$$= 1\cdot257 \frac{\{AT_g + AT_t + ac\,(\beta Y/2 - c_\lambda)\}}{Kl_g + \mathfrak{R}_t''} \qquad (174)$$

while the normal $B_g = 1\cdot257 \dfrac{AT_g + AT_t}{Kl_g + \mathfrak{R}_t}$.

The ratio of the two is therefore

$$\frac{B_g''}{B_g} = \frac{\{AT_g + AT_t + ac\,(\beta Y/2 - c_\lambda)\}\,(Kl_g + \mathfrak{R}_t)}{(AT_g + AT_t)\,(Kl_g + \mathfrak{R}_t'')}$$

If this is not to exceed a value k, we have

$$\frac{2\,(AT_g + AT_t)}{ac\,.\,Y\,(\beta - 2c_\lambda/Y)} \geq \frac{1}{(k-1) + k\,\dfrac{\mathfrak{R}_t'' - \mathfrak{R}_t}{Kl_g + \mathfrak{R}_t}}$$

The fraction $\dfrac{\mathfrak{R}_t'' - \mathfrak{R}_t}{Kl_g + \mathfrak{R}_t}$ is usually about 0·35 to 0·4, so that if k is not to exceed 1·25, the right-hand side becomes 1·45 to 1·33. If λ_e be reckoned as 15 electrical degrees in the non-commutating-pole machine, (or $c_\lambda = 0{\cdot}0835Y$), and as zero in the commutating-pole machine,

$$\frac{2\,(AT_g + AT_t)}{ac\,.\,Y\,(\beta - 0\cdot167)} \geq 1\cdot61 \text{ to } 1\cdot47.$$

[1] Dr. R. Pohl, "The Development of Continuous-Current Turbo-Generators," *Journ. I.E.E.*, Vol. 40, p. 239, and the discussion thereon.

[2] The great desirability of lower values not exceeding 20 volts has led to the devices for subdividing each armature loop into two sections by auxiliary commutator connexions to be mentioned in Chapter XX, § 35.

Approximately,[1] therefore,

$$\frac{AT_g + AT_t}{\frac{JZ}{2p} \times \frac{\beta}{2}} = \frac{AT_g + AT_t}{ac\,.\,\beta Y/2}$$

$$= \frac{\text{field ampere-turns for single air-gap and tooth}}{\text{armature ampere-turns acting at one pole-tip}}$$

or alternatively expressed

$$\frac{X_g' + X_t}{ac\,.\,\beta Y} = \frac{\text{field } AT \text{ for double air-gap and teeth}}{\text{armature ampere-conductors under pole-face}}$$

must be $= 1{\cdot}25$ to $1{\cdot}5$ . . . . . (175)

and if $\beta = 0{\cdot}735$ in the non-commutating-pole machine and $0{\cdot}66$ with commutating poles

$$\frac{X_g + X_t}{ac\,.\,Y} = \frac{\text{field } AT \text{ for double air-gap and teeth}}{\text{armature ampere-conductors per pole}} = 0{\cdot}92 \text{ to } 1{\cdot}1$$

. . . (176)

While the above quantitative figures admit of considerable latitude in their application according to the circumstances of the case, they serve at least clearly to show that the higher the normal value of B_g or of $X_g + X_t$, the greater may be the armature ampere-conductors per pole-pitch. Further, their derivation will have indicated that the increase in the saturation of the teeth under the trailing pole edge plays an appreciable part in limiting distortion.

§ 19. Advantage of the multipolar machine. The advantage of a large number of poles from the present point of view is evident. For the same value of JZ, the value of the ampere-conductors per pole $JZ/2p = ac\,.\,Y$ is reduced in proportion to an increase in the number of poles. There is, therefore, theoretically no difficulty in limiting distortion of the field to a reasonable amount.

Next, as already shown (Chap. XIII, § 39), there is a certain maximum value towards which B_g tends and, with a more or less constant length of air-gap, a certain maximum value for $AT_g + AT_t$ or $X_g + X_t$ which practical considerations fix. Thence it

[1] The armature ampere-turns per pole are $\frac{1}{2} \times \frac{JZ}{2p} = \frac{1}{2} ac\,.\,Y$, but the cross-magnetizing ampere-turns as acting on a pole between its edges are $\beta \frac{JZ}{2p} = ac\,.\,\beta Y$, or at their maximum across a whole pole-pitch are $ac\,Y$. If expended in equal proportions over the two air-gaps, the cross ampere-turns acting on one pole-tip are therefore $\frac{\beta}{2} \times \frac{JZ}{2p} = \frac{1}{2}\,ac\,\beta Y$, or at the extreme limit $= \frac{1}{2}\,ac\,.\,Y$, the armature ampere-turns per pole. Since there is a liability for confusion between the cross ampere-turns "acting on a pole" and "per pole," it is best to speak solely of ampere-conductors instead of ampere-turns, which avoids all ambiguity.

follows that there is a certain maximum value which $ac \cdot Y$ must not exceed even in large machines, and such values in machines without and with commutating poles are respectively $ac \cdot Y = 15{,}000$ or 18,000 ampere-conductors per pole-pitch. Lastly, economy in manufacture dictates that ac should be pushed to its possible maximum until limited by heating and sparking. Finally, therefore, making $ac = 900$ per inch, a limiting value for the pole-pitch is reached, viz.

$$Y = 16{\cdot}7 \text{ to } 20 \text{ inches}$$

as already indicated in Chapter XV, § 17, and usually it is less.

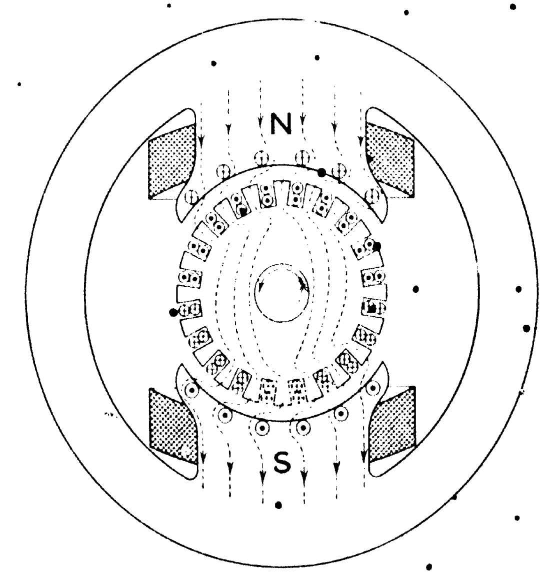


FIG. 332.—The principle of compensating field winding.

§ 20. Compensating field winding.—In order to prevent the displacement of the field by the action of the armature current on load, and so to lessen the likelihood of "flashing-over" under difficult conditions such as occur in the design of turbo-dynamos, the cross ampere-turns of the armature must be neutralized. This is effected by an equal number of *compensating ampere-turns* carrying current in the opposite direction to the turns of the armature and distributed over the pole-face. So that the neutralizing action of the compensating turns may be proportioned to the armature current, they are in series with the main circuit; they are wound

either in holes pierced through the poles close to their bored faces as first proposed by Professor Ryan, or in slots uniformly disposed over the pole-face. The principle is indicated in Fig. 332, the compensating conductors in the 2-pole machine being joined up into a coil enclosing the armature. The ordinary field winding of the poles may be retained at the back of the laminated pole-shoes which carry the compensating winding. In the multipolar machine the arrangement becomes simpler, since with four or more poles the compensating coils become flatter, and their ends can be more conveniently bent to clear the armature. The two sets of coils, exciting and compensating, are then, so to speak, in quadrature, and the winding bears a

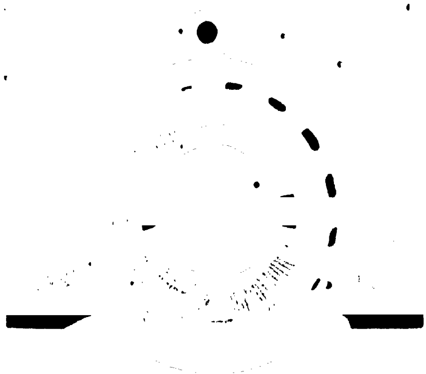


FIG. 333. Field frame of 300-kilowatt turbo dynamo with compensating bar-winding in place.
(Allgemeine Elektricitäts Gesellschaft.)

resemblance to the winding of the stator of a quarter-phase alternator.

Fig. 333 shows the compensating coils before the four large shunt coils embracing the poles are in place for a continuous-current 300-kilowatt turbo-dynamo at 230 volts, built by the Allgemeine Elektricitäts-Gesellschaft; the low voltage causes a bar-winding to be adopted.

Fig. 334 shows the field-magnet of a 600-h.p. 110 revs. per min. compensated motor built by the British Thomson-Houston Co., Ltd., also with a bar compensating winding; the field coils are here between the yoke and the compensating coils, and there are also commutating poles. In some compensated machines the field winding proper is itself distributed among the same slots with the compensating coils but in quadrature with them. The compensating and field windings can also be combined with local

commutating coils surrounding reversing teeth in the centre of the gaps between the several fields.[1]

An interesting question arises as to how the armature in a compensated machine is subjected to torque when the displacement of the field is prevented. The answer is that, as shown by Prof. Gutton,[2] the lines of flux thread round the conductors of the compensating winding and of the armature in opposite directions so

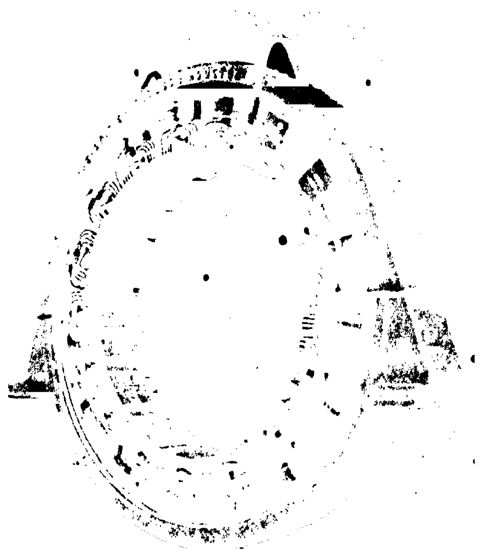


FIG. 334.—Field magnet of 600-h.p. 110 r.p.m. compensated motor. (The British Thomson-Houston Co., Ltd.)

that, although inclined to the armature surface at much the same angle as in the machine without compensating winding, there is no displacement as a whole and the neutral axis retains the same position as at no-load, as indicated in Fig. 332.

Since the compensating ampere-conductors are usually only disposed on the pole-face, it follows that if they compensate for the maximum cross ampere-turns with diametric armature winding

[1] For further illustrations of compensated machines, see Chap. XXII, Fig. 421, and W. Hoult, "Direct Current Turbo-Generators," *Journ. I.E.E.*, Vol. 40, p. 625; G. Stoney and A. H. Law, "High Speed Electrical Machinery," *Journ. I.E.E.*, Vol. 41, pp. 289-295.

[2] Quoted by A. Mauduit, *Recherches Expérimentales et Théoriques sur La Commutation*, p. 64. (Dunod et Pinat, Paris.)

and no angle of lead, the actual ampere-turns under the poles must be over-compensated; with chord winding of such degree that the short-circuited wires are brought to the pole-tips, there need be no over-compensation, but of course such a winding is unsuitable for commutating poles, which are usually combined with compensating winding.

The following particulars of an 8-pole compensated dynamo, 500 volts, 2500 amperes, 750 revs. per min., are given by J. Rezelman. The armature was 43·4 ins. diam. by 18·1 ins. long with 168 slots, each containing 2 bars. The winding being a simple lap, $ac = 770$

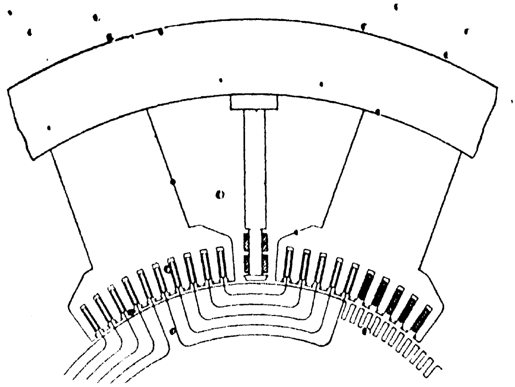


FIG. 335. Compensating field winding and commutating pole.

per inch. The main air-gap was 0·256 in. The polar arc was 13 ins. or 76·5 per cent. of the 17 ins. of the pole-pitch. The main poles were laminated and on each were 10 slots, each carrying one compensating bar (Fig. 335). Four coils of 5 turns were in series, and two such sets in parallel, so that each compensating bar carried 2500/2 = 1250 amperes. The ampere-conductors of the compensating winding were, therefore, $ac_c = 960$ per inch. The armature bars under a main pole-face were, therefore, over-compensated, and the compensating winding balanced 95·5 per cent. of the total armature ampere-conductors per pole $\frac{JZ}{2p} = \frac{312 \times 336}{8} = 13{,}100$, *i.e.* 12,500 ampere-turns of compensating winding per pole = 95·5 per cent. of 13,100. The solid commutating poles, therefore, were wound with two turns only, each carrying 1250 amperes. The commutating air-gap was 0·394 in.

Compensating winding is especially valuable in the case of turbo-dynamos where the high speed and small number of sectors per pole lead to high values of the average volts per sector.

Messrs. Parsons & Co. have partly on the same account made use in turbo-dynamos of a special arrangement of compensating coil which also produces a commutating field with almost a complete air-path; not only can a perfect balance up to a heavy overload current be thereby secured, owing to the absence of iron saturation, but also the absence of hysteresis and eddy-currents enables the quickest changes of current to be simultaneously followed.[1]

§ 21. Effect of eddy-currents on field ampere-turns.—In conclusion it must be added that any eddy-currents in the armature exert a demagnetizing effect on the field analogous to that of the back ampere-turns, and inaccurate commutation of the short-circuited loops, causing large parasitic currents therein, may also have an appreciable effect on the ampere-turns required on the field. If open wide slots in the toothed armature set up eddy-currents in the pole-pieces, these again must be counterbalanced and may call for a small increase to be added to AT_b in reckoning out the field winding.

The Homopolar Dynamo

§ 22. Armature reaction in homopolars.—A single bar mounted axially on the surface of a cylindrical rotor and carrying current would by its magneto-motive force cause a reduction in the density of the resultant flux ahead of itself and an increased density behind itself. This unequal distribution of the flux would reach into the mass of the rotor and stator, and travelling over the face of the latter would give rise to losses by hysteresis and eddy-currents in the iron. This effect would, however, disappear with a complete copper cylinder, which by itself would be surrounded by a flux concentric with the axis of the cylinder. The M.M.F. of such a cylinder is at all points at right angles to the original radial inducing flux and to its M.M.F., so that we have what may be termed a "rectangular intersection" of M.M.F.'s. Two fluxes at right angles to each other are physically impossible, and the actual result in nature is that the flux is twisted round, causing it to follow a *longer* and perhaps *more saturated* path; if at_1 and at_2 are the two M.M.F.'s per cm. length of the two paths, the reluctivity ξ_f is dependent on $1{\cdot}257\sqrt{at_1^2 + at_2^2}$, and the latter divided by the former yields the resultant flux density B_f. It is, however, perfectly legitimate mentally to resolve the resultant flux density again into two rectangular components, or in our case into a radial and a concentric component. When so resolved, the component fluxes must be regarded as carrying with them the permeability of the medium in its *final* state; that is, the two component densities are given by the relation $B_f = \sqrt{B_{1f}^2 + B_{2f}^2}$ where B_{1f} and B_{2f} are the densities that would be produced by the two M.M.F.'s acting separately on the medium in its *final* state, or $1{\cdot}257\, at_1/\xi_f$ and $1{\cdot}257\, at_2/\xi_f$.[2]

In the present case, everything being symmetrical in circles concentric with the shaft in machines of the axial type, the resultant flux is given a slant towards the armature instead of being truly radial to it (Fig. 338). Whether in air or iron, not only is the length of path increased, but owing to the slant of the lines, the density at right angles to the flow is increased, as

[1] G. Stoney and A. H. Law, "High-speed Electrical Machinery," *Journ. I.E.E.*, Vol. 41, p. 291.

[2] *Cp.* J. E. Noeggerath, "Acyclic (Homopolar) Dynamos," *Trans. Amer. I.E.E.*, Vol. 24, p. 8. A different view has, however, since been expressed by him, *Trans. Amer. I.E.E.*, Vol. 31, Part II, p. 1838.

shown by comparing the sections shown black in Fig. 337. In air, with its constant reluctivity, this does not imply any reduction of the total flux entering the armature or of the induced E.M.F., since the increased M.M.F. which is required is exactly that which is supplied by the M.M.F. of the rotor cylinder. But in iron the increased saturation and lessened permeability will, according to the view expressed above, lead to a value B_1 for the radial component which is less than the value of the original radial density, and the induced E.M.F. is accordingly decreased. So long as at_1 has any value the resultant flux will never become concentric, but will always slope into the armature. The gradual twisting round of the initial flux (even if its value decreases) and its increasing density at right angles to its path as the armature current is increased exactly corresponds to the gradual growth of a component concentric flux from zero up to a high value proportional to at_2. It would only become truly concentric if at_2 were itself to become infinite.

In order, therefore, to check the reduction of E.M.F. which occurs under load and which has above been attributed to armature reaction and to saturation of the pole faces, it has been the practice to compensate the M.M.F. of the armature. This would be secured by a similar copper cylinder on the face of the stator connected in series with the armature and carrying the same

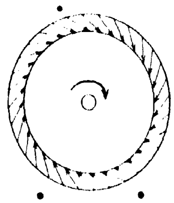


FIG. 336.—Path of flux in air-gap of homopolar dynamo under load.

FIG. 337.—Diagram to illustrate increased resultant density in air-gap.

current but in the opposite direction, or with several inducing sectors by corresponding flat sectors on the face of the stator.

But, as already described (Chapter VII, § 6), in practice it is more convenient to depart somewhat further from the ideal case, and instead of sectors to employ a number of equally spaced bars sunk into the surface both of the stator and the rotor. If these are sufficiently numerous, an approach is made to the simple concentric shell and its purely concentric M.M.F. Owing to the local separation of the bars some of the unequal distribution of the flux which accompanies the single bar still persists with consequent loss by hysteresis and eddy currents, and with some reduction of the E.M.F. owing to unequal saturation of the iron. Its amount is not, however, sufficient to render it necessary to laminate the rotor core.

§ 23. End-ring reaction in homopolars.—The reaction of the armature current has, however, not yet been exhausted, and about the effect now to be described there is no question. With a single bar and a single brush pressing on the collecting ring, except when the connexion of the bar to the ring is exactly underneath the brush contact surface, the current divides into two portions encircling the ring in opposite directions and reaching equality when the bar connexion is diametrically opposite to the brush. The net result is therefore an alternating M.M.F. embracing the main magnetic circuit. In the axial type with double magnetic circuit, this acts at each end of the armature on its own magnetic circuit, causing hysteresis and eddy currents in any solid metal.

Such an unequal division of the current in each collector ring may very largely be obviated by dividing the single bar into four or more conductors in parallel which are connected at equidistant points round the circle of the collector ring, and this method has in practice been followed.

To balance the ring reaction completely, each end must separately be rendered neutral, and the aim must be to reduce to zero the total algebraic sum of the currents in any radial section through the rings at every moment when their directions, clockwise or counter-clockwise, are duly taken into account. This result cannot be entirely secured with a finite number of rings, but is approximately attained if with x bars connected consecutively to adjacent rings, say, in a *clockwise* direction, the brushes are set consecutively at angles of $360/x$ degrees apart, but in a *counter-clockwise* direction. Thus while the connexions of the bars to the rings describe a screw thread round the shaft in one direction, the brushes describe a screw thread round the shaft in the opposite direction (Fig. 338).[1] The objection then arises that with x

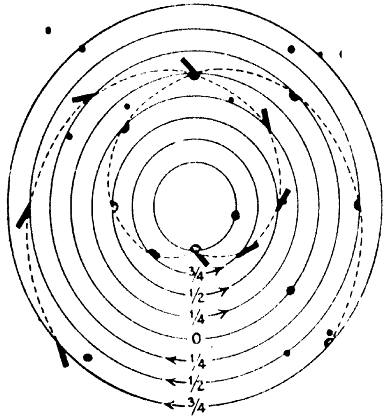


FIG. 338. Arrangement for reducing end ring reaction.

bars there are x points of collection. Dr. J. E. Noeggerath[2] has therefore developed the method further, by increasing the pitch of the brushes so that they describe more than one spiral round the shaft. Thus with 8 rings at one end, the brushes are made to describe 2 threads round the shaft, or with 12 rings 4 threads, and thereby the points of collection are brought into 4 groups, with the consequent advantage that only four openings are required in the magnet frame to give access to the brushes for inspection and attention.

If the connexions of the rotor bars are not taken straight into the rings in lines parallel to the axis of the shaft, but slant across the armature so that the bars and the connexion points have an angular displacement, the currents partially or totally encircle the main magnetic circuit, and, with the right direction of displacement to increase the flux, a compounding effect can thereby be produced. The same effect is similarly produced if the leads connecting the brushes to the compensating bars in the stator-frame are carried partially round the shaft instead of in straight lines, with the additional advantage that the compounding effect can be adjusted in amount by shifting the brushes more or less round the rings.

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[1] The proportion of the total current carried is marked for the lower paths from bars to brushes, and is the inverse of the relative lengths of the paths.

[2] *Trans. Amer. I.E.E.*, Vol. 24, p.

CHAPTER XX

COMMUTATION AND SPARKING AT THE BRUSHES

§ 1. Sparking at the brushes.—All who have had practical experience of the working of dynamos giving a continuous current will know that in most cases the brushes by which the current is collected are so mounted as to permit of their being shifted round the cylindrical surface of the commutator at least through some small angle. They will also be aware that if the position of the tips of the brushes, as they press on the commutator, be not properly adjusted, the result will be *sparking at the brushes*. The waste of energy involved in such sparking is but small, but its presence always tends to shorten the life of the commutator and brushes, so that its suppression, so far as possible, is in every way desirable. The presence or possibility of sparking at the brushes is, in truth, the peculiar bane of machines with commutators, as contrasted with alternators; currents in the latter may require to be collected by brushes or rubbing contacts such as have been shown in many previous diagrams, but the nicety of adjustment required by the brushes of hetero-polar continuous-current machines, if sparking is to be minimized or entirely avoided, is a disagreeable characteristic of their whole class, and is entirely due to the presence of the commutator as opposed to the simple collecting rings of the alternator.

Unless an approximately correct adjustment of the brush position can be obtained, a row of sparks will appear, leaping across between the moving sectors and the stationary brushes. These sparks, which are virtually small arcs between sector-edge and brush-edge, may be small, bluish-white in colour, and comparatively harmless; or if the inexactness of the adjustment be considerable, they may be of a reddish colour and extremely violent. But in either case, if allowed to continue, they will sooner or later pit the surface of the commutator sectors, destroy their smoothness and evenness, and heat the brushes. Once started, the effects are cumulative, and the mischief grows apace: the commutator becomes untrue and worn into deep and rugged grooves; increased sparking is caused by the "jumping" of the brushes as they pass from sector to sector, and perhaps the tips of the brushes become partially fused; thus the commutator is gradually eaten away until its state is past all remedy. To check the evil it is necessary to trim the brush-tips continually and "true up" the surface of the commutator by turning it in a lathe or by grinding it with an emery wheel, and these often-repeated processes result in a greatly reduced life of both commutator and brushes.

§ 2. Means for adjustment of the brush position.—Sparking in closed-circuit armatures is therefore an unmitigated evil, to be overcome almost at any cost, and the possibility of moving the brushes so that the position of their tips may be adjusted to suit the normal working load is usually a necessity. It should further be possible to do this easily and steadily while the machine is running, and without in any way interrupting the passage of the current. In small multipolar dynamos this is effected by mounting the brushes on insulated spindles projecting horizontally from a cast-iron star frame or "rocker." This latter is made in two pieces, bolted

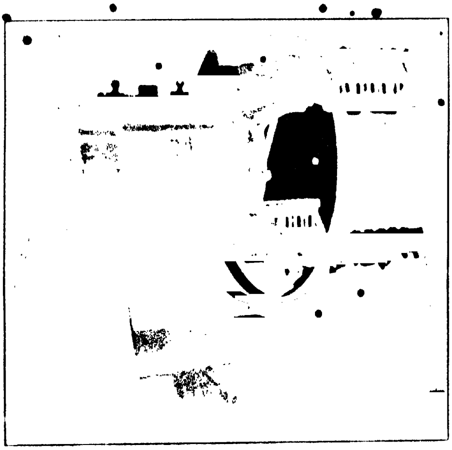


FIG. 339. Four-pole brush gear.

together and fitting into a groove turned in the surface of the plummer-block next to the commutator, so as to admit of its being swung through an arc or rotated by a hand-wheel actuating a link connected to the rocker (Fig. 339) or by a worm gearing into worm-wheel teeth on a sector fastened to the rocker (Fig. 340). Rigidly fastened to each arm of the star frame is a gun-metal spindle, entirely insulated from the iron by means of micanite or bakelite plates and ferrules, and the brush boxes which hold the brushes are threaded on the spindles and firmly fixed in place. Fig. 339 shows the rocker mounted in position on the bearing and carrying four sets of carbon brushes, opposite sets being connected together by semicircular copper rings, to which are attached the main armature brush leads. Each brush box is fitted with a spring, by which the brushes are kept pressed down on the commutator.

In large multipolar machines a cast-iron ring is usually mounted

on the face of the magnet yoke-ring, and rotated within the circular groove which forms its seating by similar mechanical gearing. From this ring project as many arms as there are poles, and each

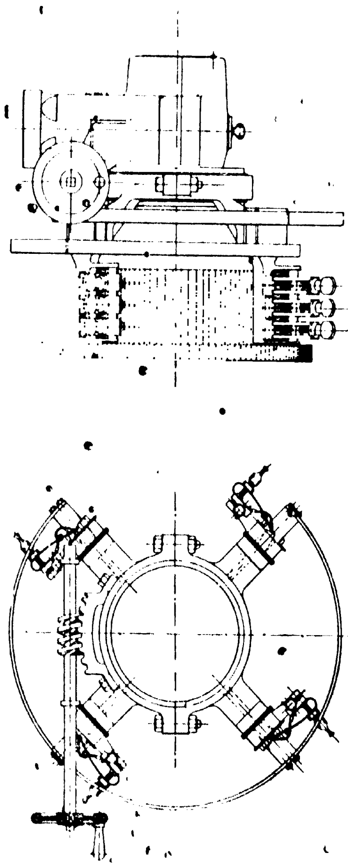


FIG. 340.—Four-pole brush gear.

arm either forms the frame carrying a set of brushes or has attached to it a brush spindle (*cp.* Figs. 341 and 342).

§ 3. The process of short-circuiting a section.—The exact nature of the process by which the current is commuted in a section of a closed-circuit armature winding during the time when it is *short-circuited* has hitherto only been generally described. It has been shown (Chapter X, §§ 2, 6) that the brushes must be placed so

that they sum up the E.M.F.'s induced in the groups of sections forming the two or more parallel paths into which the winding is divided, and at the same time so that they short-circuit each separate section when it is passing approximately through the interpolar gap ahead of the neutral line in the non-commutating-pole machine, or under a commutating pole, so that in either case there is a small reversing E.M.F. generated in it. Hence, if the field be bipolar, their position will be at opposite ends of a diameter, corresponding roughly with

FIG. 341.—Ten-pole brush gear mounted on magnet frame. (Messrs. W. H. Allen, Sons & Co., Ltd.)

a position of the short-circuited coils on a line of symmetry at right angles to the general direction of the field. This preliminary description now requires to be further amplified.

Considering any one section of the armature winding (whether a single loop or a coil of many loops), terminated by connexion to a commutator sector at either end, let that sector which first enters under the edge of the stationary brush be termed the "leading" sector of the coil, just as that edge or corner of a pole-piece under which a coil first enters after passing through the gap between two pole-pieces has already been called the "leading" edge, these being opposed respectively to the "trailing" sector and the "trailing" edge or corner.

Usually the width of contact of the brushes on the circumference of the commutator is greater than the width of a sector, so that two or more sections are short-circuited at one time at each set of

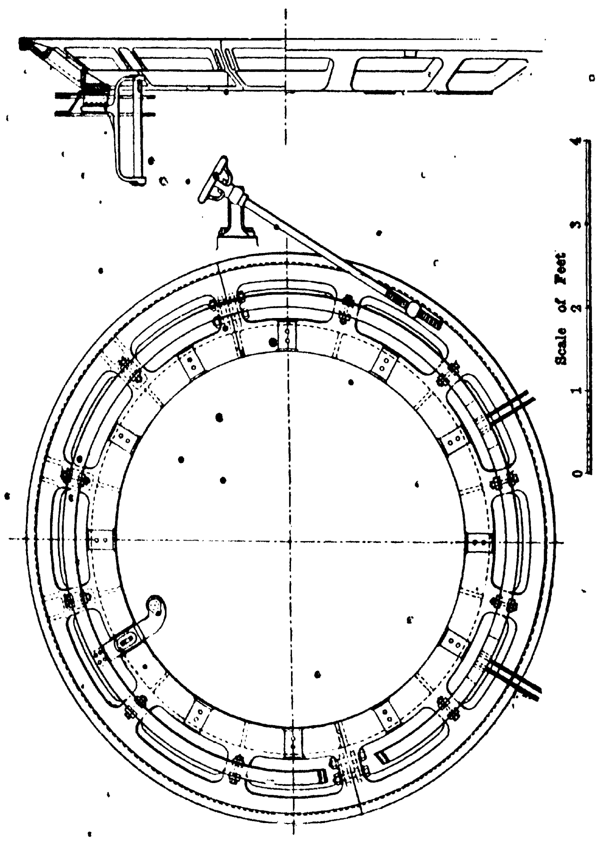


FIG. 342.—Twelve-pole brush carrier.

brushes. At any time during short-circuit, let i be the instantaneous value of the current in a short-circuited coil, and let J be the full current then flowing in any one branch of the armature winding, $= I_a/q = I_a/2a$, according to the number a of pairs of parallel paths in the armature, both i and J being reckoned positive when in the direction of the current before commutation. Then i will progressively take various values as indicated by the upper row of

symbols in Fig. 343, viz. i'', i, i', according to the position of the coil or the time which has elapsed since short-circuit began. In Fig. 343 section 3-4 is just going to be short-circuited as sector 4 enters under the brush, and at this initial moment when $t = 0$, the starting condition is always $i = +J$ as shown. The current through a commutator connector or a sector, reckoned positive when towards the brush, is always equal to the difference between the currents in the coil which immediately follows and in that which immediately precedes it.

The period of short-circuit being, in the ordinary case of a simple lap-wound drum, the time that elapses between entrance of the trailing sector under the brush and emergence of the leading sector

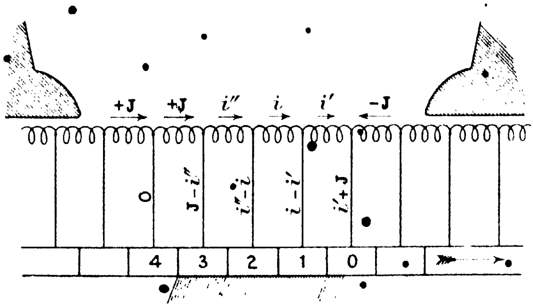


FIG. 343. Commutation of current.

on the other side, is directly proportional to the thickness of the brushes less one strip of mica, and inversely proportional to the peripheral speed of the commutator, *i.e.* it is the time taken by the edge of a mica strip to pass the brush; or in seconds,

$$T = \frac{b_1 - b_m}{v_c} = \frac{(b_1 - b_m) \times 60}{\pi D_c N} \qquad . \quad . \quad . \quad . \quad (177)$$

where b_1 = the width of the brush contact in the direction of rotation, b_m = the thickness of a mica strip, D_c = the diameter of the commutator, all expressed in the same units, and N = the number of revolutions per minute. If b = the pitch of the sectors on the surface of the commutator, *i.e.* the width of one sector and one mica strip in the same unit as that in which $b_1 - b_m$ is expressed, and C = the number of sectors, $bC = \pi D_c$, so that we also have $T = \frac{b_1 - b_m}{b} \times \frac{60}{CN}$. It is usually but a small fraction of a second, averaging from [illegible] to $\frac{1}{1000}$th; *e.g.* with carbon brushes [illegible] thick, set so as to give practically the same width of contact (after deducting

the thickness of a mica strip), on the circumference of a commutator $7\frac{1}{2}''$ in diameter and running at 900 revs. per minute,

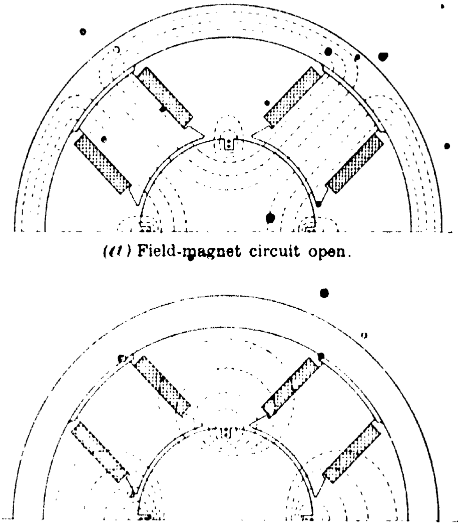
$$T = \frac{0{\cdot}075 \times 60}{3{\cdot}14 \times 7\frac{1}{2} \times 900} = 0{\cdot}00212$$ second, or if v_c = the peripheral speed of the commutator in feet per minute, *i.e.* $= \frac{\pi D_c N}{12}$, where D_c is the diameter of commutator in inches, and $b_1 - b_m$ is in inches, $T = 5 \cdot \frac{b_1 - b_m}{v_c}$ seconds. During this brief time the current i must sink from $+J$ to zero and rise again in the reverse direction, *i.e.* with changed sign. If the reversed current be raised to the same value at the end of short-circuit as it had at the beginning, or $i = -J$, the opening of the short-circuit will find the coil carrying the exact current flowing in the coils of the branch of the armature which it is to join. Thus, in Fig. 343, when the sector marked 0 has moved a little further and is at the point of emerging from under the brush-tip, the current i' in the coil should for perfect commutation be exactly $-J$. The current in the leading commutator connector and sector will then again be $-J-(-J) = 0$, the whole of the current which they have been carrying during short-circuit having been withdrawn.

Thus, *e.g.* with 50 sections per armature path and an armature winding over the resistance of which 5 volts are expended, a net effective voltage of $\frac{1}{10}$th of a volt must be acting on the short-circuited coil as it leaves the brush, in order to correspond with the passage through it of the normal current. The commutation will then be effected without any violent change, and consequently without any sparking. It is evident that the transference of a section of the armature from one path of the winding into another on the leading side of the brush can only be sparkless if the original current in the section is stopped, reversed, and further is reversed to exactly the same value in the opposite direction within the period of short-circuit.

§ 4. Apparent inductance of short-circuited section.—Now the current-turns of the short-circuited coil, since they surround a portion of the magnetic circuit, react on the field system, so that the number or distribution of the lines of the resultant field is different from what it would be if the short-circuited coils were absent. For the consideration of the problem of commutation, then, the complete resultant field must be mentally resolved into (1) the flux and its distribution as due to all the armature ampere-turns with the exception of the short-circuited coils which are, at any time under the brushes, and (2) the flux and its distribution as due to the short-circuited coils when imagined to be isolated from the rest of the armature winding. System (1) and system (2) each have their own appropriate amount of magnetic energy stored

in their fields, as components of the resultant field, and the value of this energy in the case of system (2) can be more or less accurately determined from the approximate distribution assigned to its own fluxes with which it is separately credited (for the latter see Fig. 344 (*a*) and Fig. 387 (*c*)).

Since the short-circuited coils are situated at or near to the line of symmetry between the poles, most of the lines of the supposed flux linked with them cross the double air-gap and enter or leave the iron pole-faces, and thence pass onwards through the field-



(*a*) Field-magnet circuit open.

(*b*) Field-magnet circuit closed.

FIG. 344. Self-induced flux due to short-circuited sections of armature.

magnet bobbins to complete their circuit. The field-magnet bobbins are practically short-circuited by the armature winding, and as the double air-gap is the chief item in the magnetic reluctance, the system is so far roughly equivalent to a transformer with an air-core and a short-circuited secondary. Under these circumstances, for rapid changes of current comparable with the frequency of commutation in a dynamo, the effect of the secondary, if of negligible resistance, nearly counterbalances the action of a short-circuited coil which is the primary, and the latter apparently has very little self-inductance. Any change in the total flux linked with the exciting coils and the armature as a whole is, in fact, damped by the great mutual inductance existing between a short-circuited coil regarded as a primary and the field coils as a secondary.

Only a few lines from the short-circuited sections still continue to pass *viâ* the field-magnet, sufficient, that is, to provide the E.M.F. which causes the damping variation in the exciting current. This damping action is especially marked if the exciting coils are wound on metal formers or cases, which in themselves constitute a closed secondary of one turn.

There is, however, a certain smaller number of lines in the component field of system (2) which do not pass through the field coils, but which circle round through the air between the pole-tips, or pass immediately across the tops of the slots in which the short-circuited coils lie in the toothed armature. Further, when the field-magnet winding is closed on the armature and the current in the short-circuited coils varies, some of the lines which would otherwise pass through the main magnetic circuit are repelled into the tips of the poles, so that they avoid the exciting coils of the magnet. The interpolar flux circling round the short-circuited coil-sides is thus increased (Fig. 344 (*b*)), although not to any great extent. In virtue of the total of these lines a certain " apparent " self-inductance may be attributed to a short-circuited coil, and the value of this is very much less than its true self-inductance. When tested with alternating current, the apparent inductance of an armature section with the field-magnet circuit closed on itself may be less than half the true inductance with the field-magnet circuit open.

The question is, however, still more complex, and demands further analysis. Taking a single section of a drum armature, *i.e.* a coil with two sides, when undergoing short-circuit, there are always other coil-sides adjacent to it which are also short-circuited either at the same brush or at adjacent brushes on either side of it. In the case of a machine in which the number of commutator sectors per pole or $C/2p$ is a whole number, whatever is taking place at one set of brushes is also taking place at the adjacent brushes, so that close to the sides of the considered coil A, in which there is a varying current i, there must be the sides of two other coils B and B_x short-circuited at the adjacent brushes and each carrying an identical current, since they are at precisely the same stage in the process of commutation (Fig. 345). In addition, therefore, to the E.M.F. from the apparent self-inductance of coil A, or $-\mathcal{L} \cdot di/dt$, there is also present in A the E.M.F. from its mutual inductance with coil-sides B and B_x, or $-\mathcal{M}\, di/dt$. Next, if as usual each brush set covers more than the width of one section, there are other sections $A_{,}$, $A_{,,}$, etc., lying alongside A which are undergoing short-circuit at the same brush, but which have reached a different stage of commutation, and which are therefore carrying other currents $i_{,}$, $i_{,,}$, etc. Lastly, there are other coils $B_{,}$, $B_{,,}$, etc., of which one side is in close neighbourhood to coil A, and which are short-circuited at adjacent brushes on either side ; from both of these groups there

is mutual inductance $\mathcal{M}_{\mathrm{I}}$, $\mathcal{M}_{\mathrm{II}}$, etc., giving E.M.F.'s in A which may be concisely summed up as $-\Sigma\,(\mathcal{M}_1 di_1/dt)$. In this case also we need only consider the *apparent* mutual inductance, due to such lines as do not pass through the pole-faces and onwards through the yoke where the variations would be damped out by the exciting coils. Thus the total E.M.F. *of apparent self and mutual inductance* in the considered coil is

$$-(\mathcal{L}+\mathcal{M})\frac{di}{dt}-\Sigma\left(\mathcal{M}_1\cdot\frac{di_1}{dt}\right) \quad . \quad . \quad . \quad . \quad (178)$$

To illustrate the quantitative effects, if $\mathcal{L}$ be the apparent self-inductance of the one section A, the armature being in air, the addition of a second section A_1, if in the same slots and short-circuited

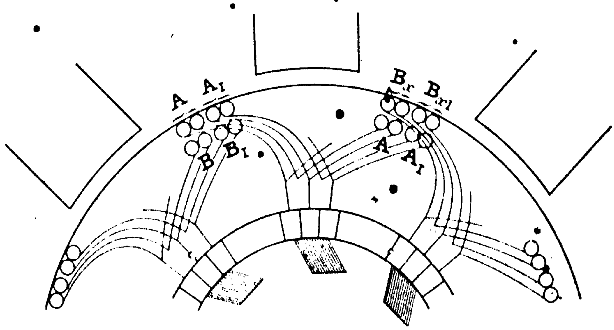


FIG. 345.—Coil-sides having mutual inductance with considered coil A.

by the *same* brush, will almost double the flux, and the apparent self and mutual inductance of A will not be far short of $2\,\mathcal{L}$. But on the other hand, when the added coil-sides are in different layers from those of A, as *e.g.* B and B_r, being parts of sections short-circuited at *adjacent* brushes, the total mutual inductance from the two coil-sides B and B_r at the bottom and top of slots with the coil-sides of section A at the top and bottom of the same slots will be about 70 per cent. of the entire self-inductance of the latter. When the added coil-sides are in slots adjacent to those of A, their position in those slots is immaterial; if then the end-connexions follow the same path, as *e.g.* when section A_1 is in slots adjacent to A and is short-circuited by the same brush, its mutual inductance will be about 33 per cent. of $\mathcal{L}$; if the end-connexions follow different paths and only the slot inductance is affected, as with B and B_r in slots adjacent to A, this will sink to about 22 per cent. The apparent self and mutual inductance of A alone in these two cases is therefore $1{\cdot}33\,\mathcal{L}$ and $1{\cdot}22\,\mathcal{L}$.

In the case of a machine in which $C/2p$ is an uneven number, M is absent, but the number of coils to be taken into account under $\Sigma(M_1 di_1/dt)$ is increased. What may be the effect of the second term in the above expression (178) entirely depends upon the sign of di_1/dt as compared with the sign of di/dt; whenever the rate of current change in any one or more of the coils A_1, B_1, etc., is opposite in sign to the rate of change in the considered coil A, then the coils of which this is true simply have a damping effect, and take up some of the energy which is being freed from coil A, or if the current has reversed render its growth more rapid. But when the rate of current change, say in coil B_1, has the same sign as di/dt, then they mutually support one another, and owing to the proximity of the coil B_1 the energy connected with the coil A and its apparent inductance are increased.

A further complication is that the apparent inductance will, strictly speaking, vary with the position of the short-circuited coil, according to whether it is in the centre of the interpolar gap or nearer to one pole-tip than to the other, and thus will change somewhat during rotation. Yet since the movement of the coil during the period of short-circuit is but small, the *apparent self and mutual inductance* of a section of the winding may approximately be regarded as constant for a given position of the brushes.[1]

But however this may be, and leaving for the present the further examination of the effect of $M_1 di_1/dt$, the coil under consideration, at the moment of arrival at the point where short-circuit begins, is possessed of a certain amount of electromagnetic energy stored in its field equal to $\frac{(S+M)I^2}{2}$. The question of securing sparkless commutation turns, then, entirely upon our ability to dissipate and to re-store this amount of energy within the brief space of time of a few hundredths of a second.

It can be dissipated in any or all of four ways—

(1) through transference of the energy by transformer inductive effect into other short-circuited coils;

(2) in heat by passage of the current over the ohmic resistance of the coil and contact resistance of the brushes, the heating being then in excess of the normal;

(3) by motor action, the short-circuited coil assisting to drive the armature whenever its current is opposed in direction to the E.M.F. induced in it by the main external field; and

(4) by sparking.

It can be re-stored in three analogous ways: (1) by transformer action, (2) through the action of the contact resistance of the brushes

[1] Since the percentage of the lines which do not pass through the field-magnet bobbins is greater when the coil is moved away from the symmetrical line, the apparent inductance increases.

applying a difference of potential to the ends of the short-circuited coil which assists the growth of the reversed current in it, and (3) by generator action, the E.M.F. induced by the reversing external field exceeding the amount lost over the ohmic resistance by the passage of the current existing at the moment.

Under ideal conditions of perfect balance between the effects of rotation and magnetic energy-change and no sparking, the resultant magnetic field would remain constant and unmoved. But even in this case the means of liberating the energy and of re-storing it must be present, so far as each short-circuited coil is considered in isolation from all the rest, when the current in it sinks from J to zero and is again raised to the same value in the reverse direction, *i.e.* to $-J$.

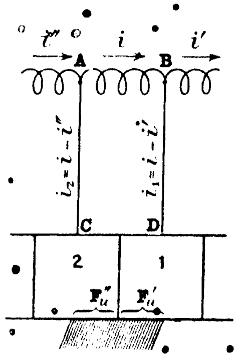


FIG. 346. The short circuit of a single coil.

§ 5. The equation of short-circuit.—The liberation of the stored energy in an electrical form or its re-storage gives rise to the induced E.M.F. $-(\mathcal{L}+\mathcal{M})di/dt$ which forms the first term of (178); *e.g.* if the current begins at once to fall towards zero, and then rises to $-J$ without over-reversal, di/dt is throughout the process negative, and therefore the induced E.M.F. is positive and retards commutation by maintaining the current in the old direction.

At the same time the coil is moving through an external field. This field may be either that from the fringe of the main field lines within the interpolar gap, or a special field provided for the purpose from commutating poles. In either case it is the resultant due to the magnet-winding or windings as modified by the presence of the armature ampere-turns that are carrying the load current and are not themselves undergoing short circuit. The value of the impressed E.M.F. due to the movement through this field will vary as short-circuit proceeds, and may be expressed as a function of the time, $= f_e(t)$. In order to complete the differential equation for the conditions of a section during short circuit, let a single coil AB be considered separately (Fig. 346), the positive direction round the whole circuit $ABCD$ being taken to be that of the current in the coil before commutation begins. The current in the leading commutator connector is then

$$i_1 = i' - \text{the current in the preceding coil}$$

and in the trailing commutator connector is

$$i_2 = i - \text{the current in the following coil.}$$

There must now be introduced the ohmic loss of volts over the resistance of the short-circuited coil AB and its commutator connectors, and over the brush contact resistance. Let r be the resistance of the coil AB, and r_c be the resistance of one commutator connector, *e.g.* BD. Let the contact area between the brush and the leading sector be at any time during the period of short-circuit F_u', and simultaneously that between the brush and the trailing sector be F_u''. The contact resistance of either portion is equal to its specific contact resistance per square inch divided by the area in square inches, and it must be borne in mind that the specific contact resistance of the two portions need not necessarily be precisely the same, but may depend upon the current density; the latter may vary on the two sides of the dividing mica in relation to time, and if so the specific contact resistance may also have a temporal variation. Hence if R_k' and R_k'' are the instantaneous specific contact resistances of the leading and trailing portions respectively, the loss of volts over the contact with the leading sector is $\frac{R_k' \cdot i_1}{F_u'}$, and over the contact with the trailing sector is $\frac{R_k'' \cdot i_2}{F_u''}$. Let every ohmic loss be now reckoned as negative; then by Kirchhoff's laws the algebraic sum of all the E.M.F.'s acting round one short-circuited section must be zero. The internal resistance within the brush or the sectors may be neglected entirely in comparison with the other resistances. The complete equation is therefore

$$(\mathcal{L}+\mathcal{M})\frac{di}{dt} - \Sigma\left(\mathcal{M}_1 \cdot \frac{di_1}{dt}\right) + f(t) - ri - r_c \cdot i_1 - r_c \cdot i_2 - \frac{R_k' \cdot i_1}{F_u'} - \frac{R_k'' \cdot i_2}{F_u''} = 0$$

or

$$-(\mathcal{L}+\mathcal{M})\frac{di}{dt} - \Sigma\left(\mathcal{M}_1 \cdot \frac{di_1}{dt}\right) + f(t) - ri - r_c(i_1 + i_2) - \left(\frac{R_k' \cdot i_1}{F_u'} + \frac{R_k'' \cdot i_2}{F_u''}\right) = 0 \quad . \quad . \quad (179)$$

The two expressions bracketed together in the last term of the above equation contain the current densities s_u' and s_u'' in the leading and trailing sector respectively, so that they may also be put in the form $R_k' \cdot s_u' + R_k'' \cdot s_u''$, and it will be found that they are not without importance in the problem of commutation. The direction of the fall of potential by the three last terms entirely depends upon the algebraic signs which i_1, i_2, and i are found to have, and as these are determined in relation to the short circuit and not to the external circuit, so also are the potential drops.

The above equation in its general form is necessarily true of each and every coil at any point of time during short-circuit, but will again take certain special forms under particular conditions. When,

as usual with carbon brushes, $b_1 - b_m > 2b$ and several coils are simultaneously short-circuited by each set of brushes, there are three stages of the process—

(1) At the beginning of short-circuit between $t = 0$ and $t = T\frac{b}{b_1}$, the leading sector 1 is entirely under the brush and the trailing sector is gradually passing under the brush ; the areas of the sectors which are covered by the brush are then $F_u' =$ practically $l_b b$ where $l_b =$ the joint length of a set of brushes measured parallel to the axis of rotation, and $F_u'' = l_b b \frac{t}{Tb/b_1} = l_b b_1 \frac{t}{T} = F_u \cdot \frac{t}{T}$, where F_u is the total area of contact of one set of brushes.

(2) Next from $t = T\frac{b}{b_1}$ to $t = T\left(1 - \frac{b}{b_1}\right)$, both the leading and the trailing sectors are completely covered by the brush, and both areas F_u' and $F_u'' = l_b b$.

(3) Finally towards the end of short-circuit from $t = T\left(1 - \frac{b}{b_1}\right)$ to $t = T$, there is a stage when the leading sector is emerging from under the brush, and $F_u' = F_u \cdot \frac{T - t}{T}$, and $F_u'' = l_b b$.

When $b_1 - b_m < 2b$ but is $> b$, the intermediate stage disappears.

Finally, when $b_1 - b_m = b$, the area of contact between brush and leading sector continuously diminishes as the area of contact with the trailing sector continuously increases. The diminution and increase proceed simultaneously and in correlative degree, so that $F_u' = F_u \cdot \frac{T - t}{T}$ and $F_u'' = F_u \cdot \frac{t}{T}$. Further, in relation to the single coil of Fig. 346, i' is then $= -J$ and $i'' = J$, so that $i_1 = i + J$ and $i_2 = i - J$.

§ 6. The ideal case of linear commutation.—The ideal case of commutation may be regarded as that in which the varying current i when plotted in relation to time yields an inclined straight line passing from the full normal value $+J$ to the reversed value $-J$ with a constant rate of change $-2J/T$ (Fig. 347), so that its instantaneous value at any time t is

$$i = 2_J\left(\frac{1}{2} - \frac{t}{T}\right)$$

When this is the case, it also follows that s_u' is always $= 2J/F_u$, and $s_u'' = -2J/F_u$, where F_u is the total area of contact of one set of brushes. The signs of the current densities are therefore different in relation to the short circuit but their numerical values are equal, and in each case this is equal to the normal current density s_u if

the current $2J$ passed uniformly through the area F_u. The characteristic feature of such commutation is then that the current density over the brush face is throughout constant and uniform at its normal value. The specific contact resistances R_k' and R_k'' *must*

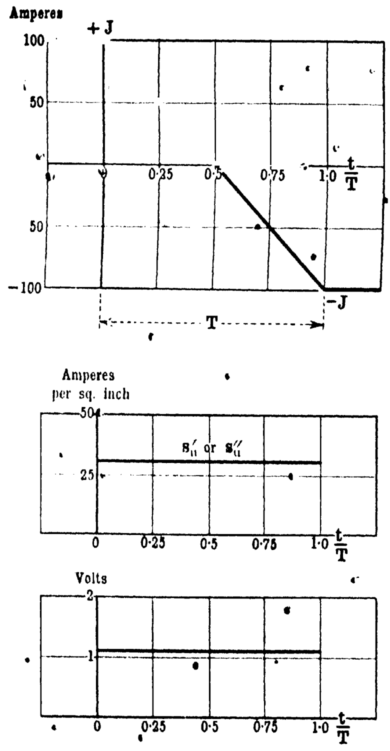


FIG. 347. Ideal linear current change.

therefore, be alike and equal to the normal specific resistance R_k, and the difference of potential between the brush and the sectors when plotted in relation to time yields a straight line similar to that of the current density as shown at the foot of Fig. 347. In consequence the two expressions in the bracket of equation (179) cancel out, showing that the normal brush contact resistance $R_1 = R_k/F_u$, although affecting the total voltage of the machine, has under these conditions no effect whatever upon the process of commutation.

Further, the watt density is uniform, and the loss of watts over the brush contact resistance has its minimum value.'

§ 7. The reversing field required for linear commutation.—When there are several coils simultaneously short-circuited, let it still be assumed that in each case the change of current from $+J$ to $-J$ is a linear function of the time, so that the rate of change, not only in coils A, B and B_x, but also in coils A_{I}, A_{II}, . . . and coils B_{I}, B_{II} . . . etc., is constant and throughout the same in each $= -2J/T$. Then the E.M.F. from the mutual inductance with the coils simultaneously short-circuited is simply additive to the inductance of the considered coil A, and

$$-(\mathcal{L}+\mathcal{M})\frac{di}{dt}-\Sigma\left(\mathcal{M}_1\frac{di_1}{dt}\right)=-(\mathcal{L}+\mathcal{M})\frac{di}{dt}-\Sigma(\mathcal{M}_1)\frac{di}{dt}$$
$$=(\mathcal{L}+\Sigma\mathcal{M})\frac{2J}{T}$$

Assuming $\mathcal{L}+\Sigma\mathcal{M}$ to be constant, it is now of interest to consider what must be the value of the external impressed E.M.F. as a function of the time which will produce such a straight-line change of current; this, which may be regarded as the correct value, will be symbolized as $f(t)_c$. It has already been shown that in such a case the two last terms of (179) cancel out. When several sectors are covered by the brush there are three stages, as explained in § 5, and corresponding to the different values of the brush contact areas with the leading and trailing sectors, the current in the leading and trailing commutator connectors respectively will be first $2J\frac{b}{b_1}$ and $2J\frac{t}{T}$, then $2J\frac{b}{b_1}$ and $-2J\frac{b}{b_1}$, and lastly $2J\cdot\frac{T-t}{T}$ and $-2J\cdot\frac{b}{b_1}$.

The current i in the coil is now throughout $2J\left(\frac{1}{2}-\frac{t}{T}\right)$. Thence from equation (179)—

(1) between $t=0$ and $t=T\cdot\frac{b}{b_1}$

$$(\mathcal{L}+\Sigma\mathcal{M})\frac{2J}{T}+f(t)_c-r\cdot 2J\left(\frac{1}{2}-\frac{t}{T}\right)-r_c\cdot 2J\frac{b}{b_1}+r_c\cdot 2J\frac{t}{T}=0$$

$$f(t)_c=-2J\left\{\frac{\mathcal{L}+\Sigma\mathcal{M}}{T}+r\left(\frac{t}{T}-\frac{1}{2}\right)+r_c\left(\frac{t}{T}-\frac{b}{b_1}\right)\right\}.$$

(2) from $t=T\cdot\frac{b}{b_1}$ to $t=T\left(1-\frac{b}{b_1}\right)$, in the same way as above we find

$$f(t)_c=-2J\left\{\frac{\mathcal{L}+\Sigma\mathcal{M}}{T}+r\left(\frac{t}{T}-\frac{1}{2}\right)\right\}$$

(3) from $t = T\left(1 - \frac{b}{b_1}\right)$ to $t = T$

$$f(t)_c = -2J\left\{\frac{\mathscr{L} + \Sigma . \mathscr{M}}{T} + r\left(\frac{t}{T} - \frac{1}{2}\right) + r_c\left[\frac{t}{T} - \left(1 - \frac{b}{b_1}\right)\right]\right\}$$

Corresponding, therefore, to the three stages, the reversing field should have three portions of different slope (Fig. 348).

From the initial and final values of $f(t)_c$, namely—

$$f(t)_{t=0} = -2J\left\{\frac{\mathscr{L} + \Sigma . \mathscr{M}}{T} - \frac{r}{2} - r_c \cdot \frac{b}{b_1}\right\} = -2J\left\{\frac{\mathscr{L} + \Sigma . \mathscr{M}}{T} - \frac{R}{2}\right\}$$

$$f(t)_{t=T} = -2J\left\{\frac{\mathscr{L} + \Sigma . \mathscr{M}}{T} + \frac{r}{2} + r_c \cdot \frac{b}{b_1}\right\} = -2J\left\{\frac{\mathscr{L} + \Sigma . \mathscr{M}}{T} + \frac{R}{2}\right\}$$

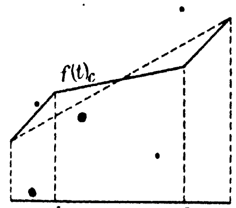


FIG. 348. Reversing E.M.F. required for linear commutation.

we obtain an average expression with uniform slope, as shown by the dotted line (Fig. 348), for which the equation is

$$f(t)_c = -2J\left\{\frac{\mathscr{L} + \Sigma . \mathscr{M}}{T} + R\left(\frac{t}{T} - \frac{1}{2}\right)\right\} \quad . \quad . \quad . \quad . \quad (180)$$

where $R = r + 2r_c \cdot \frac{b}{b_1}$

This holds strictly when $b_1 - b_m \geqq 2b$, and the intermediate stage disappears.

If the brush width is less than or equal to that of a sector, or $b_1 - b_m \leqq b$, the currents in the leading and trailing commutator connectors become respectively $2J\left(\frac{T-t}{T}\right)$, and $-2J \cdot \frac{t}{T}$; the same equation then results, save that we have $\mathscr{M}$ instead of $\Sigma . \mathscr{M}$, and R is $r + 2r_c$.

$\frac{2J\,(\mathscr{L} + \Sigma . \mathscr{M})}{T}$ is evidently the average value of the E.M.F. from self and mutual inductance, or the constant instantaneous value which results when the commutation is actually performed uniformly; it may therefore be called the "inductive voltage" as opposed to

the ohmic voltage JR. The density of the main field from the field excitation or of the reversing field from commutating poles, with allowance for all the reacting ampere-turns of the armature save those of the coils actually undergoing short-circuit, ought then to rise in an inclined straight line by an amount proportional to JR. The sign of the correct commutating field at the beginning of short-circuit may require to be either positive or negative, according as $\frac{R}{2}$ is greater or less than $\frac{\mathscr{L} + \Sigma . \mathscr{M}}{T}$; usually $\frac{R}{2}$ is the lesser quantity, so that the impressed field density must be throughout negative or reversing. At the beginning it balances the difference between the inductive and ohmic voltages, and at the end it balances their sum.

The above assumes that $\mathscr{L} + \Sigma . \mathscr{M}$ is constant for any one coil, and therefore that it is the same for each coil which is simultaneously short-circuited. This assumption is not strictly correct, especially in the case of toothed armatures with several coil-sides in each slot, when the short-circuit curves cannot be exactly alike owing to the different positions of the coil-sides relatively to the commutator sectors. But the curve of the commutating field must in any case be smooth, and cannot have abrupt changes of inclination, so that only an average adjustment of it is possible. A more correct shape for it in the case of a number of sectors simultaneously short-circuited can be theoretically deduced, but its closer determination is not of practical value owing to the many secondary effects from imperfect brush contact, oscillation of the field, etc., that enter into the problem. The equation (180) and the value of the correct reversing density B_{rc} for the resultant field which can be thence deduced are, however, sufficiently accurate to be of value as guides in practical work.

§ 8. The importance of the case of linear commutation.—When the complexity of all the numerous secondary effects are borne in mind, to secure such an absolutely exact balance between $f(t)_c$ and the inductive and ohmic voltages that the current does actually follow a straight-line law of change could only be the rarest of accidents. It may therefore be asked whether the case of linear change, although ideally the best for commutation, deserves special consideration from a practical point of view. The answer is that in all cases when it is not fulfilled the quantity $R_k' . s_u' + R_k'' . s_u''$ comes into play, and it will be found that this has the effect of checking any divergence from the straight-line law, although it may be perhaps to only a small extent.

It is, however, evident that any such effect must depend on the law governing R_k in relation to current-density. A digression must therefore now be made in order to investigate the specific contact-resistance of brushes. This will be considered from the point of

view of ordinary working conditions with constant current-density, as well as from the special point of view of rapidly varying current-densities as a necessary preliminary to the question of how far the corrective effect mentioned above can be realized in practice. In the latter connexion it will be found that a marked distinction exists between metallic brushes of copper gauze or brass leaves and those of carbon.

§ 9. The contact-resistance of copper brushes.—The specific contact-resistance of brushes per unit area of bearing surface is affected in general by the current-density, the pressure, the peripheral speed of the commutator and the state of its surface, and more especially in the case of carbon brushes by the direction of the current and by the temperature of the working surfaces. The effects of these various conditions have been investigated by a number of experimenters, and especially by Professor Arnold, from whom the following results are mainly derived.[1]

While the true specific resistance of the contact between brush and commutator when the latter is stationary may be taken to be a constant quantity, giving a drop of potential which rises in a straight line with increasing current-densities, in all cases the actual curve of the difference of potential when the commutator or slip-ring rotates bends over as the current-density is increased, more or less suddenly or gradually according to the nature of the material. This shows that the apparent contact-resistance on a rotating surface progressively decreases as the current-density rises, and it is the value of the specific running contact-resistance, or R_k defined simply as the quotient of the loss of volts divided by the current-density when the dynamo is running, with which alone we are concerned, the true specific contact-resistance having but little interest in practice. Further, for a given pressure and current-density the specific contact-resistance is always greater when the commutator or slip-ring rotates than when they are at rest.

Taking first the case of *copper* brushes, it is found that with normal brush pressures and conditions of surface the contact resistance with a rotating commutator decreases but slowly after a current-density of about 40 amperes per square inch is exceeded, and gradually becomes almost constant. It is practically independent of the peripheral speed when once this has passed a low value, but this result is of course dependent upon the commutator surface being smooth, and its running free from vibration so that there is no corresponding vibration set up in the brush-holders. Increase of the pressure for any given speed causes better contact and decreases the resistance.

Coming to numerical data, with copper brushes a density of 40 amperes per square inch is almost always exceeded, and 200 amperes per square inch may be regarded as the maximum limit. The peripheral speed of the commutator is sometimes as high as 3000 feet per minute, but preferably does not exceed 2500 ft. per minute, and in all cases the lower its speed the better.

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[1] For carbon brushes, see also especially P. Hunter-Brown, "Carbon Brushes," *Journ. I E E.*, Vol. 57, p. 193; and Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 302-8.

In a good dynamo running under ordinary conditions, the brush pressure should range from 1¼ to 1¾ lb. per square inch of contact area; this may be tested by noting the pull required to lift the brush from the commutator surface with a small spring balance. Even under normal conditions the specific contact-resistance of copper brushes shows great variations; on a slip-ring it may fall as low as 0·0002 or on a commutator to 0·0007 ohm per square inch, while, if the periods of vibration of the commutator and brush-holder happen to coincide, it may be as much as 0·0025 ohm per square inch. As an average value for R_k may be taken 0·0016, and in no case is it likely to exceed 0·003 ohm per square inch.

§ 10. The contact-resistance of carbon brushes under permanent conditions.—The specific contact-resistance of *carbon* brushes falls much more rapidly with increasing current-density than that of copper brushes, and, after a medium current-density is reached, almost in inverse proportion thereto. The curve of fall of potential in relation to current-density therefore bends over fairly sharply and becomes nearly flat. It resembles an equilateral hyperbola, becoming asymptotic to a limiting value. The passage of the current between brush and collector thus differs radically from true metallic conduction. On a smooth slip-ring the curve for ΔP after reaching a maximum may even slowly descend with increasing current-densities, showing that R_k is then decreasing faster than the current-density is rising. This phenomenon is closely involved with the property of carbon, by which its resistivity falls as it becomes hotter. Owing to the negative temperature coefficient of the carbon, as the current-density is increased the expenditure of energy in heating the contact is checked; and this effect can only be eliminated by artificially maintaining the commutator at a constant temperature. The true fall of resistance with increasing current-density is to be explained as mostly due to the small carbon particles which are worn off the brushes, especially under a high current-density, when a blackening of the commutator surface results. A more intimate contact between brush and commutator is obtained by this wearing away of the carbon, which proceeds rapidly when the brush is heated and, becoming softer, disintegrates more rapidly.

Increase of the pressure lowers the contact-resistance, but the true effect is again partially masked by the fact that the increased friction loss raises the temperature and assists in lowering R_k. But as soon as a pressure of about 2 lb. per square inch is reached there is little further improvement, and the pressure that can be advantageously employed is strictly limited by the mechanical friction and consequent heating that results.

The alteration of the cooling power of the slip-ring or commutator is also a disturbing factor when the effect of different peripheral speeds is to be examined.

A curve connecting specific contact-resistance with current-density should therefore presuppose either that in every case the

passage of a given current was maintained long enough for its corresponding constant condition of temperature to have been reached with some normal peripheral speed, or that some given temperature such as may be found in practice is artificially maintained constant throughout.

The temperature of the contact is thus a factor of the greatest importance, and Professor Arnold found marked differences in its effect between different varieties of carbon and between the positive and negative brushes.[1] Between temperatures of 20° C. and 35° C., or say 70° F. and 95° F., there is no great change, but beyond this point there is as a general rule a progressive and more or less rapid fall, which is especially marked when the current flows from carbon to metal, *i.e.* at the negative brushes. *E.g.* at temperatures of 55° C. (131° F.) and 75° C. (167° F.) the contact-resistance and loss of potential at the positive brushes with a current-density of 35 amperes per square inch may be respectively only about 88 and 50 per cent. of the value at 35° C., while at the negative brushes they may be only about 70 and 25 per cent. Efficient ventilation of the commutator to keep it cool is therefore of great assistance in suppressing sparking, and this is amply borne out by the well-known fact that machines which when cool run quite sparklessly, yet may begin to give trouble when they become hot after a prolonged run.

At low temperatures the contact-resistance at the anode brushes (negative brushes of a dynamo, positive of a motor) is, as a rule, greater than at the cathode brushes (positive in a dynamo, negative in a motor), but at high temperatures this is often reversed, so that the curves intersect. The divergence between + and - brushes may be as much as 50 per cent., but varies much with different qualities and different temperatures.[2]

A. Mauduit[3] observed no noticeable differences between the voltage drop under + and - brushes, nor between the voltage limits at which sparking began, but he found that as soon as appreciable sparking occurred, while the sparks were drawn out beyond the brush tip at the anode brushes (negative of a dynamo), they, as it were, retreated inwards under the cathode brushes (positive in a dynamo),[4] so as to become nearly invisible, and gave to the commutator a characteristic black roughened surface. In this phenomenon he finds the explanation of the usual observation that sparking and bad commutation occurs soonest and to a greater degree at - than at + brushes in a dynamo, although in reality the damage

[1] *Cp.* Arnold and Pfiffner, *Arbeiten aus dem Elektrotechnischen Institut zu Karlsruhe*, Vol. 1, p. 299.

[2] *Cp. Journ. I.E.E.*, Vol. 57, p. 223.

[3] *Recherches Expérimentales et Théoriques sur la Commutation*, p. 220.

[4] *Cp. Journ. I.E.E.*, Vol. 57, p. 219, for somewhat similar phenomena observed in relation to motor brushes, and the explanation there put forward.

done to the commutator is the same by either form of spark. The lengthening of the spark under high inductance until it reached across a mica strip from sector to sector and thereupon became of white colour and destructive was observed with negative brushes of Le Carbone's former QS_4 type, although this type in itself was found to possess a special power of quenching a spark of small energy.

The condition of the metal rubbing surface has considerable effect. If it be newly polished, the contact-resistance is low; as the surface becomes oxidized, and acquires a brown skin, the resistance rises. But on the other hand, when blackened by the presence of small conducting particles of copper, the resistance falls. Though blackening is to be avoided, a commutator should be allowed to retain the dull brown colour which continuous running will produce in the absence of sparking. An oily surface may more than double the resistance, especially if the current-density be low, but must be strictly avoided owing to the danger of carbonization by sparking. Paraffin wax, on the other hand, although in itself an insulator, when used sparingly and thoroughly spread on a warm commutator, has little or no effect on the loss of volts under average conditions; it very much reduces the friction and noise of the carbon brushes, and is therefore frequently used as the basis of commutator compounds.[1]

The effect of speed has been left to the last on account of its great importance, and in this connexion the two cases of the smooth slip-ring and the commutator must be clearly distinguished. The former shows lower contact-resistances which are practically independent of the speed (although with some tendency to increase at higher speeds), and correspondingly higher current-densities are permissible than in the commutator. The reason for the difference is to be found in the fact that, even with a commutator which may be regarded as practically smooth, carbon brushes are periodically subjected to momentary vibrations as they pass the mica strips dividing the sectors. The natural elasticity of the copper gauze brush suffices to take up the minute mechanical shocks, but the unyielding carbon brush is kept in a state of continuous vibration. In consequence the curve connecting the loss of potential over the contact with the current-density does not so quickly become flat in the case of the commutator as in the case of the slip-ring. In the latter the potential difference between brush and ring, or $R_k \cdot s_u$ reaches a maximum at which within wide limits of current-density it remains nearly constant, and according to the nature of the material this constant loss is about 0·75 volt with hard carbons or 0·45 volt with very soft carbons of high conductivity. But on a commutator the curves usually continue to show a gradual increase with

[1] Prof. F. G. Baily and Mr. Cleghorne, *Journ. I.E.E.*, Vol. 38, p. 158.

increasing current-density, and for the same varieties of carbon the figures would rise, say, to 1·5 and 1 volt respectively.

But the real difference is rather that with the commutator 2, 3, or even 4 volts between brush and commutator are easily reached with sparking hardly, if at all, perceptible, the amount increasing as the speed is increased and the pressure weakened. If the commutator surface is rough, or untrue, and the brush-holders are insufficiently damped, this effect is magnified, and if at some particular speed the natural period of vibration of the brush-holders coincides with the period of the shocks it reaches a maximum, and the loss of volts may be 6 or more (*cp.* § 29). The same effect may be imitated on the slip-ring by so weighting the brush-holder as to produce resonance at the particular speed employed. In practice it is generally found that the lighter the moving parts of the brush-holders are, the better for all ordinary conditions of speed. Even with the same peripheral speed, the higher the actual number of revolutions per minute the greater the likelihood of vibration, so that small high-speed machines are more liable to give trouble than large machines with commutators of large diameter running at a low number of revolutions per minute.

Thus a continuous-current dynamo with its commutator may with but little exaggeration be said to run always with its carbon brushes in a stage not far removed from incipient arcing[1]; owing to the slight percussion of the brush as the sectors pass beneath its face, even when the commutator is practically quite smooth and sparking is not perceptible, higher differences of potential are possible than are found experimentally with slip-rings.

With carbon brushes a pressure of $1\frac{1}{4}$ to $1\frac{3}{4}$ lb. per square inch should suffice, although at high peripheral speeds over 2000 feet per minute where there is vibration, it may become necessary to increase it to 2 lb. per square inch. For peripheral speeds from 1000 to 2000 feet per minute, and average values of the brush pressure from $1\frac{1}{4}$ to 2 lb. per square inch, Fig. 349 shows for hard and soft carbon brushes the values of the specific contact-resistance which correspond to good conditions of working on a smooth commutator without sparking. The full-line curves presuppose that the running is maintained long enough for the constant temperature to be reached that is proper to the particular current-density, while the dotted curves indicate the effect of a commutator which is maintained at one and the same constant temperature corresponding to the current-density at which the curves cross. In the curves of Fig. 349 the values at the + and − brushes have been averaged, and it will be seen that for normal current-densities, such as from 30 to

[1] It has been suggested that even when the collection is sparkless, it may in fact be proceeding by minute arcs of insufficient energy to be visible to the eye (*cp.* Dr. M. Kahn, *Journ. I.E.E.*, Vol. 57, p. 222).

40 amperes per square inch, the specific contact-resistance under ordinary conditions of pressure and temperature may be taken as varying between 0·03 and 0·015 ohm, or on an average for 30 amperes per square inch and hard carbon brushes, $R_k = 0·03$ ohm.

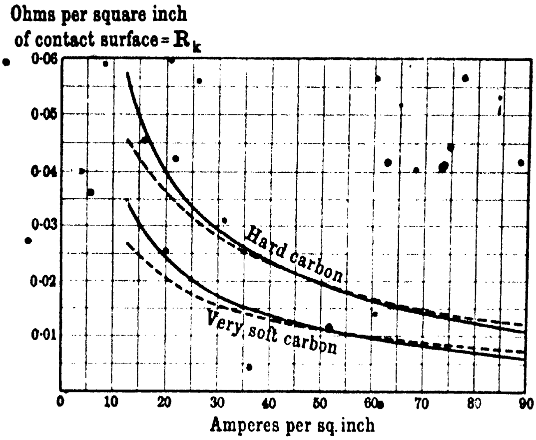


FIG. 349.—Contact resistance of carbon brushes.

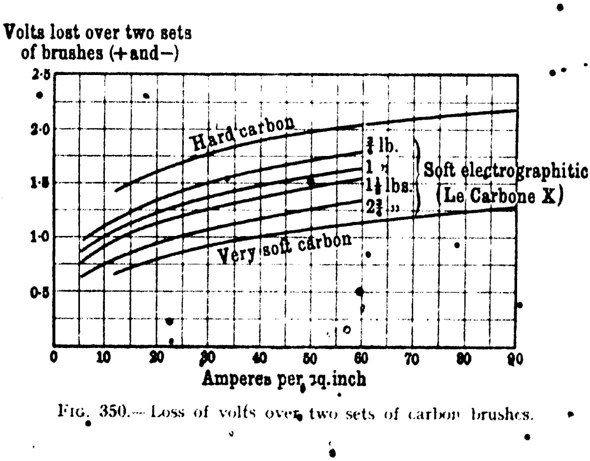


FIG. 350.—Loss of volts over two sets of carbon brushes.

In Fig. 350 are given curves of the loss of volts over two sets of brushes (*i.e.* positive and negative) for different kinds of carbon, and with an allowance for about ½″ of length down the two carbons, the commutator being assumed to have reached its final natural

state of temperature after prolonged running at each current-density.[1] They may therefore be used to determine the total drop of volts and loss of watts over the brushes and commutator so far as the current-density is uniform over the contact area.

Although the selection of the best grade of carbon to suit given conditions of voltage, current-density, and speed is to some extent a matter of experience proceeding by trial and error, yet it may be said that for voltages of 250 and upwards the coarser and harder (non-graphitic) varieties, such as Battersea carbon Link *A* or Link *C*, or Le Carbone *F* or *S*, with densities of 30 to 40 amperes per square inch, in general give good results. With the admixture of increasing percentages of graphite, the brush becomes softer and more conductive, but its contact-resistance is lessened. With low voltages, favourable conditions, and in cases where it is imperative to shorten the commutator as much as possible, hard *graphitic* brushes, such as Battersea carbon Link *B*, Morganite Link 1, hard Morganite Link *HM3*, or Le Carbone *BB* are frequently used with current-densities up to 65 amperes per square inch. The *electrographitic EG* series of the Morgan Crucible Co. and Le Carbone *X*, *EG*, and *Z* grades are artificially graphitized by being baked in an electric furnace, and are also used for 50 to 65 amperes per square inch. Other graphitic brushes, such as the *LFC* series of Le Carbone, are again softer, and as the content of graphite is increased, we approach natural graphite. The thermal conductivity of graphite (whether natural or artificial) is higher than that of carbon, and in this respect the graphitic brush is at an advantage, inasmuch as it can conduct away more quickly the heat developed by any unequal current-density which is highly localized, before the temperature reaches the glowing point and disintegration begins.[2] But the

[1] The intermediate curves for soft electrographitic brushes of Le Carbone X quality are derived from Prof. F. G. Baily and Mr. W. S. H. Cleghorne's experiments (*Journ. I.E.E.*, *loc. cit.*), who deduced as an equation for the loss of volts over two sets of brushes, positive and negative, expressing closely the shape of the curves and the effect of different pressures

$$2E_b = \frac{(\text{amperes per sq. inch})^{0\cdot 28}}{1 + 0\cdot 88\sqrt{p}}$$

where p is the pressure in lb. per square inch.

Between 20 and 65 amperes per square inch, the loss of volts over positive and negative brushes according to the Société Le Carbone (quoted by J. Rezelman, *Phénomènes de la Commutation*) may be taken approximately as

$$2E_b = 0\cdot 16\sqrt{s_u} \text{ for their brand } LFC_2$$

$$= 0\cdot 066\sqrt{s_u} \text{ ,, ,, ,, } CG4 \text{ (carbon mixed with copper)}$$

while for low current-densities

$$2E_b = 0\cdot 035 s_u \text{ for } LFC_2$$

$$= 0\cdot 0093 s_u \text{ ,, } CG4.$$

Such expressions are sometimes useful for detailed calculations, even though only approximate.

[2] P. Hunter Brown, *Journ. I.E.E.*, Vol. 57, p. 196.

softer graphitic brushes with a low friction coefficient disintegrate more readily, blacken the commutator, and in proportion to their increased conductivity oppose less contact-resistance to sparking; further, they should not be used when several sectors are to be covered by the brush simultaneously. They may therefore be regarded as intermediate between copper and the harder varieties of carbon which suppress sparking more thoroughly.

Various types of brushes have also been brought out, in which the high conductivity of copper has been combined with the high contact-resistance of the carbon brush to a more or less extent, by mixing carbon and copper in various proportions as in the Copper-Morganite Link *CM* series, and the *CG* and *MC* grades of Le Carbone. The latter are only suitable for the most favourable conditions of very low voltage, or on slip-rings, where commutation does not enter into the problem and current-densities of 150 amperes per square inch or over are required.

§ 11. The contact-resistance of carbon brushes under rapidly varying currents.—So far the effect of various current-densities permanently maintained has alone been considered at length. But during the running of a dynamo or motor the carbon brushes are subjected to a very rapid sequence of varying current-densities in the different portions of their surface, the sequence being continually repeated as the sectors pass under and away from the brushes. The effect of rapidly varying current-densities upon the contact-resistance requires, therefore, to be considered, and this has been investigated by passing a periodic alternating or a pulsating current through the brush into a rotating ring; the simultaneous momentary values of current and voltage across the contact-surface are then obtained, so as to determine corresponding values of the instantaneous current-density, and of the specific contact-resistance. The curves so obtained do not repeat the full-line curves of Fig. 349 for long-continued current-densities, but as might be expected resemble the dotted curves. For a given virtual current-density the resistance when the current falls to lower values is lower than for similar constant current-densities, owing to the carbon being really at a higher temperature than would correspond to them, while for instantaneous densities above the virtual value the resistance is higher than for similar constant current-densities owing to the carbon being cooler. Further, the experiments of Dr. Kahn and Professor Arnold have shown that the curves in relation to instantaneous current-density intersect the curves for permanent current-densities at the point of the virtual current-density in the alternating case, *i.e.* where the square root of the mean square of the instantaneous current-densities is equal to the continuous current-density. The virtual current-density or R.M.S. value marks, in fact, the point of equal heating effect over the contact area in

the two cases. The curves are also independent of the periodicity, so long as this is high.

§ 12. The general effect of the current-density and contact resistance.—We are now in a position to revert to the consideration of the effect of $R_k'.s_u' + R_k''.s_u''$ in equation (179). It is evident that if a true physical law could be formulated to express the connexion between R_k and s_u, a number of interesting mathematical deductions could be made from the differential equations of (179). Unfortunately no such definite law holds in practice. Thus for different current-densities permanently maintained, R_k is not a constant; with the commutator artificially maintained throughout at a constant temperature it may be represented approximately over some range by an equation of the form

$$R_k = \frac{a}{s_u} + b$$

where a and b are constants, and the curve is a rectangular hyperbola. The loss of volts would then be $R_k . s_u = a + b . s_u$, *i.e.*, it would rise as an inclined straight line. But it is evident that any such law cannot be pressed very far, since the curve of loss of volts does not cut the vertical axis but passes through the origin; again at very high current-densities it does not progressively increase, and R_k does not become constant but still continues to fall. Thus it is more nearly true to say that the curve of the drop in volts E_b in relation to current-density is an equilateral hyperbola[1] approaching asymptotically to a maximum value E_{max} with a law $E_b = E_{max} \frac{s_u}{s_u + c}$ where c is the current-density for which $E_b = \frac{1}{2} E_{max}$.

It can, however, in general be said that with carbon brushes when the current-density is increased the contact-resistance does not fall quite in inverse proportion, so that on the whole the drop in volts with a higher current-density is greater than with a lower current-density, yet on the other hand that the contact-resistance does fall very rapidly so that the curve of the drop in volts gradually approaches a limiting maximum.

Bearing in mind the first of these generalities, let the simplest case be considered, when $b_1 = b_m = b$ and only one section is short-circuited, and let the current-change diverge from the straight line.

§ 13. Divergence from linear commutation.—The cases of divergence from a uniform rate of commutation may be grouped under four principal kinds.

(1) Retarded commutation, yielding a curve which when plotted as in Fig. 351 from a starting-point of $+ J$ is on the whole convex.

[1] A. Mauduit, *Recherches Expérimentales et Théoriques sur la Commutation*, p. 286.

Suppose that at some time t seconds from the commencement of short-circuit the current in the coil has not fallen by its correct amount proportional to t, then the current-density in the leading sector $s_u' = i_1/F_u'$ is greater than that in the trailing sector $s_u'' = i_2/F_u''$. The former is positive, and the latter is negative; for simplicity's sake it is best to regard the densities in themselves as having no sign, since we are not here dealing with them as continuously varying quantities, and to add their signs when required. Hence the expression in the bracket of equation (179) becomes $R_k' . s_u' - R_k'' . s_u''$. Since the specific contact-resistance of carbon brushes does not vary so much as in inverse proportion to the current-density, there results a positive difference, the fall of potential between sector 1 and the brush toe being greater than that between

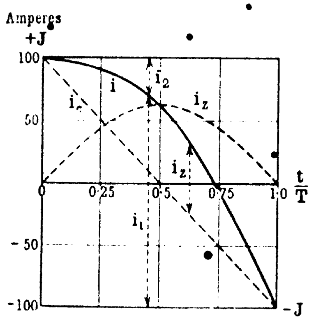


FIG. 351 Retarded commutation.

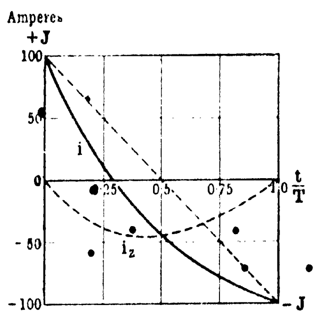


FIG. 352. Accelerated commutation.

sector 2 and the rest of the brush. The difference of potential which thence arises $p = -(R_k'.s_u' - R_k''.s_u'')$ is thus a *negative* E.M.F. in relation to the short circuit or acts round the circuit in the negative direction against the old current, tending to reduce its value; *e.g.* in Fig. 346 the E.M.F. in question which is solely due to the unequal current densities in the two portions of the brush would be directed from D to C through the coil BA.

Thus the unequal brush contact-resistance acts as an outlet through which the stored energy may expend itself in heating not only the coil, but also the commutator surface; while after reversal of i it is the means by which the energy that has to be re-stored is derived from the electrical output of the rest of the winding, for the voltage of the external circuit is during this stage temporarily lowered.

(2) Accelerated commutation, giving a concave curve (Fig. 352). In this case the current in the short-circuited coil is too quickly reduced, and the density in the leading sector is less than that in the

trailing sector. In relation to the short circuit the former is positive and the latter negative, so that the expression within the bracket is again $R_k'.s_u' - R_k''.s_u''$, but as opposed to the former case yields a negative difference. The resulting difference of potential $\Delta p = -(R_k'.s_u' - R_k''.s_u'')$ is therefore a *positive* E.M.F. checking the fall of the old current and opposing the rise of the reverse current.

(3) If the current is at first actually increased in its original direction above $+J$ (Fig. 353), the current-density in the trailing sector so long as this is the case becomes positive as well as that in the leading sector; in other words, the excess current actually flows through the brush from one side to the other and round the short-circuit. The two expressions in the bracket have now therefore to be added,

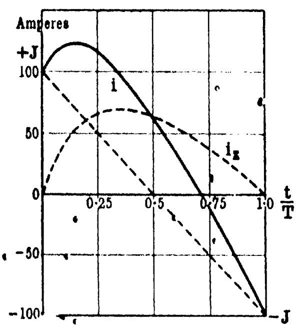


FIG. 353.—Increase of current in original direction.

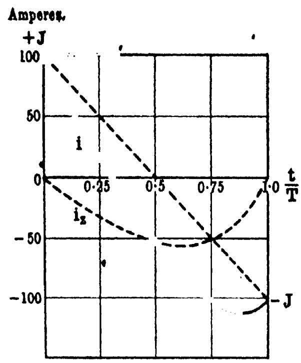


FIG. 354.—Over-reversal.

and it is $\Delta p = -(R_k'.s_u' + R_k''.s_u'')$, which acts negatively to limit the short-circuit current and to bring it back again to its correct amount proportional to t.

(4) If the current towards the end of the period of commutation is *over-reversed* to a value above $-J$ (Fig. 354), s_u' itself becomes negative as well as s_u'', and it is again the sum of the two which is effective, their corrective difference of potential being $\Delta p = -(-R_k'.s_u' - R_k''.s_u'')$, and therefore positive or checking the over-reversal.

Combinations of the above leading cases are also of frequent occurrence, the curve of i crossing the inclined straight line of uniform commutation. In generators the case is often met with in which the commutation is much retarded at first, and is then followed by over-reversal, due to the coil moving through too strong a reversing field.

Thus if there be any divergence from proportionality between the

change of current and the time, in every case whether of excess or deficiency, over- or under-reversal, the effect of the *brush contact-resistance* is to keep the current in each sector more nearly proportional to the area of contact, and so to assist in promoting the straight-line change which corresponds to a uniform current density.

§ 14. The amount of the corrective action.—The next question is, To what extent can the corrective action from the brush contact-resistance be actually relied upon in practice? That is to say, up to what limits can the voltage drop between brush and commutator rise without sparking, when the contact resistance is checking divergence of the short-circuit current i from the straight-line value?

With carbon brushes it has already been shown that the value of $R_k \cdot s_u$ approaches a maximum value, and this value may be set at about 1 volt for soft carbon to 1·5 volts or perhaps 2 volts for hard carbons.[1] As soon as such values are reached, the true contact-resistance is lost, and a spark is on the point of birth.

With metallic brushes there is not so definitely a maximum value for $R_k \cdot s_u$, but sparking begins at about 0·4 volts.

Hence for the purpose of taking up any difference between the actual reversing field and the correct field required to give a straight-line change ending at the final moment with $i = -J$, the effectiveness of carbon brushes is at least four times that of copper gauze or brass brushes.

With the latter, as the armature current varies and in the absence of commutating poles, the brushes must be shifted in order that the reversing field may be very closely of the right strength to keep the current-density uniform, and the corrective action of the contact resistance can only be relied on to prevent sparking when the brushes are but little removed from the natural position of exact reversal. The proper setting is found in practice by shifting the rocking bar slightly backwards and forwards until a position is observable on either side of which the sparking becomes greater. In all cases, therefore, where the fluctuations of load are large and rapid, recourse must be had to carbon brushes. But even a carbon brush cannot be counted on to cope with more than about 1·5 volts divergence without sparking, since $R_k'' \cdot s_u''$ must itself have some appreciable value.

§ 15. Comparison of metallic and carbon brushes.—The advantage of carbon over metallic brushes is in fact closely related to the normal loss of volts which occurs with the two types under ideal conditions when commutating perfectly with a constant current-density $2J/F_u$. With copper brushes it is not practicable to employ a higher normal current-density than about 175 to 200 amperes per sq. inch, and R_k, as before stated, may be reckoned on the average as $= 0·0016$ ohm per sq. inch. The maximum

[1] A. Mauduit, *loc. cit.*, p. 283.

normal loss of pressure over one set of brushes is therefore about $0{\cdot}0016 \times 190 = 0{\cdot}3$ volt, or 0·6 volt over the two sets of opposite sign, and in practice the loss is more often 0·5 volt or lower owing to the current-density being less.

The necessity for limiting the normal current-density in carbon brushes to such an amount as to leave a considerable margin to meet actual inequality of the density has led in practice to the adoption with hard carbon brushes of a normal value not exceeding some 35 to 40 amperes per square inch. A high local current-density, even though not accompanied by sparking, will cause the brush, perhaps locally, to become red-hot or to glow, its resistance falls, which increases the trouble, and disintegration of the material ensues. Assuming the specific contact resistance at the above limiting density to be 0·03 ohm, the normal loss of pressure over the two sets of brushes is from 2 to 2·2 volts. Comparison of this loss with that for copper brushes gives a rough measure of the degree of the practical superiority of the carbon brush from the point of view of suppressing sparking, and it also shows that it is necessarily purchased at the expense of the efficiency of the machine. Such sacrifice is, however, but small as compared with the advantage that carbon offers; although this material may not enable the brushes to be retained on the line of symmetry, yet with it an intermediate angle of lead can be found between the best possible positions for zero and full-load, such that neither the too rapid reversal in the first case nor the insufficient reversing field in the second case will cause an excessive current-density and over-heating or serious sparking. The brushes can then be retained in this position through all changes of load perhaps up to an overload of 30 per cent.

Furthermore, the great advantage of the carbon brush is that, being non-metallic, if some sparking does take place it does not become fused at the tip and adhere to the surface of the commutator. By reason of its physical nature, carbon is able to quench the spark quickly, so long as the power expended in the spark does not exceed a few watts, and little harm is then done to the commutator.

With low-voltage machines for large currents, a softer carbon may advantageously be employed, and the normal current-density raised to 50 amperes per square inch with a consequent decrease in the necessary size of commutator (*cp.* § 10). The same also applies to the case of dynamos fitted with commutating poles, which supply a reversing field nearly proportioned to the load.

§ 16. The external reversing field without commutating poles.—It has above been shown that even carbon brushes cannot be expected by their own unaided action to effect the commutation of current-turns with considerable inductance without sparking, but must be assisted by an external impressed E.M.F. to balance

at least partially the self- and mutually-induced E.M.F.; further, it has been shown that with metallic brushes the balance must be much more nearly exact than with carbon brushes.

The two methods of producing $f(t)$ which have been already mentioned in § 5 must therefore now be more fully described, and first that which is due to movement through the fringe of lines in the interpolar gap in the absence of special (commutating) poles.

If the reader refers to any diagram, such as Fig. 84 or 105, it will be evident that the exact position relatively to the poles which the loops or coils of the winding occupy, at the time when they are short-circuited, depends upon the position of the tips of the brushes as they rest upon the commutator, and that this position of the short-circuited coil can be altered if the brushes are shifted slightly backwards or forwards round the circle of the commutator sectors.

If in Fig. 343 the brushes are shifted backwards away from the line of symmetry or against the direction of rotation, the loops at the commencement of their short-circuit will be moving through the fringe of the trailing pole-tip; hence an E.M.F. is set up in them in the original direction of the current prior to passage under the brushes. The initial value of the impressed E.M.F. $f(t)$, or E_0, is thus positive; although, as rotation continues, it falls and eventually, if short-circuit lasts long enough, will be reversed, yet at the outset at least the commutation will be retarded as in Fig. 351 (case 1). If the angle of trail of the brushes and the initial E_0 be large, it may even raise the amperes which the coil is carrying above the normal current being carried by the other armature coils (case 3, Fig. 353). In either case it will largely increase the difficulty of commutation by causing an excessive current-density in the heel of the brush, and, if short-circuit ends with the coil still behind the neutral line of zero field, in the toe, so that the whole work of reversal will be thrown on to the electrical action of the brush contact-resistance. Sparking will then ensue between the trailing edge of the sector which has just emerged from under the brush and the tip of the brush itself. Thus the consequence if short-circuit ends while the coil is still moving on the trailing side of the line of symmetry is in general destructive sparking, so that any angle of trail is so far as generators are concerned quite inadmissible.

If the brushes are brought into such a position that they have neither trail nor lead, the diameter of commutation will coincide with the neutral line at no-load, but when current is taken out of the armature the cross flux from the armature ampere-turns will, as explained in Chapter XIX, have the same sign as the flux under the trailing pole-tip. The short-circuited loops will therefore still be moving in a field of the wrong sign, the resultant neutral line having moved forward.

But now if the brushes are so far advanced that short-circuit

does not end until $f(t)$ has become negative, there will be a reversing E.M.F. acting on the coil and assisting in the production of a current in it, the same in direction as that which the coil will be called upon to carry as soon as it emerges from under the brush. The case is illustrated by Figs. 101 or 105, where the coil-sides short-circuited by a brush are slightly in advance of the interpolar lines of symmetry, and are assumed to be just moving in the fringe of lines from a leading pole-edge when short-circuit ends. Even if the E.M.F. impressed by the external field in the required new direction does not suffice by itself to raise the current to equality with the normal current of the other coils, yet much will have been done to prevent the rise of the current-density, in any portion of the brush and at any time during short-circuit, from reaching an excessive amount.

When the external field becomes negative or reversing while the current in the short-circuited coil is still in its original direction, the E.M.F. $f(t)$ is in the opposite direction to the current, and the coil is itself driving the armature forward as in a motor. We thus have at once a ready means by which the bulk of the initial stored energy which has to be dissipated may be conveniently absorbed in the form of mechanical work instead of as heat and with any degree of rapidity. When the current has been reversed, the prime mover expends mechanical energy with equal rapidity depending upon the value of the reversing field, and this appears not only as heat in the coil but also as stored electro-magnetic energy.

§ 17. The necessity for an angle of lead in the non-commutating-pole machine. Thus to supplement the corrective action of the varying brush-contact area, the brushes must be given a *forward lead* in the direction of rotation, and further the amount of this lead (λ_s in electrical degrees) must be such as to cause the diameter of commutation to overtake and pass the neutral line where the resultant flux changes its direction relatively to the armature surface. How this is effected has already been described in Chapter XIX, end of § 7, but it may here be added that it is secured more rapidly in the compound-wound than in the shunt-wound machine, since in the former additional ampere-turns are provided by the field winding as the armature current increases, which take effect on the air-gap and assist in maintaining a distribution of flux more nearly resembling that which holds on no-load.[1]

But in both the compound-wound and shunt-wound dynamo, if the total effect of $f(t)$ is to be exactly proportioned to the armature current that has to be commuted, it is evident that the angle of lead must be varied in accordance with the variations of the load.

[1] For the effects from rocking the brushes, cp. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 253-265.

§ 18. Disadvantages of a large angle of lead.—With a large angle of lead the demagnetizing turns of the armature increase the necessary weight of copper on the field, and in the case of compound-wound machines, as explained in Chapter XVII, § 20, render the regulation for constant potential less perfect than it would otherwise be. In an extreme case, if the diameter of commutation has to be advanced much beyond the neutral line in order to reach the requisite strength of reversing field, the machine further becomes inefficient as a generator of E.M.F., since some of the active conductors are then inducing a back E.M.F. Such disadvantages are, however, of but small moment. The real objection to an angle of lead so great as to bring the diameter of commutation up to the leading pole-tips lies in the steepness of the gradient of the flux-density near to the pole-tip. This implies that small movements of the brushes or small changes in the armature ampere-turns will produce great variations of the reversing E.M.F. much more so than when the angle of lead is small and the coils are short-circuited near the middle of the interpolar gap. Hence, not only must the brushes then be very accurately situated to suit the exact load on the armature; but even when they can be so placed as to secure entirely sparkless collection for each value of the load, they become very sensitive to small changes of load, a slight alteration in the armature current materially altering the distribution of the field near the pole-tip. If the output fluctuates between wide limits as in practice is often the case—continuous attention may become necessary, and even then it may not be possible to shift the brushes quickly enough to meet rapid fluctuations. The greater the angle of lead at full-load, the greater is the percentage inaccuracy of the adjustment under varying load, and the more forcibly does the objection apply. It is therefore of the greatest importance to keep the angle of lead within small limits, and, if possible, to secure a fixed position of the brushes for all loads, so that no appreciable sparking results, however widely and rapidly the load may vary. To attain this in the absence of special (commutating) poles, the utmost use must be made of the action of the brush contact-resistance to correct either under-reversal or over-reversal.

If the brushes are so far advanced that the reversing field becomes too strong, cases (2) and (4) of accelerated commutation and over-reversal (Figs. 352 and 354) arise. But since it is, as shown above, of great importance to minimize the angle of lead as far as possible, such cases are at once remedied in generators at full-load by adopting a lesser angle of lead. They are not therefore of such frequent occurrence, except at no-load or intermediate loads, when the brushes are retained in the correct position for full-load. But in nearly every case, if the effect of sparking on the brushes and commutator be carefully examined, it will be found in generators

that it is the trailing edges of the sectors that are first pitted and worn by the sparks, and the leading edge of the brush that first deteriorates, showing that it is the final current-density and the final rate of change which have the greatest importance in generators.

§ 19. The reversing field from commutating poles.—The production of a reversing field by means of a commutating pole wound with ampere-turns AT_r exceeding and opposed to $JZ/4p$ has been already described in Chapter XV, § 7, and Chapter XIX, §§ 10-14. The curve of flux-density round the armature surface now shows a hump in the middle of each interpolar gap, the same in sign as the flux ahead of the commutating pole, as shown in Fig. 329 or 330.

Some variation in the adjustment of the strength of the reversing field thus obtained is given by shifting the diameter of commutation slightly to one or other side of the centre of the commutating pole.[1] But it will be seen that, speaking generally, the diameter of commutation must be close to the centre in order that the actual positions of the sides of the short-circuited loops may fall within the direct influence of the pole, and exact coincidence was assumed in the example of Chapter XIX, § 14.

§ 20. The division of the short-circuit current into two components.—In all cases when the current diverges from the straight-line, as in Figs. 351-354, it is instructive mentally to consider the actual current i as resolved into two components, i_c and i_z, where i_c is the part which corresponds to a straight-line change from $+J$ to $-J$, and i_z is an "additional" part which takes account of any excess or deficiency of the actual current as compared with the straight-line current. The two components i_c and i_z are indicated by dotted lines in Figs. 351-354. In the practical case of a brush covering a number of sectors, as the current i_z in any one coil assumes its different values, we may also picture a nearly stationary system of i_z currents as flowing through the coils, of varying amount from end to end and closed through the brush material. An imaginary instance when i_z reaches its maximum in a coil near the centre of the brush is indicated roughly in Fig. 355, and it is seen that this current is progressively fed into the winding from one end of the brush and tapped off at the other end through the commutator connectors.

It must, however, be emphasized that such a resolution of the actual current $i = i_c + i_z$ is entirely fictitious, in the sense that it is not legitimate to construct an independent differential equation for i_z (even if this were possible) to trace the complete curve

[1] Cp. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 257-267.

of i_z and then to superpose it on i_c.[1] The reason is that the brush contact resistances are affected by the actual values of the total current passing. Hence if the impressed E.M.F. from the field $f(t)$ be similarly divided into two components $f(t)_c + f(t)_z$, the former, corresponding to a straight-line change under the actual conditions, will no longer have the value deduced in § 7, when the drop of volts between leading and trailing sectors and brush cancels out.

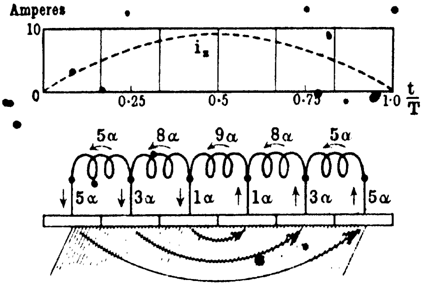


Fig. 355. Additional currents with brush covering a number of sectors.

§ 21. The inductance in relation to the i_z components.—But the advantage of the above resolution is that it clearly brings out the damping effect from several coils simultaneously short-circuited, so far as the i_z component is concerned. *E.g.* in the case of a dynamo as in Fig. 351, if three coils are simultaneously short-circuited at each brush, and coil A is close to the end of its short-circuit period, while the remaining coils A_I and A_{II} are situated at the points shown in Fig. 356, the rates of change di_{zI}/dt and di_{zII}/dt as shown by the tangents to the curve of i_z are of opposite sign to di_z/dt and help to *reduce* the apparent inductance of A.

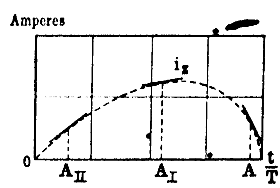


Fig. 356. Rates of change of "additional" currents.

On the other hand, for the same reason the apparent inductance

[1] The division of the short-circuit current in the above manner has been condemned by A. Mauduit in *Recherches sur la Commutation*, p. 285, but the criticisms apply rather to the illegitimate use of the division as a basis for mathematical deductions, and do not affect the usefulness of the conception as a means of picturing more clearly in the mind the process of commutation. In this valuable work, to which reference is frequently made in the present chapter, the reader will find a reasoned account of previous theories and experiments on commutation, as well as the very instructive experiments of M. Mauduit himself.

of coil A, when itself at the earlier stages of commutation, is actually increased, since it is then also carrying the damping current set up by another coil which is approaching the end of its short-circuit. The case is not, therefore, one of simple damping, and the inductance of coil A in relation to i_z is not a constant. But for the sake of simplicity it may be assumed that when the coil A is approaching the end of short-circuit, it adds to the current in the coils A_{I}, A_{II}, etc., damping components i_{I}', i_{II}', etc.; then by Kirchhoff's laws and assuming the coils to be similar, so that $\mathcal{L}_1 = \mathcal{L}$

$$-\mathcal{M}_1\frac{di_z}{dt} - \mathcal{L}\frac{di_1'}{dt} - r_1 . i_1' = 0.$$

Neglecting $r_1 . i_1'$ as comparatively small,

$$\frac{di_1'}{dt} = -\frac{\mathcal{M}_1}{\mathcal{L}}\cdot\frac{di_z}{dt}$$

The secondary current i_1' induces in the primary coil A an E.M.F.

$$-\mathcal{M}_1\frac{di_1'}{dt} = \frac{\mathcal{M}_1^2}{\mathcal{L}}\cdot\frac{di_z}{dt}$$

The self and mutually induced E.M.F. in the considered coil A is then

$$-(\mathcal{L}+\mathcal{M})\frac{di_z}{dt} + \Sigma\left(\frac{\mathcal{M}_1^2}{\mathcal{L}}\right)\frac{di_z}{dt}$$

or the apparent self and mutual inductance is

$$\mathcal{L} + \mathcal{M} - \Sigma\left(\frac{\mathcal{M}_1^2}{\mathcal{L}}\right) = \mathcal{L}_{sz}$$

Thus from the figures given in § 4 the apparent self-inductance of section A of Fig. 345 on an armature in air when coil-sides B and B_x in the same slots form a short-circuited secondary and act as dampers will be $\mathcal{L} - \frac{(0.7\mathcal{L})^2}{\mathcal{L}} = 0.51\mathcal{L}$. When the neighbouring section A_1 in *adjacent* slots acts as a damper, the inductance of A will be

$$\mathcal{L} - \frac{(0.33\mathcal{L})^2}{\mathcal{L}} = 0.89\mathcal{L}$$

and if both the section A_1 and the coil-sides B and B_x act as dampers

$$\mathcal{L} - 0.49\mathcal{L} - 0.11\mathcal{L} = 0.4\mathcal{L}.$$

At the final moment, then, the value $\mathcal{L}_{sz}$ which is less than $\mathcal{L} + \Sigma . \mathcal{M}$ is "apparent" for a double reason, since it takes into account not only the damping of any flux passing through the exciting coils, but also the damping action from the other sections simultaneously short-circuited.

§ 23. Importance of the quantity $\mathcal{L} + \Sigma . \mathcal{M}$. — In order to reduce the strength of reversing field necessary to give straight-line

commutation and a uniform current-density under the brushes, $L + \Sigma M$ in equation (180) should be small. The coefficients of mutual induction M_1, M_{11}, etc., between the considered coil and others which are simultaneously short-circuited should therefore be reduced as far as possible. On the other hand in order that the final rate of change of the additional current should not be unduly high, L_{s2} should be as small as possible, and the mutual inductance of other coils in close proximity should be high so as to increase their damping effect. Thus the more L_{s2} is reduced, the greater becomes $L + \Sigma M$, and reductions in both quantities are incompatible. A reduction in $L + \Sigma M$ should, however, take precedence over any reduction in L_{s2}. Any gain from damping at the end of short circuit can only be obtained at the expense of greater divergence from the straight-line ideal in the initial or intermediate stages. A straight-line current-change not only in coils A and B, but also in all the coils simultaneously short-circuited both at the same brushes as coil A (Fig. 345) and at adjacent brushes, is the necessary condition that they may all throughout have the same rate of change; they then all assist in retarding commutation, but by this alone can a constant and uniform current-density under the brushes be secured. Thus $L + \Sigma M$ is the chief factor in the required reversing field, and determines the necessary angle of lead in those cases where we are dependent upon shifting the brushes. The greater the required density of reversing field, the more sensitive becomes the machine to changes of load or to any dissymmetry of the various sections. Further, if there should be no reversing field present and the actual external field is small and negligible, the product of $L + \Sigma M$ with $2J/T$ becomes a sound guide to the amount of work that will be left to the corrective action of the brush contact-resistance to perform.

In order, therefore, to forecast the behaviour of a dynamo as regards sparkless running, it is imperative for the designer to estimate, even if only approximately, the value of $L + \Sigma M$. He must so dispose the coils which are simultaneously short-circuited that they have minimum mutual inductance so far as this is not forbidden by other considerations, and in each coil the number of turns must be so small, or, which amounts to the same, the number of commutator sectors for a given number of active conductors must be so large, that $L + \Sigma M$ is reasonably low.

It has already been stated that the self and mutual inductance of a short-circuited coil, strictly speaking, varies according to its position on the armature core relatively to the poles and therefore varies during short-circuit as rotation proceeds, yet that it may approximately be regarded as constant for a given position of the brushes. But now, further, for the purpose of approximate calculation the exact position of the brushes must be ignored, and

$\mathcal{L}+\Sigma\,\mathcal{M}$ must practically be identified with the value obtained when the group of coil-sides which are short-circuited at all brushes are as nearly as possible in the centre of the interpolar zone or directly under a commutating pole. The $\mathcal{L}+\Sigma\,\mathcal{M}$ of any one considered coil varies according to its position within the group, but in order that any error may be on the safe side it is best to take that position of the coil which leads to the maximum value of the inductance, *i.e.* when it is as nearly as possible in the centre of the group, and to regard this as the constant value. Even then the quantity $\mathcal{L}+\Sigma\,\mathcal{M}$ is by no means easily calculated with any great accuracy, and much could be done by direct experiment in the technical laboratory to bring the methods of calculation into closer relation to the facts. Thus the extent to which the flux entering the pole-faces also passes through the exciting coils, and there has its variations damped out, so that it adds little to the inductance, varies as between laminated and solid pole-shoes, and the amount of the flux entering the extreme pole-tips and deflected by the main excitation into an air-path to complete its circuit (Fig. 344 (*b*)) can only be determined by direct experiment.

§ 23. Method of calculating the self and mutual inductance of armature coils.—The *inductance* of a coil in absolute units is equal to the number of linkages of its component turns with the lines which thread through them when the current carried is one C.G.S. unit. Let w be the number of wires in one side of a coil, *i.e.* the coil has w turns. The M.M.F. of these wires when carrying one C.G.S. unit of current is $4\pi w$, and this acts upon a magnetic circuit of which the permeance has to be determined. But since the wires must necessarily occupy some space, they do not all act upon precisely the same circuit, so that all the flux to which they give rise is not completely linked with all the wires. The inductance of the group of wires is therefore usually given by obtaining an expression for a single equivalent permeance $\mathcal{P}$ such that if it were acted upon by the M.M.F. of all the wires, *i.e.* by $4\pi w$, the resulting flux $4\pi w\mathcal{P}$ linked with all the w wires gives the actual total number of linkages. The general expression for the inductance of the group of wires in practical units is then

$$\mathcal{L} = 4\pi w^2\mathcal{P} \times 10^{-9} \text{ henrys} \quad . \quad . \quad . \quad . \quad (181)$$

Usually $4\pi\mathcal{P}$ is grouped together as Λ in conventional units, so that $\mathcal{L} = w^2\Lambda \times 10^{-9}$, and if l be the length under consideration

$$\mathcal{L} = w^2 l\lambda \times 10^{-9} \text{ henrys} \quad . \quad . \quad . \quad . \quad (182)$$

The coefficient λ is therefore essentially an expression for 4π times the equivalent permeance per centimetre of the length which is under consideration in the given coil.

In the case of the toothed drum armature which has the greatest

practical interest, the permeance relatively to a half-coil will be divided into two main portions, corresponding to

(1) the *side of the coil* embedded in the iron of the armature core, and proportional therefore to the core-length ; and

(2) the *end-connexions* at one end from slot to slot, and proportional therefore to the length of an end-connexion as fixed by y_s[1] measured in slots.

The former will require to be again subdivided into—

(*a*) the portion corresponding to the flux *within the slots*, and this again into (*a'*) corresponding to the flux within the slot proper up to the bottom of the wooden wedge, and a part (*a''*) corresponding to the flux across the wedge and across the opening of the slot which may have overhanging edges ; and

(*b*) *the surface-of-the-core permeance.*

In the case of both (*a*) and (*b*) it may approximately be assumed that the iron of the armature core or teeth is infinitely permeable as compared with air. All the lines of (*a*) will then pass through the iron at the root of the tooth and round the bottom of the slot without loss of magnetic potential in their passage through this portion of their path.

Within the slot, three leading cases may be distinguished and are easily calculated upon the assumption that the flux passes practically in straight lines across the slot from side to side between the walls.

I. *Both the flux-density across the slot and the conductors with which it is linked are increasing* (*cp.* Fig. 358 (i)).

For any considered height h within which the conductors increase uniformly from 0 to w, the M.M.F. acting across a parallel-sided slot of width w_s with a current of one C.G.S. unit per conductor and at a distance x from the bottom of h is $4\pi w \frac{x}{h}$. The permeance of an infinitely thin strip of air across the slot is $\frac{dx}{w_s}$ per cm. length of the slot axially. The flux in the strip is therefore $\frac{4\pi w\,.\,x dx}{h\,.\,w_s}$ linked with wx/h conductors. An element of the inductance is therefore

$$d\lambda = 4\pi w^2 \frac{x^2 dx}{h^2 w_s}$$

and the integral $\int_0^h x^2 dx$ being $h^3/3$,

the total linkages $= 4\pi w^2 \frac{h}{3w_s}$ per cm. along the slot, and

$$\lambda = \frac{4\pi}{3} \times \frac{h}{w_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (183)$$

II. *The flux-density* from another similar coil in the same slot *is uniform* over the height h, *but the conductors of the considered coil with which the flux is linked are increasing* (*cp.* Fig. 358 (ii), where the wires of the considered coil are shown black, while those of the second coil-side in a lower layer are shaded).

The uniform flux-density for one C.G.S. unit of current per conductor of the second coil-side across the upper half of the slot is $\frac{4\pi w}{w'_s}$ and in the thin strip of air the flux is $4\pi w\frac{dx}{w'_s}$ linked with $w\frac{x}{h}$ wires of the considered coil-side at the distance x from its bottom. Therefore an element of the mutual inductance from the second coil-side is

$$d\mathcal{M} = 4\pi w^2\frac{x\,dx}{h w'_s}$$

and the integral $\int_0^h x\,dx$ being $h^2/2$,

the total linkages are $4\pi w^2\frac{h}{2w'_s}$ per cm. length, and

$$\lambda = \frac{4\pi}{2}\cdot\frac{h}{w'_s} = 2\pi h/w'_s \quad . \quad . \quad . \quad . \quad . \quad (184)$$

III. *All the flux is linked with all the conductors.*

In this, the simplest case, $4\pi w\frac{h}{w'_s}$ lines are linked with w conductors; the total linkages are therefore $4\pi w^2\frac{h}{w'_s}$ per cm. length, and

$$\lambda = 4\pi h/w'_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (185)$$

The inductance in the three cases, therefore, rises in the proportion $\frac{4\pi}{3}:\frac{4\pi}{2}:4\pi$, or as $1:1\frac{1}{2}:3$.

If there are present j coil-sides, each with w conductors, lying near by or alongside of one another, and the self and mutual inductance of one only of the coil-sides is to be calculated, different methods of procedure can be followed, dependent upon different ways of grouping the various permeances and fluxes. Thus either (1) the entire self-inductance of the considered coil is first calculated, and to this is then added the mutual inductance, an equivalent permeance being found which when acted on by the remaining $(j-1)$ coil-sides will give the actual number of linkages between the considered coil and the flux of the remaining coils. *E.g.* from the figures given in § 4, if $\mathcal{L}$ is the entire self-inductance of one of two neighbouring sections side by side in the same layer of the winding but in different slots, $\mathcal{M}$ is 0·33 of this value, so that $\mathcal{L}+\mathcal{M} = 1{\cdot}33\,\mathcal{L}$. Or when it is not necessary to know the value of $\mathcal{M}$ separately, it is usually more convenient (2) to take separately

the self-inductance of the considered coil so far as it embraces a local magnetic circuit which is entirely independent of the remaining coils; and then to calculate an equivalent permeance $\mathfrak{P}'$ such that, when acted upon by a M.M.F. of $4\pi jw$ from *all* the wires, the flux which results if linked with all the w wires of the considered coil will give the actual number of linkages from both the self and mutually induced flux in that portion of the magnetic circuit which is common to them all; *e.g.* in the same case as above, the separate flux due to the considered coil will yield $0{\cdot}66\ L$, and the joint flux, due to double the number of current turns will yield $2 \times (0{\cdot}33\ L)$, making in all $(0{\cdot}66 + 2 \times 0{\cdot}33)\ L = 1{\cdot}33\ L$. We thus have

$$L + \Sigma . M = 4\pi w^2 (\mathfrak{P} + j\mathfrak{P}').$$

Or if the number of coil-sides affecting the considered coil varies at different parts, the second term is again subdivided, j being given its correct value from one upwards for each of the several subdivisions, so that in general

$$L + \Sigma . M = 4\pi w^2(\mathfrak{P} + j_1\mathfrak{P}_1 + j_2\mathfrak{P}_2 + \ldots)$$
$$= w^2(l_1\lambda_1 + l_2\lambda_2 + l_3\lambda_3 + \ldots) \times 10^{-9} \text{ henrys.}$$

§ 24. The apparent inductance of coils of toothed drum armatures. In the case of a *drum* armature the number of wires in a coil-side forming an element of the armature winding is equal to the number of turns in the coil, that is, $w = Z/2C$. It must especially be remarked that in the multipolar drum armature the coil-sides undergoing short-circuit are not necessarily alike in each interpolar zone: when the number of commutator sectors per pole or $C/2p$ is *whole*, if the position of the short-circuited coil-sides in a pair of consecutive interpolar zones is set out, the remaining interpolar zones of the multipolar machine are merely repetitions of the first pair, but this is not the case when $C/2p$ is a *fractional* number. Further, unless the pitch of the coil is equal to the pole-pitch, so that it may by analogy from the 2-pole case be called "diametric," the positions of the two sides of any one coil among the groups of short-circuited coil-sides are dissimilar.

In the toothed armature, apart from the causes mentioned above, a dissimilarity in the two consecutive interpolar zones in which a considered coil may lie also results, even when $C/2p$ is a whole number, if the number of slots is not exactly divisible by the number of poles without remainder, *i.e.* if $S/2p$ is not a whole number.

The first portion (1) of the inductance which is proportional to the core length l cm. must therefore be determined separately for each of the two sides of a complete coil, as λ_1 and λ_2, the situations of the two sides often differing so radically that it is best to treat them separately and to add their inductances subsequently. The second portion (2) deals with the two end-connexions, each of length l' cm.

Thus for the drum coil

$$\mathcal{L} + \Sigma . \mathcal{M} = w^2 [l(\lambda_1 + \lambda_2) + 2l'\lambda'] \times 10^{-9} \text{ henrys} \quad . \quad (186)$$

In order to calculate its value, the first operation must be to set out clearly the position of the short-circuited coil-sides in all the interpolar zones so far as these differ, and to choose a certain pair of zones and a particular position for the considered coil therein which it is estimated will give the maximum number of linkages and maximum inductance. The three items (a′), (a″), and (b) which are proportional to the length of the core will then require to be estimated separately for each of the two zones in which lie the sides of the coil, and the corresponding values of λ to be indicated as λ_1', λ_2', λ_1'', λ_2'', λ_1''' and λ_2'''. For each of the several portions, again, the correct value for j must be taken.

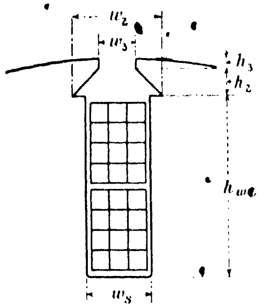


FIG. 357. Double layer winding in slot.

The usual case of a barrel winding in two layers being assumed with parallel-sided slots, the first item λ' is proportional to the ratio of the depth of the slot h_w below the wedge[1] to its width w_s (Fig. 357); the second item λ'' to the ratio of the heights above the coil-sides to the mean widths between them.

The surface-of-the-core permeance is divisible into the portions corresponding to the joint flux linked with all the simultaneously short-circuited sections, and to the local fluxes immediately linked with the section under consideration, or with two or more out of the total number. These several permeances, when corrected to suit the assumption that they are acted on by j_b, the total number of coil-sides short-circuited in the zone, may be grouped together as $b . j_b$.

The more developed expression for the apparent inductance, self and mutual, of a drum coil on a toothed armature is therefore

$$\begin{aligned}\mathcal{L} + \Sigma . \mathcal{M} &= w^2 [l(\lambda_1' + \lambda_1'' + \lambda_1''' + \lambda_2' + \lambda_2'' + \lambda_2''') \\ &\qquad + 2l'\lambda'] \times 10^{-9} \text{ henrys} \\ &= w^2\Big[l\Big(k_1'\frac{h_w}{w_s} + a_1'' j_{a1}'' + b_1 j_{b1}\Big) \\ &\qquad + l\Big(k_2'\frac{h_w}{w_s} + a_2'' j_{a2}'' + b_2 j_{b2}\Big) + 2l' c_c j_c\Big] \times 10^{-9} \quad . \quad (187)\end{aligned}$$

in which as in (186) $w = Z/2C$, the turns per coil.

[1] The copper of the coil-sides may with sufficient accuracy be assumed to meet in the centre of the slot and to reach up to the bottom of the wedge and down to the bottom of the slot.

(*a*) *The slot inductance* is easily deduced from the rules given in § 28 and on the assumptions named above, since in every case it is only necessary in the two-layer winding to write $h_w/2$ for h, and

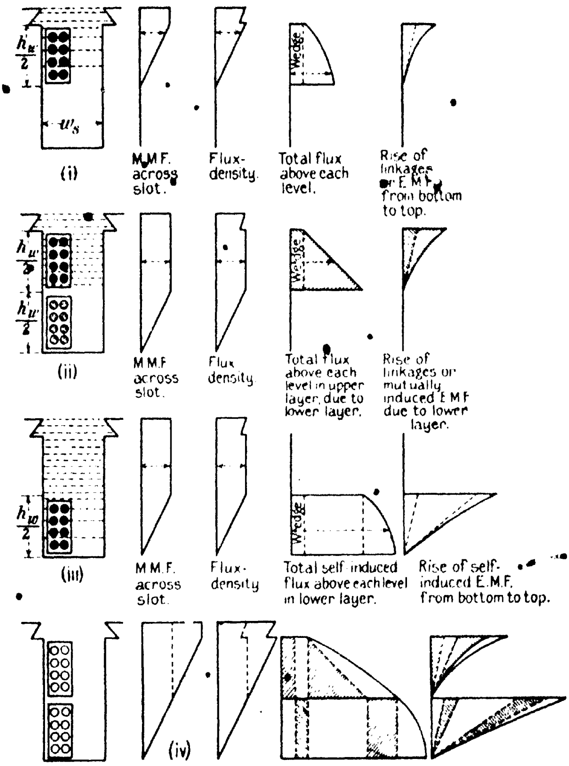


Fig. 358. Calculation of slot inductance in a double-layer winding.

(i) Self-inductance of upper coil side.
(ii) Mutual inductance of upper coil-side from lower coil side.
(iii) Self-inductance of lower coil side.
(iv) Self and mutual inductance of coil-side in both layers.

Mutually linked flux and mutually induced E.M.F. shown shaded.

to add up the component items for the (a') portion. The inductance in the three cases then rises when single coil-sides are alone in question in the proportion $\frac{2\pi}{3} : \pi ; 2\pi$, while between the bottom

of the wedge and the mouth of the slot for the (a'') portion, the inductance is proportional to 4π.

(a') Thus, for a single coil-side at the top of a slot (Fig. 358 (i)) by (I), $a' = \frac{2\pi}{3} \cdot \frac{h_w}{w_s} = 2{\cdot}09 \frac{h_w}{w_s}$. With a short-circuited coil-side added in the lower layer (Fig. 358 (ii)), the additional item from its mutual inductance by (II) is $\pi \frac{h_w}{w_s}$, so that for the upper coil-side per cm. length the total $a' = \frac{5\pi}{3} \cdot \frac{h_w}{w_s} = 5{\cdot}23 \frac{h_w}{w_s}$. If the considered coil-side is at the bottom of the slot (Fig. 358 (iii)), to the same flux and the same inductance due to it within the lower half of the slot up to the height $h_w/2$, as in our first case, has now to be added a further self-induced flux of uniform density in the upper half, for which by (III), $a' = 2\pi \frac{h_w}{w_s}$; the total is therefore $a' = \left(\frac{2\pi}{3} + 2\pi\right) \frac{h_w}{w_s} = 8{\cdot}37 \frac{h_w}{w_s}$.

If in either of the two layers there are j coil-sides short-circuited instead of the single one assumed above, it is only necessary to multiply the corresponding item by j. The three leading cases when expressed in a form immediately applicable to our main equation are therefore as follows—

$$k' = a'j_a' \div \frac{h_w}{w_s} = 2{\cdot}09\, j_a \text{ in case I of § 23}$$
$$= 3{\cdot}14\, j_a \quad ,, \quad ,, \quad \text{II} \quad ,, \quad ,, \quad ,,$$
$$= 6{\cdot}28\, j_a \quad ,, \quad ,, \quad \text{III} \quad ,, \quad ,, \quad ,,$$

Into these elements any more complicated case may be resolved, and

$$\lambda' = \Sigma\,(a'j_a') = k' \frac{h_w}{w_s}$$

where k' can be tabulated beforehand for any given arrangement of coil-sides in a slot (*cp.* Fig. 359) without a knowledge of the ratio h_w/w_s. Thus the value of k' for the considered coil-side (marked black) with the different positions of Fig. 359 for the short-circuited coil-sides in the slot containing it are—

(1)	$k' = 2{\cdot}09 + 6{\cdot}28$	$= 8{\cdot}37$	(7)	$k' = 2 \times 2{\cdot}09 + 3{\cdot}14$	$= 7{\cdot}32$
(2)	$= 2{\cdot}09 + 6{\cdot}28 + 3{\cdot}14$	$= 11{\cdot}51$	(8)	$= 2{\cdot}09 + 2 \times 3{\cdot}14$	$= 8{\cdot}37$
(3)	$= 2{\cdot}09 + 3{\cdot}14$	$= 5{\cdot}23$	(9)	$= 2(2{\cdot}09 + 3{\cdot}14)$	$= 10{\cdot}46$
(4)	$= 2(2{\cdot}09 + 6{\cdot}28)$	$= 16{\cdot}74$	(10)	$= 2(2{\cdot}09 + 6{\cdot}28 + 3{\cdot}14)$	$= 23{\cdot}02$
(5)	$= 2 \times 2{\cdot}09$	$= 4{\cdot}18$	(11)	$= 3(2{\cdot}09 + 6{\cdot}28) + 3{\cdot}14$	$= 28{\cdot}25$
(6)	$= 2(2{\cdot}09 + 6{\cdot}28) + 3{\cdot}14$	$= 19{\cdot}88$	(12)	$= 3 \times 2{\cdot}09 + 3{\cdot}14$	$= 9{\cdot}42$

Strictly speaking, the first items arising under case I only hold for a coil-side divided into a large number of laminæ or layers of small wires. For solid conductors in each layer, of a height corresponding to $h_w/2$, there is no self-inductance in the conductor as a whole from the flux within the conductor's own height; the effect from this for reasons given in Chapter XXVIII, § 6, causes a non-uniform distribution of the current-density at any moment over the cross-section of the solid conductor, and an apparent increase in its ohmic resistance, which should appear in the full formula for the short-circuit condition. If an approximate correction in the

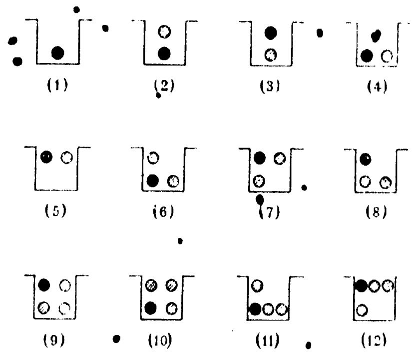


FIG. 359.

total inductance is made by halving the constant 2·09 or even by neglecting it, the increased resistance should be introduced. Yet since this is difficult to calculate[1] and it is not otherwise allowed for, it suffices for comparative purposes to make use of the quantity $2{\cdot}09j_a$, as if the conductors were composed of numerous very thin laminae.

(a'') In relation to the flux above the upper layer of coils, $\lambda'' = a'' j_a''$, and j_a'' is simply the number of short-circuited coil-sides in the slot containing the considered coil. So far as the distance $h_s - h_w$ is concerned, $a'' = 4\pi\left(\frac{2h_2}{a_2} + \frac{h_3}{a_3}\right)$. But at the extreme edges of the opening, the densely crowded lines curve outwards into the air-gap, so that to the slot proper may be added a zone within a semi-circle described on the mouth of the slot (*cp.* Fig. 360). The mean area and mean length of path varying together, the permeance

[1] For attempts at such calculation, see M. B. Field, "Idle Currents," *Journ. I.E.E.*, Vol. 37, p. 112; and L. Fleischmann, *Archiv für Elektrotechnik*, Vol. 2, p. 387. But the assumption of a rectangular shape for the current wave in the conductor departs from the actual fact in an important respect.

is independent of the width of the opening, and as an approximate estimate 0·2 may be taken as its value, so that

$$a'' = 12{\cdot}57\left(\frac{2h_2}{w_2 + w_3} + \frac{h_3}{w_3} + 0{\cdot}2\right) \quad . \quad . \quad . \quad . \quad . \quad (188)$$

which reduces to $12{\cdot}57\left(\frac{2h_2}{w_2 + w_s} + 0{\cdot}2\right)$ in the ordinary case of an open slot where h_2 is the height of a wedge flush with the surface.

(*b*) *The surface of the core.*

Next must be taken the inductance from the *surface of the core*, the group of short-circuited coil-sides being assumed to be situated midway between the pole-tips for the purpose of obtaining comparative results, although such an assumption is not strictly true when the brushes are shifted away from the line of symmetry. Further, the calculation will be made for the armature removed from its field-magnet system and situated in air.

The experiments of Pichelmayer[1] and also of J. Rezelman[2] show that, when tested with alternating currents of about 100 frequency, the apparent inductance of sections on an armature with the field-magnet circuit closed is not far different from their real inductance when the armature is in air. The slot and end-connexion inductance is not thereby altered, so that the similarity of the quantitative results obtained in the two cases is simply due to the fact that the flux which continues to circle through the field-magnet coils and sets up the damping variation in them and eddy-currents in the solid iron has about the same magnitude as the flux from the surface of the core when in air. How far this remains true at higher frequencies of 500, comparable with the ordinary conditions in commutation, remains doubtful, but from the ease with which armatures can be tested with alternating currents (*vide* § 46), and the comparative figures thereby obtained can be used to check the results of calculation, the convenience of the assumption warrants its adoption in practice. It is only on this account that the inductance of a section or sections of an armature in air possesses any interest.

In air with frequencies up to 200, Rezelman's experiments with slotted armatures showed that the reactance of a section or sections remained proportional to the frequency, so that there was no perceptible damping effect in the laminated iron of the armature. With laminated poles of plates 1 mm. thick immediately above the slots occupied by the section under test with an air-gap of 3·5 mm. = 0·138″, although the inductance was greatly increased, there

[1] *E.T.Z.*, Vol. 33, p. 1100, abstracted in *Elektr.*, 28th Feb., 1913.

[2] *Recherches sur les Phénomènes de la Commutation.*

was no perceptible change of the reactance with increasing frequency.[1] Even when solid commutating poles of cast steel, unexcited and with no yoke, were presented above the slots occupied by the section under test, although the total inductance was increased 30 per cent., there was no damping effect[2] with an air-gap of 4 mm. = 0·157″; if, however, the air-gap was reduced to 1 mm. = 0·0394″, damping was shown by the apparent inductance or permeance becoming lessened with increase of the frequency and by a marked increase in the watts absorbed.

The permeance per cm. length of the armature surface within the span of the slots containing the short-circuited coil-sides is dependent on the number of these slots and on the relative widths of the slot-opening and tooth-crown. The remaining permeance corresponding to the joint flux due to the j_b coil-sides is practically independent of the slotting of the core owing to the comparatively great length of the paths in the air. The total surface permeance might therefore be divided into these two portions. A different division will, however, be found more convenient; a first portion will be made to embrace the whole of the span from the interpolar line of symmetry up to the pole-tips, a standard ratio of $\frac{\text{pole-arc}}{\text{pole-pitch}} = \beta = 0{\cdot}7$ and a standard ratio of $\frac{\text{width of tooth-crown}}{\text{width of slot-opening}} = \frac{w_t}{w_3} = 1{\cdot}5$ being adopted as fairly representing ordinary practice; the results for this portion can then be once for all calculated in terms of $D/(2p \times w_3)$ on the basis of a flat core, since the departure from a flat surface within these limits is not great.

For the second portion of the permeance from the pole-tips onwards up to the middle of the pole, but with the armature in air, it will be shown in a Note added at the end of this chapter that, whatever the number of poles or diameter of the armature, when the curvature of the convex surface is taken into account the value of $4\pi\mathcal{S}$ per cm. length of core may fairly be represented by the figure 10.

In consequence in the following formulæ and curves which are applicable to a single zone alone, the value of $4\pi\mathcal{S}$ up to the pole-tips has been calculated and to this in every case has been added a constant figure of 10 to give $b = 4\pi\mathcal{S}/l$, where $\mathcal{S}$ is the equivalent permeance for flux assumed linked with $w = Z/2C$ wires. The

[1] Very little flux in such a case is linked with the exciting coils, but when the section was on the interpolar line of symmetry, the damping effect was of course very marked, the flux not only being linked with the exciting coils but also penetrating the joint between the laminated poles and the solid steel yoke, where eddy-currents would be set up.

[2] The poles and armature being stationary, eddy-currents due to the passage of the slot-opening past the solid pole-faces are not here in question.

coefficient for insertion in our principal equation (187) is then for the single zone

$$\lambda''' = b \cdot j_b$$

The number of sections simultaneously short-circuited is $\left(\frac{b_1 - b_m}{b}\right)_+$, where the subscript *plus* sign indicates that the next larger whole number is to be taken if there is any remainder. The number j_b of short-circuited coil-sides in one zone is then either $2\left(\frac{b_1 - b_m}{b}\right)_+$ or $2\left(\frac{b_1 - b_m}{b}\right)_+ - 1$, and in every case the value of b given in the following curves is arranged so that it may be multiplied by the full number j_b of coils short-circuited in the zone in question.

The leading cases may be grouped into four or five kinds, according as the short-circuited coil-sides are concentrated in one slot or are divided between two, three, four, or five slots. A larger number than five slots containing coil-sides simultaneously short-circuited seldom occurs in practice.

(i) *Single slot.* When the short-circuited coil-sides are confined to a single slot (Fig. 360), the lines of flux are best assumed to be semicircles spanning the opening of the slot. With the standard ratio of pole-arc to pole-pitch as assumed above, the arc from the centre of the slot-opening up to the pole-tip is $0{\cdot}15\pi D/2p = 0{\cdot}471 D/2p$. The permeance in one interpolar zone per cm. length of core is then by equation (121) $\frac{2{\cdot}3}{\pi}\log\left(0{\cdot}942\frac{D}{2p \times w_3}\right)$ acted upon by a M.M.F. of $4\pi j_b w$, where j_b is the total number of short-circuited elements in the one zone and w is the number of wires in the coil-side forming an element. The resulting flux is linked with the w wires of the considered coil, so that the self and mutual inductance is

$$4\pi j_b w^2 \times \frac{2{\cdot}3}{\pi}\log\left(0{\cdot}942\frac{D}{2p \times w_3}\right) = w^2 j_b \times 9{\cdot}2\log\left(0{\cdot}942\frac{D}{2p \times w_3}\right) \times 10^{-9}\text{ henrys.}$$

Since $\log\left(\frac{D}{2p \times w_3} \times 0{\cdot}942\right) = \log\frac{D}{2p \times w_3} - 0{\cdot}0259$, the total value of b with the second division also taken into account is

$$b = 9{\cdot}2\log\frac{D}{2p \times w_3} - 0{\cdot}238 + 10$$

$$= 9{\cdot}2\log\frac{D}{2p \times w_3} + 9{\cdot}762$$

as given in the upper curve of Fig. 360.

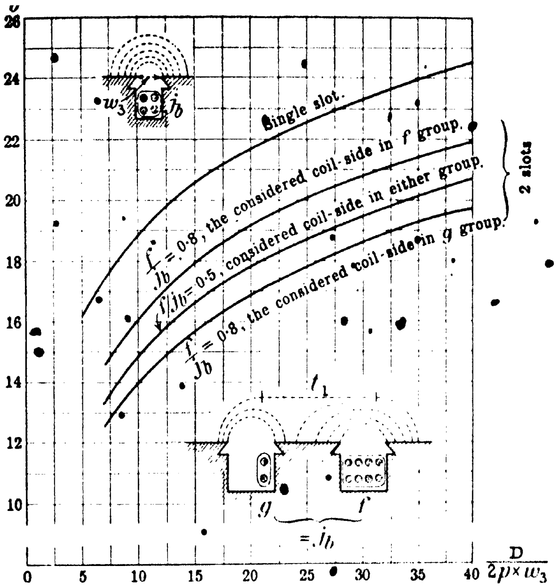


FIG. 360. Short circuited coil-sides in a single slot, or in two adjacent slots.

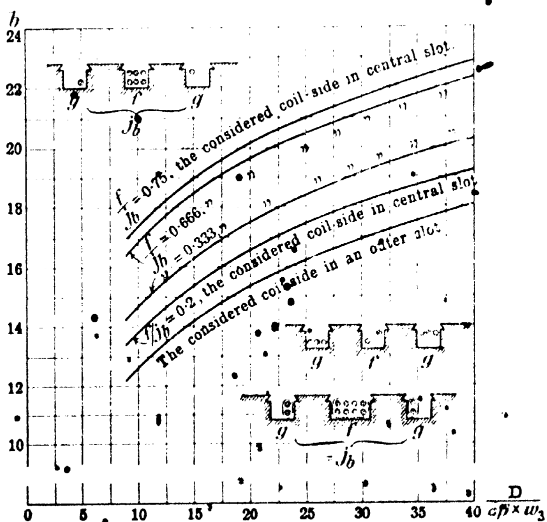


FIG. 361.—Short-circuited coil-sides in three slots symmetrical about centre.

The inductance is therefore a function of the quantity $D/2pw_s$, and the curve of Fig. 360 becomes universally applicable to different numbers of poles and different widths of slot. After having calculated $D/2pw_s$ in any given case, it is only necessary to read off the value of b from the curve and to multiply it by j_b, or the number of short-circuited coil-sides which are found to be present in the single interpolar zone.

The actual position of the short-circuited coil-sides within the slot is in the present connexion immaterial; thus in the diagram the considered coil-side is marked black, and there are three other short-circuited coil-sides shown shaded, so that $j_b = 4$.

When the short-circuited coil-sides are spread over more than one slot, in addition to the joint flux linked with all the coils, there are also in general local fluxes linked immediately with one or more slots, and the methods of calculation adopted are explained in the Note appended at the end of this chapter. The assumptions made cannot at best be entirely accurate, but by them correct relative results are approximately obtained.

(ii) *Two adjacent slots*, with the short-circuited coil-sides $f + g = j_b$ equally or unequally divided between them (lower curves of Fig. 360, calculated on the assumption given in the Note, p. 166).

As the disproportion between f and g increases, the coefficient for a coil-side situated in the slot having the larger number f approaches that for a single slot, while conversely the coefficient for a coil-side in the smaller group falls and approximates to the value for the joint flux alone.

Other curves for different values of f_g are easily interpolated.

(iii) *Three adjacent slots.* The curves of Fig. 361 apply to the case of three adjacent slots, the central containing f coil-sides, and each of the outer slots g coil-sides. But the distribution may also be unsymmetrical about the centre line, as shown in Fig. 362. In such cases it is usually an outer slot containing the greatest number in which the considered coil-side will lie, and the lower curve of Fig. 361 applies. But if it lies in the central slot, the dyssymetry may be ignored, and the same curves of Fig. 361 may be used with interpolation for the appropriate value of f/j_b.

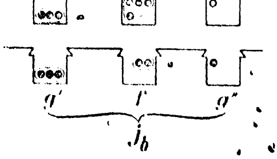


FIG. 362.—Short-circuited coil-sides in three slots unsymmetrical about centre.

(iv) *Four adjacent slots.* The curves of Fig. 363 are calculated for the symmetrical cases A and B, and by comparison with these other cases can be dealt with by interpolation.

(v) *Five adjacent slots,* each containing short-circuited coil-sides, rarely occur: in Fig. 389 of the Note to this chapter is shown the coefficient for the central slot, if all were equally filled.

In the calculation of (*a*) and (*b*), for accuracy from L, the total axial length of the armature core, should be deducted the sum of the widths of the air-ducts, to obtain l the magnetic length of the core; the difference should then be added to l', the length of the end-connexions in air. Further, the inductance from steel binding-wires, if present, has above been neglected.

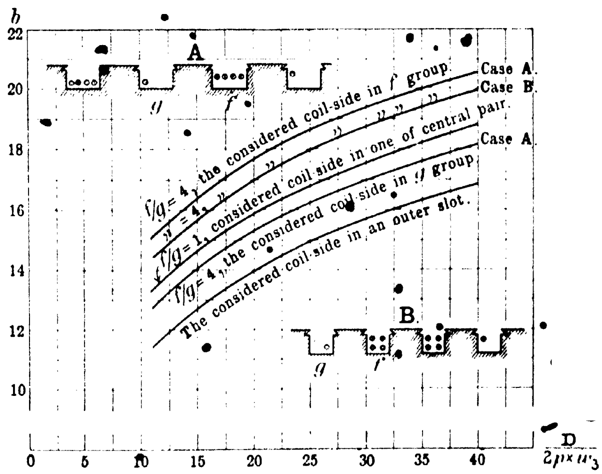


FIG. 363.—Short-circuited coil-sides in four or five slots.

The preceding curves show the gradual decline in the rate at which the permeance and the inductance rise as the diameter of armature increases for the same number of poles and width of opening of slot. Their lower terminations in all cases mark the point at which the width of all the short-circuited coil-sides approaches dangerously close to the total width of the interpolar zone between the pole-tips; as an extreme maximum the number of teeth and slots in which lie the short-circuited coils should in the non-commutating-pole machine always fall short of the width of the whole interpolar zone by at least the width of one tooth-pitch, so that for one, two, three, four, or five slots involved in the span of the sections short-circuited by the brushes, $\frac{D}{2p \cdot w_3}$ should not be less than about 5, 7, 9, 11, and 14 respectively. With commutating poles the short-circuited coils will be less widely spread.

(c) *The end-connexions.*

In accordance with the conclusions reached in the Note appended at the end of this chapter, for the V-shaped end-connexions of a barrel-wound armature, the expression adopted for insertion in the principal equation (187) is

$$\lambda' = j_e c_e \left(\frac{b_1 - b_m}{b}\right)_{\dagger} \times 3 \log \frac{l'}{(a + b)} \quad . \quad . \quad . \quad (189)$$

where l' is the length of one end, and a and b are the two dimensions, height and width, of the rectangular packet formed by the coil-ends which are simultaneously short-circuited at one brush (Fig. 391).

§ 25. Influence of pitch of armature coils.—It is instructive to note the influence which the pitch of the armature winding has upon the slot and core inductance, and in order to illustrate this a number of diagrams of two interpolar zones are collected in Fig. 364 for the particular case of three sectors or six coil-sides per slot in two layers and a ratio $\left(\frac{b_1 - b_m}{b}\right)_{\dagger} = 4$.

When the number of slots is an exact multiple of $2p$, *i.e.* $S/2p$ is an integer, true *diametric winding* is obtained when the back-pitch of the armature coil reckoned in elements is $y_B = \frac{u \times S}{2p} + 1$, where u = the number of coil-sides per slot, and S = the number of slots; or when reckoned in terms of slots, $y_B{}^1 = S/2p$. In this case each of the sides of a complete coil will also have exactly above or below it in the same slot another coil-side which is short-circuited at a neighbouring brush of opposite polarity, so that the inductance for a given value of $\left(\frac{b_1 - b_m}{b}\right)_{\dagger}$ is a maximum. To shorten y_B, its preceding diametric value must be reduced in steps of two elements, *i.e.* the next lower possible values are $\frac{u \times S}{2p} - 1$, $\frac{u \times S}{2p} - 3$, and so on. After passing through a number of such steps $= \frac{u}{2} - 1$, the rear pitch in slots is itself shortened and becomes $y_B{}^1 = (S/2p) - 1$. The winding may then be called *long chord*, the definition of this term being that with narrow brushes the two coil-sides which are then short-circuited in each zone are separated by an intervening tooth. After the same number of intermediate steps as before, the rear pitch in slots is again shortened to $(S/2p) - 2$; the two coil-sides short-circuited by narrow brushes in each zone are now separated by two teeth and one slot, and the winding may be termed *short chord*.

The intermediate cases when the amount by which the pitch is shortened is not equal to a complete slot do not yield interchangeable coils, so that in practice they are seldom used (but *cp.* p. 167). When Fig. 364 is examined, it is seen that as we proceed down

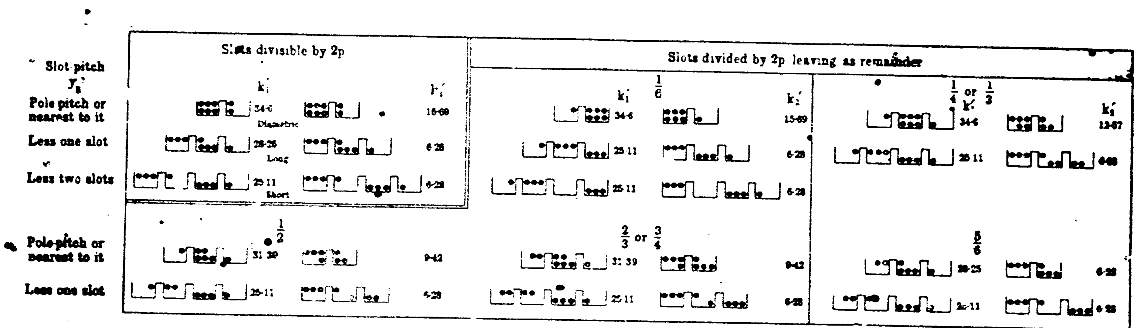


FIG. 364.—Six coil-sides per slot; $\left(\frac{b_1 - b_m}{b}\right)_+ = 4$.

the scale from diametric to long chord, and thence to short chord the two layers of short-circuited coil-sides are, as it were, gradually sheared apart, and the greatest $\mathcal{L}+\Sigma\mathcal{M}$ of any one coil out of a given number short-circuited by the brush width is reduced.

When $S/2p$ is fractional, the smaller the remainder, the nearer can the rear pitch in slots be brought to equality with the pole-pitch, so as to resemble the true diametric case. From this point a similar gradual shearing apart is produced as $y_B{}'$ is shortened by one or more complete slots. The reduction in the values of k_1' and k_2' for the slot inductance is shown by the figures in Fig. 364.

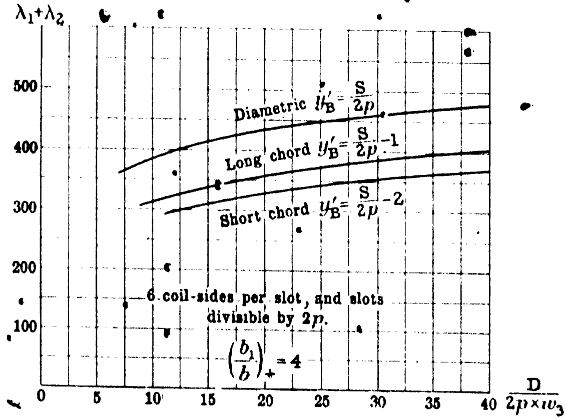


FIG. 365.—Effect of shortening the pitch of coils.

Taking any particular values of $\left(\frac{b_1-b_m}{b}\right)_+$ and of u, if the surface-of-the-core inductance is calculated and added to the slot inductance it will be found that their sum is appreciably reduced by shortening the coil-pitch and so spreading out the two layers of short-circuited coil-sides in each zone. Fig. 365 shows this for six coil-sides per slot, and $\frac{b_1-b_m}{b}=4$, and on the assumption that the slot is open and with a ratio $\frac{h_w}{w_s}=2\cdot5$. The percentage reduction is considerable for the first shortening of the slot-pitch from $S/2p$ to $(S/2p)-1$, and is much less when the process is carried still further.

With fractional windings the number of coil-sides short-circuited in one zone may be either $\frac{2(b_1-b_m)}{b}$ or $\frac{2(b_1-b_m)}{b}-1$, according to the circumstances of the brush width and pitch; in the latter case the

curves of the surface-of-the-core inductance are somewhat flatter in large armatures where the joint flux predominates than in whole windings with the same width of brush. There is therefore a slight advantage in fractional as compared with whole values of $S/2p$, but the difference is not great.

In conclusion, it may be pointed out that occasionally a coil is wound with a number of wires in parallel; but if so, they must be wound in the same slot or slots as if they were a single conductor, since otherwise eddy-currents would be set up within the coil, due to the different E.M.F.'s generated by the wires in the different slots.

§ 26. The inductance of the short-circuited sections with commutating poles. The question next arises how far the $L + \Sigma M$ of given coils is affected by the presence of commutating poles.

When the short-circuited sections are situated centrally under the iron face of a commutating pole, and assuming the latter to be unexcited, the self and mutually induced flux would very largely pass through the short commutating air-gap of length l_{gr} into the commutating pole, cross the iron pole, and emerge again through the air-gap l_{gr}; or if the short-circuited coils are not quite central under the commutating pole, but nearer one edge, some of their flux would pass directly up the commutating pole and close its circuit round the yoke and an adjacent main pole. In either case the convenient iron path thus presented to the short-circuited coils and the shortened length of the air-path would largely increase the inductance.[1]

But this is not the case when, as in practice, the commutating pole is excited, so that reversing flux passes into the armature core. The M.M.F. of the total armature ampere-turns including the short-circuited turns is overcome by the greater M.M.F. of the coils exciting the commutating poles. The M.M.F. of the short-circuited turns is then expended mainly over the commutating air-gap in driving the reversing flux aslant across the air-gap and slots along a longer path and in a backward direction towards the rear of the short-circuited coils. Thus just as the surface-of-the-core and slot inductance of the short-circuited coils without commutating poles is not actually due to the separate existence of lines round them but shows itself in the displacement of the resultant field, so now the surface-of-the-core and slot inductance with commutating poles is really represented by the distortion and displacement of the reversing field. The effect mainly makes its appearance in the air paths, and there is probably but little difference in the inductance of the same short-circuited coils, with and without commutating poles, when in each case centrally situated in the middle of the interpolar gap. The stored energy of a short-circuited section

[1] The experiments of J. Rezelman (*Recherches sur les Phénomènes de la Commutation*, p. 24) confirm this.

is virtually the quantity requiring to be calculated, and if in both cases this is stored in air, its value under different distributions of the flux is presumably not greatly different. In default, therefore, of exact experiment, the value of $l + \Sigma . h$ calculated as in § 24 may be used equally in connection with commutating poles.

One point need only here be stated in addition. In order to keep the circumferential breadth of the commutating pole within practical limits, the conductors short-circuited in each interpolar zone must be more closely concentrated than is advisable in the absence of commutating poles. The armature winding may not require to be actually diametric, but it often is so made, and it is only in this case that the two short-circuited sides of a coil will be exactly similarly situated in reference to a pair of reversing fields, and will have equal E.M.F.'s impressed upon them, so that their effect reaches a maximum. In consequence then of $y_B{}^1$ being nearly or exactly equal to the number of slots within the pole-pitch (*cp.* p. 167), the $l + \Sigma . h$ with commutating poles is naturally high as compared with the case of windings usual on non-commutating-pole machines which are more or less chorded.

§ 27. The time of commutation. While in the simple lap-wound armature $T = \dfrac{b_1 - b_m}{v_c}$, in the multiplex lap-wound armature for the same brush width the time is reduced to

$$T = \frac{b_1 - b_m + b\left(1 - \dfrac{a}{p}\right)}{v_c}$$

where $\dfrac{a}{p}$ is greater than unity.

In the simple wave-wound armature with as many sets of brushes as there are poles, a coil is first short-circuited as part of a complete zigzag round the armature which returns to a sector adjacent to that from which it starts and under the same brush. The short-circuit current can then pass not only *via* the complete zigzag, but also through all the leads connecting the brushes of the same sign; and the duration of this stage is exactly as in the simple lap armature given by the passage of a mica strip past the brush, or $t = \dfrac{b_1 - b_m}{v_c}$. The second stage consists in the successive reduction of the number of parallel paths open to the short-circuit current through the brush leads until only one is left through the nearest brush of the same sign, and finally this remaining path is opened by the adjacent brush of the same sign leaving the sector which forms one end of the coil under consideration. Thus the second stage is itself divisible into $p - 1$ successive reductions of the parallel paths open to the current of the coil. The difference between the commutator pitch

and the pitch of the brushes of similar sign, *i.e.* $y_c - C/p$, is the fraction of a sector corresponding to each of the $p-1$ changes, and this difference is equal to $\pm 1/p$. The space traversed during each of the changes is therefore b/p, and the movement throughout the second stage during the $p-1$ changes is $\frac{b}{p}(p-1)$. The total time of short-circuit at one brush and between brushes of the same sign is thus extended to $T = t + \frac{b(p-1)}{pv_c} = \frac{b_1 - b_m + b\left(1 - \frac{1}{p}\right)}{v_c}$.

This extension of the time is in itself advantageous to the commutation,[1] so that the use of as many sets of brushes as there are poles is to be recommended so long as they are easily visible and accessible, and do not give too much friction in small machines. What percentage increase in the time of commutation is obtained entirely depends upon the width of the brush; thus if the width of the brush expressed in terms of the width of a sector is xb,

$T = t\left(1 + \dfrac{1 - \frac{1}{p}}{x}\right)$; *e.g.* with four poles and $x = 1$, $T = 1\frac{3}{4}t$, but if the brush width is twice that of a sector, $T = 1\frac{3}{8}t$. With wide brushes therefore the increase becomes of less and less importance.

With multiplex wave-wound armatures the case is slightly different. If $b_1 - b_m < b$, a complete zigzag round the armature is never short-circuited; there are then, as above, $p-1$ successive stages of reduction in the number of paths through the leads connecting brushes of the same sign, each lasting a time corresponding to $\frac{a}{p} \cdot b$, and the total time corresponds to $b \cdot \frac{a}{p}(p-1)$. So small a brush width would, however, not be used in practice. As soon as $b_1 - b_m > (a-1)b$, an initial stage is added on during which a complete zigzag is short-circuited, and the duration of this depends upon the amount by which $b_1 - b_m$ exceeds $(a-1)b$. The time of this stage is therefore proportional to $b_1 - b_m - (a-1)b$, and the total time of all the stages is proportional to $b_1 - b_m - (a-1)b + b \cdot \frac{a}{p}(p-1)$ $= b_1 - b_m + b\left(1 - \frac{a}{p}\right)$. Thus the general formula

$$T = \frac{b_1 - b_m + b\left(1 - \frac{a}{p}\right)}{v_c} = \left\{\frac{b_1 - b_m}{b} + \left(1 - \frac{a}{p}\right)\right\} \times \frac{60}{CN} \quad (190)$$

[1] *Cp.* J. K. Catterson-Smith, *Journ. I.E.E.*, Vol. 35, p. 430; and Professor F. G. Baily and Mr. W. S. H. Cleghorne, *Journ. I.E.E.*, Vol. 38, p. 168.

holds in all cases, whether the armature be simplex or multiplex, wave or lap-wound, so long as there are as many sets of brushes as there are poles. Or if $b_1 - b_m$ and b are in inches, and v_c' in feet per minute,

$$T = \frac{5}{v_c'}\left\{b_1 - b_m + b\left(1 - \frac{a}{p}\right)\right\} \text{ seconds} \quad . \quad . \quad . \quad (190a)$$

§ 28. The true criterion of sparking.—It remains to estimate quantitatively the factors upon which sparking at the brushes ultimately rests, for the guidance of the designer as to the degree in which sparkless running is likely to be attained in any given machine.

The true measure of the limitation of the output by sparking is not simply the maximum current-density which occurs at any time during short-circuit at the face of the brush, nor is it the maximum value which the difference of potential between brush and commutator reaches. Neither current nor voltage in themselves are to be feared; it is the integral effect of their product, namely, electrical energy, which alone can give rise to any destructive effect from sparking or over-heating. Yet even this statement must be qualified at least in the case of sparking by a further proviso. As Professor Arnold[1] has pointed out, a certain voltage must be present between the surfaces of brush and commutator before even a high watt-density becomes detrimental in a dynamo.

That the actual expenditure of energy *in the air* by a visible spark *need* not in itself damage the surfaces is shown by the discharge points of an induction coil which are uninjured by the sparks. And on this account Mr. Thorburn Reid[2] has given as the criterion of the sparking limit the maximum energy-density or rather watt-density at the trailing edge of a sector. But actual facts show that very high watt-densities which would lead to no objectionable consequences with plain slip-ring contacts would quickly cause sparking on the commutators of direct-current machines. The difference lies in the value of the voltage which forms one of the components of the watts, and which for the same value of the watt-density will be higher in the commutator than with the slip-ring. The two cases are, in fact, decisively differentiated by the presence of the slight percussions and vibration to which the brushes are subjected on the commutator as they traverse the mica and copper strips, even when the surface is as smooth as it is possible to attain in practice. The predisposing cause is therefore to be found in a mechanical consideration, which for the same current-density raises the voltage at the point of contact, or the apparent contact-resistance, in the commutator as compared with the slip-ring.

[1] *Die Gleichstrommaschine* (2nd edit.), Vol. 1, p. 403.

[2] *Trans. Amer. I.E.E.*, Vol. 14, p. 611.

The abrupt rise of voltage after the normal maximum value of $R_k . s_u'$ is reached, indicates that the true conducting contact has been ruptured and that an arc or spark, however minute, has formed.

Accompanied, then, by voltages exceeding certain limits which may vary according to the conditions, the watt-density which finally causes the visible spark does materially damage the surface of both commutator and brush. Especially has a *rapid variation* of the current-density to be guarded against; for it is this which, through the action of self-induction, supplies the E.M.F. necessary for damage to the surface to result under the conditions of the continuous-current dynamo, and apart from any electrical or magnetic cause for it, such rapid variation may itself be solely due to mechanical jarring. Too high a watt-density alone may cause carbon brushes to glow, and this must be avoided as causing deterioration of their surface, but comparatively low values of the current-density, if accompanied by voltages increasing roughly in inverse proportion to the current, will still suffice to produce injurious sparking. The slight mechanical chattering of the brushes as modifying their true steady contact-resistance, and the effect of rapid current variation, are therefore in combination the primary causes of the sparking difficulty.

A calculation of the total energy which may have to be expended in the spark at the moment of opening the short-circuit of a section suggests itself, in the first place, as a guide to the relative merits of different machines. If $e_1 =$ the instantaneous P.D. between the leading sector and brush-tip at any time after the rupture of the true contact-resistance, and $i_1 =$ the current then passing through the leading sector, the energy expended in the spark during the time t for which it lasts is $\int_0^t e_1 i_1 \, dt$. Experiment indicates that the mean power $\frac{1}{t}\int_0^t e_1 i_1 \, dt$ per cm. width of brush-tip is of the order of 5 to 25 watts and should not exceed 10 watts per cm. if the sparking is to be permissible.[1]

When breaking a current in an inductive circuit at a switch, the energy stored in virtue of its inductance, viz. $\int \mathcal{L} i di = \frac{1}{2} \mathcal{L} I^2$ is liberated and is expended partly over the resistance of the circuit and partly in the spark or arc between the blade and jaws of the switch. The case of a dynamo in which the current i in the leading short-circuited section is over-reversed to some value $J + i_x$ exceeding the full value J is then analogous. At the moment of rupture of the true contact-resistance before the spark appears when the tip of the brush leaves the leading sector, let i_x' be the value of the additional current; the current in the section under consideration

[1] A. Mauduit, *loc. cit.*, p. 193.

is then reduced to J in the time t during which the arc lasts, and the energy liberated is

$$\int_{J+i_z'}^{J} \mathcal{L}_{sz} \, . \, idi = \mathcal{L}_{sz} \, . \, i_z' \, . \, \frac{2J + i_z'}{2}$$

It is more than simply the amount corresponding to the disappearance of i_z', since the flux due to i_z' is linked not only with i_z' but also with J. The whole of this energy appears in the spark when the resistance of the circuit is neglected, and

$$\mathcal{L}_{sz} \, . \, i_z' \, . \, \frac{2J + i_z'}{2} = \int_0^t e_1 i_1 \, . \, dt$$

But when the current is under-reversed, the case is not so simple. Energy is now absorbed by the self-induction, to the amount

$$\int_{J-i_z'}^{J} \mathcal{L}_{sz} \, . \, idi = \mathcal{L}_{sz} \, . \, i_z' \, . \, \frac{2J - i_z'}{2}$$

but this bears no direct relation to the energy expended in the spark which is, as it were, in parallel with the absorbing circuit and is only the intermediate mechanism through which energy is enabled to be supplied to the section. It is not therefore possible in this case to lay down any general rule expressing the magnitude of the energy expended in the spark or the damage done to the commutator in terms of the energy required to be stored under given conditions of mal-adjustment. It can only be approximately estimated for each special case, and great variations may exist. A. Mauduit[1] has shown that in the case of the negative brushes after the rupture of the true contact surface as the brush-tip leaves the trailing edge of the leading sector, the first action while it passes over the mica is to draw out the arc to a length proportional to the time of passing over the thickness of a mica strip. During this time the spark shows the usual characteristics of an arc, namely, a resistance that varies inversely to the current and directly as the length. Since here the current decreases linearly with the time and at the same time the arc lengthens in proportion to the time, the resistance rises rapidly, and its curve is practically a hyperbola, so that the P.D. rises in proportion to the time.

But this may not exhaust the action: the spark may pass across the mica to the leading edge of the trailing sector, thus still further extending the period of virtual short-circuit. This arc does not lengthen with time and only slowly becomes extinguished; further it appears to be especially harmful. This affords a powerful argument in favour of thick mica strips, and J. Burke[2] has emphasized

[1] *Loc. cit.*, pp. 180, 216.

[2] *Trans. Amer. I.E.E.*, Vol. 36, Part III, p. 2418 (1911).

the advantage to be gained in critical cases from the use of thick mica strips.

But in all cases the uncertainty in the calculation of i_s' renders it practically impossible thence to deduce the energy expended in the spark.

§ 29. Practical voltage criterion of sparking.—Abandoning, therefore, the calculation of the energy as the product of voltage and current, we must perforce fall back upon the calculation of a voltage only, and in the first place the final value of the difference of potential between the brush-tip and the leaving sector suggests itself as a criterion of sparking. In a dynamo which has been built and can be tested the curve of brush potential relatively to the commutator does enable us to gain much information as to the probable behaviour of the machine in regard to sparking, and a maximum limit of 3 to 4 volts between commutator and brush is a valuable guide.

But even such a rule may require to be considerably modified according to the absolute value of the current as the other factor which determines the energy expended. By means of oscillographic experiments, Professor F. G. Baily and W. S. H. Cleghorne[1] have measured the actual sparking E.M.F. between brush and commutator at the last moment, and have found such values as 15 to 20 volts with carbon brushes without very violent sparking. Moreover, there remains a very similar objection to that which meets us in an attempt to calculate the energy of the spark; during the process of design a calculation of the final sparking potential, even if only approximate, requires that we should know the contact-resistance R_k as dependent upon the effective current-density, and also ζ_{12}. But R_k is extremely variable at brushes of different sign, especially under high temperatures and when sparking is likely to occur, while the value of ζ_{12} is not easy of calculation, owing to the difficulty of predicting the damping effect of other short-circuited coils, especially when not situated in the same slot.

Finally, therefore, we are led to adopt as a criterion of sparking a more easily calculated quantity ΔE, or the additional E.M.F. set up between the outer sectors lying at the extreme edges of the brush, so far as commutation departs from the straight-line law. To the importance of this quantity Professor Arnold has directed especial attention.[2] To it is due the difference of the potentials between the brush and the trailing and leading sectors at its two edges.

§ 30. The shape of the potential curve.—The processes which are occurring in the short-circuited coils are brought to light by the shape of the curve which is obtained by plotting the difference of

[1] *Journ. I.E.E.*, Vol. 38, p. 163; *cf.* Liska, *E.T.Z.*, Vol. 30, pp. 82-4.

[2] "The Commutation of Direct and Alternating Currents," by Prof. E. Arnold and J. L. La Cour, *Trans. Intern. Electr. Congress, St. Louis* (1904), Vol. 1, p. 808.

potential between brush and commutator for a number of points along the face of the brush.[1] When the current is commuted at a uniform rate, and i_z is throughout $= 0$, such a potential curve becomes a straight line at a uniform height above the zero line at the positive or below at the negative brush, the brush itself being in each case reckoned as at zero potential (Fig. 347); this height corresponds to the normal loss of volts due to the passage of $2J$ over the resistance of contact. But when additional currents are

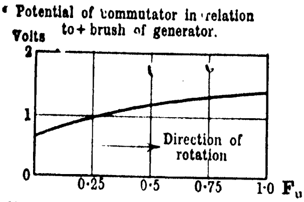


FIG. 366.—Retarded commutation.

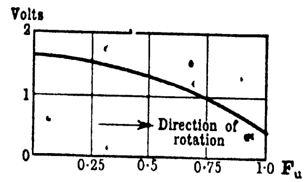


FIG. 367.—Accelerated commutation.

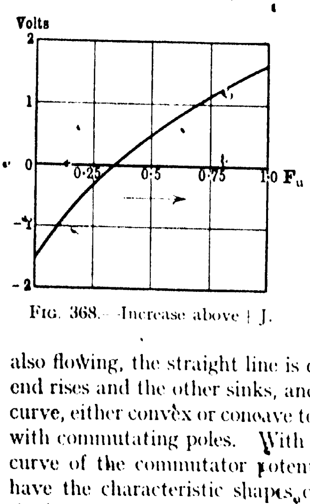


FIG. 368.—Increase above + J.

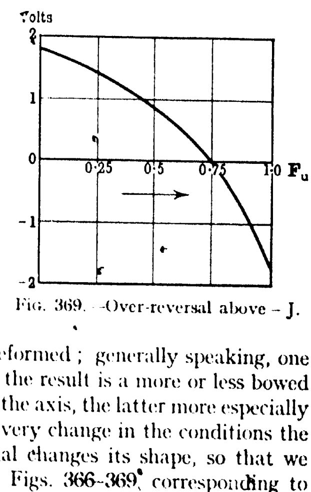


FIG. 369.—Over-reversal above − J.

also flowing, the straight line is deformed; generally speaking, one end rises and the other sinks, and the result is a more or less bowed curve, either convex or concave to the axis, the latter more especially with commutating poles. With every change in the conditions the curve of the commutator potential changes its shape, so that we have the characteristic shapes of Figs. 366–369 corresponding to the four cases of Figs. 351–354. With simply retarded or accelerated commutation in a generator the curve is bowed, rising in the former case (Fig. 366) and sinking in the latter (Fig. 367); but if the current is either increased above $+J$ or over-reversed to a value exceeding $-J$, the curve crosses the zero line (Figs. 368–369), and the direction of the potential difference changes, since the

[1] For means for taking readings of the voltage between brush and commutator, see Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 336.

actual current is in different directions under the same brush. But in every case, unless the commutation is first retarded and then accelerated (Fig. 370), or *vice versa*, so that the two errors practically balance one another, one end of the curve rises and the other sinks; on the whole, therefore, the curve is inclined, and the greater the error in the field, the steeper the slope. There thus arises a difference of potential Δp between the two sectors situated at the extreme edges of the brush, *i.e.* Δp, the algebraic difference of the potential

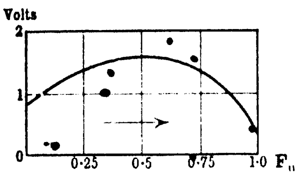


FIG. 370. Retardation followed by acceleration.

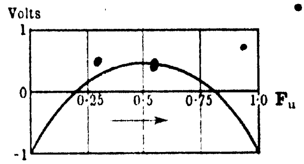


FIG. 371 Potentials at no load with wide brushes on line of symmetry.

at the two edges in the case of a wide brush covering several sectors, is practically a definite quantity depending upon the load and position of the brushes. When the curve of potential departs from

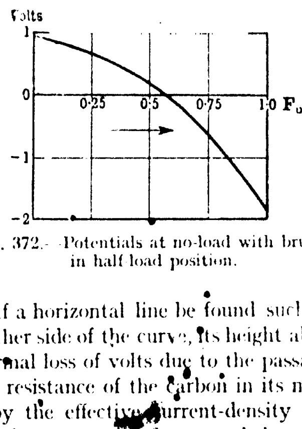


FIG. 372. Potentials at no-load with brushes in half load position.

a uniform level, if a horizontal line be found such that it encloses equal areas on either side of the curve, its height above the zero line measures the normal loss of volts due to the passage of the normal current with the resistance of the carbon in its modified condition as determined by the effective current-density under the brush. The special case of commutation first retarded and then accelerated as found in a non-commutating-pole generator at no-load with the brushes fixed in the geometrical centre, gives such a curve as Fig. 371, while if the brushes are fixed in the correct position, say for half-load, and so are too far forward at no-load, we have Fig. 372; in both of these cases it will be seen that equal areas are enclosed with the zero line, since there is practically no normal current passing.

Next, the potential curve is itself flatter than the E.M.F. curve to which it is due, since the latter also contains the ohmic drop over the resistance of the coils and their commutator connectors. The drop of volts across the substance of the carbon brush itself is practically negligible. Corresponding, therefore, to the difference of potential Δp between the sectors at the extreme edges of the brush, there is a still greater difference of E.M.F. ΔE acting through the coils, and the curve of the E.M.F. impressed by the incorrect field is steeper than that of the potential. The system of i_z currents is therefore primarily dependent upon the quantity ΔE, and this may be regarded as constant in point of time when several sectors are covered by the brush. It is certainly an experimental fact that the steeper the gradient of the curve of potential between brush and commutator, *i.e.* the greater the value of Δp, the more the likelihood of sparking, so that the magnitude of the quantity ΔE will serve indirectly as the required criterion of sparklessness in the design of a dynamo.[1]

The number of coils concerned in the production of ΔE is not simply equal to the number of adjacent sections simultaneously short-circuited by the brush width, but also is proportional to the reciprocal of the creep m, *i.e.* to p/a, so that, *e.g.* in the simple wave-wound armature p coils lie in series between two adjacent sectors touched by the brush; it may therefore be expressed generally[2] as

$$C_k = \left(\frac{b_1 - b_m}{b}\right)_+ \frac{p}{a} \quad . \quad . \quad . \quad . \quad . \quad (191)$$

where the $+$ sign indicates that when $\frac{b_1 - b_m}{b}$ is a fraction, the next higher whole number must be taken, and $\frac{p}{a}$ may be either whole or fractional.

§ 31. The assumption of fixity in the component fields.—If $b_1 - b_m$ so much exceeds b and so many sections are simultaneously short-circuited that the passage of one sector under a brush, or its exit therefrom, may be regarded as taking place instantaneously, the armature cross ampere-turns which are not short-circuited would remain perfectly constant and fixed in space, and so also the magnetic cross field to which they would give rise. Under the same conditions the system of currents in the simultaneously short-circuited coils, and their magnetic field would also become fixed in space.[3] The component E.M.F.'s set up within the short-circuited

[1] *Cp.* C. E. Wilson in discussion on B. G. Lamme's paper on Commutation, *Trans. Amer. I.E.E.*, Vol. 30, Part III, p. 2423.

[2] E. Arnold, *Die Gleichstrommaschine*, Vol. I, pp. 463–4 (2nd edit.).

[3] Any flux-swing due to the presence of teeth and slot openings in the toothed armature is in the present section disregarded.

coils could then be traced by considering them as due to (1) rotation through the external field from commutating poles or from the main excitation as modified by any direct or back reaction due to an angle of lead, (2) rotation through the cross field B_q due to the cross effect of the remaining full-current armature turns, yielding e_q, and (3) rotation through the stationary field due to their own current-turns, yielding e_s. A possible case of this occurs when the commutation proceeds proportionately to the time or is truly linear; the constant self and mutually induced E.M.F. is then $(\mathcal{L} + \Sigma . \mathcal{M})2J/T$ which would equally be given by movement through a constant field of uniform density fixed in space. Now item (2) may be grouped with (1) so that, as in §§ 16 and 17, $f(t)$ is the impressed E.M.F. due to the field resulting from the main excitation and armature ampere-turns so far as they are not short-circuited; or item (2) may be grouped with (3) as arising from the field due to *all* the ampere-turns of the armature, both short-circuited and not short-circuited. In either case the same result is reached.

But now, if we pass to the opposite extreme, when $b_1 - b_m = b$, and only one section is short-circuited at a brush, the axis of the cross ampere-turns which are not short-circuited advances during the period of commutation through the angle corresponding to a passage of a sector past a fixed point or edge of a brush. This leads to very little difference in the distribution of the armature field under the poles, but it does lead to a slightly different distribution of the field within the interpolar gap, and this in effect amounts to the cross field near the short-circuited coils being carried round with the armature; consequently so far there is no E.M.F. e_q.[1] But now if the current has not been commuted up to $-J$ at the end of short-circuit, it is abruptly and forcibly commuted with occurrence of a spark, and at the same moment the original distribution of the cross ampere-turns not short-circuited is reproduced, so that the cross field swings back abruptly and sets up an additional E.M.F. assisting the spark. This E.M.F. e_q is then a more or less sudden effect, and is not the same as that due to steady movement through B_q. Of course, so far as the current-change is linear, the displacement of the cross field is only the difference between its forward movement due to rotation and its backward movement due to the variation of the ampere-turns in the short-circuited sections, and when the current-change is perfectly linear, then again even with $b_1 - b_m < b$ we can return to the same components as above with $(\mathcal{L} + \mathcal{M})$ in term (3), or with the total armature field due to all turns as fixed in space.

Now the usual case of $b_1 - b_m > 2b$ lies intermediate between

[1] Mauduit, *Recherches Expérimentales et Théoriques sur la Commutation*, p. 262.

the two; the spacial variation of the cross field B_q is only that due to the continual entries and exits of one sector, the period of oscillation of the cross field being shorter, and its amplitude less in proportion to the total number of coils short-circuited. The amount of the back E.M.F. added by the abrupt swinging back of the cross field is, however, practically impossible of calculation, and in this lies one of the difficulties of devising any formula which will completely express the conditions that may lead to sparking.

The best practical basis therefore is probably to assume that the change of current actually *is* linear, and that the short-circuited coils endowed with inductance $\mathcal{L} + \Sigma . \mathcal{M}$ are moving through a stationary external field and a stationary cross field of density B_q which together yield $f(t)$.

§ 32. The calculation of ΔE.—In regard to the quantity ΔE, a marked distinction exists between the two cases (A) when commutation is due mainly or wholly to an actual reversing field suited to different loads and obtained either by so shifting the brushes as to reach such a field or by means of special (commutating) poles, and (B) when the brushes must be retained in some one fixed position without the assistance of commutating poles, and commutation is as it were "forced" by the action of the brush contact resistance.

In either case, for perfect commutation in a straight line, the E.M.F. to be impressed on the coil by the resultant external field should be from equation (180)—

$$f(t)_c = -2J\left\{\frac{\mathcal{L} + \Sigma . \mathcal{M}}{T} + R\left(\frac{t}{T} - \frac{1}{2}\right)\right\}.$$

Further, if a drum coil is moving in any field of density B which holds over the length L of one active inducing side in centimetres, the E.M.F. actually produced in that one coil-side is $wBLv \times 10^{-8}$ volts where v is its peripheral speed in centimetres per second and $w = Z/2C$.

The several alternatives under case (A) are then as follows—

A (i).—*In a dynamo without commutating poles*, let B_o = the density of the main symmetrical field at the given brush position which would result from the field excitation. Let B_q = the density at the same brush position which would be due to all the ampere-turns of the armature which are not short-circuited and which are therefore carrying the full current J. Then each side of the drum coil (assumed to be full-pitch) is moving through a similar field of density $B = B_o + B_q$ over its whole length, and for the coil as a whole

$$f(t) = 2w(B_o + B_q)Lv \times 10^{-8} \text{ volts}$$

Equating this to $f(t)_c$, we find that for perfect commutation the correct value of B_o is

$$B_{oc} = -\left[\frac{J}{wLv \times 10^{-8}}\left\{\frac{\mathcal{L}+\Sigma.\mathcal{M}}{T}+R\left(\frac{t}{T}-\frac{1}{2}\right)\right\}+B_q\right] \quad (192)$$

This, on the whole, must yield a negative or reversing effect as shown by the negative sign.

A. (ii).—*With commutating poles.*

(*a*) If these are as many in number as there are main poles, and their axial length L_r is equal to the length of the armature core L, $B = B_r$, the density of the reversing flux, and for perfect commutation

$$B_{rc} = -\frac{J}{wLv \times 10^{-8}}\left\{\frac{\mathcal{L}+\Sigma.\mathcal{M}}{T}+R\left(\frac{t}{T}-\frac{1}{2}\right)\right\} \quad . \quad . \quad . \quad (193)$$

(*b*) If $L_r < L$, then over the length $L - L_r$ in each interpolar gap the armature or cross induction B_q again holds, and

$$B_{rc} = -\left[\frac{J}{wL_rv \times 10^{-8}}\left\{\frac{\mathcal{L}+\Sigma.\mathcal{M}}{T}+R\left(\frac{t}{T}-\frac{1}{2}\right)\right\}+B_q\frac{L-L_r}{L_r}\right] \quad (194)$$

(*c*) If there are only half as many commutating poles as there are main poles

$$B_{rc} = -\left[\frac{2J}{wL_rv \times 10^{-8}}\left\{\frac{\mathcal{L}+\Sigma.\mathcal{M}}{T}+R\left(\frac{t}{T}-\frac{1}{2}\right)\right\}+B_q\frac{2L-L_r}{L_r}\right] \quad (195)$$

Neglecting, then, the ohmic resistance term and its variation with time as of comparatively small influence, if either B_r or B_o is on the whole of such amount as to balance the inductive voltage $(\mathcal{L}+\Sigma.\mathcal{M})2J/T$ and the cross field if present, the only remaining cause that may still set up sparking lies in the difficulty of securing such exact grading of the reversing field as to suit the true variations of $\mathcal{L}+\Sigma.\mathcal{M}$. Both B_o and B_q vary as movement of the coil proceeds by rotation, and so also does $\mathcal{L}+\Sigma.\mathcal{M}$ not only as the distance from the main pole-tip or commutating pole-face varies, but also from the different location of the short-circuited coils on either side or all on one side of the considered coil with which they are linked by mutual inductance. An average rate of change of the current or $(\mathcal{L}+\Sigma.\mathcal{M})2J/T$ with $\mathcal{L}+\Sigma.\mathcal{M}$ regarded as constant, and an average adjustment of the reversing field is all that can be taken into account. The divergence from the ideal condition is at each moment the difference between the real $f(t)$ and the required $f(t)_c$, and if each of the C_k coils acted upon between the edges of a brush happened to be exactly similarly affected, the total E.M.F. between the two sectors at the extreme edges of the brush would be

$$C_k\{f(t)_c - f(t)\} = \Delta E.$$

Its actual amount hardly, however, admits of practical calculation; it can only be said that the greater the necessary value of B_r or of the angle of lead when the brushes are shifted, the greater the difficulty of obtaining a balance between the reversing and the inductive voltages, so that under all conditions of load ΔE may remain $= 0$.

But in case (B) *with the brushes fixed in one position and without the assistance of commutating poles*, a more definite calculation of ΔE can be made.

(1) If the fixed position corresponds exactly to the interpolar line of symmetry, *i.e.* to the *no-load position*, then at full-load there is present in each short-circuited coil an E.M.F. in the wrong direction from B_q, and also an E.M.F. in the same incorrect direction from the inductance. These must cause the difference of E.M.F. between the brush and sectors at its two edges, and as a first approximation it is sufficiently accurate to assume that at full-load the inclined curve of E.M.F. between brush and commutator rises in a straight line. If B_q be reckoned for a point midway between the edge of the brush and its centre, the latter being also in our present case the geometrical centre between the poles, an average value is obtained for the E.M.F. due to the cross field, which each short-circuited coil may be assumed to give throughout the period of commutation, namely, $2wB_qLv \times 10^{-8}$ volts.

The E.M.F. therefrom summed up between the edges of a brush through the C_k coils will be C_k times that of a single coil, and similarly the total for the average E.M.F. from the self and mutual inductance will be $C_k(\mathcal{L} + \Sigma . \mathcal{M})2J/T$.

The ohmic resistance term which should be present for ideal commutation rises as an inclined straight line from $+JR$ to $-JR$, changing sign midway during the period of commutation, *i.e.* under the centre of the brush. So also the small symmetrical field which is present changes its sign midway under the brush and is similar on either side of the centre. Hence, whatever the degree of divergence between these two for any point on the one side of the centre, the same divergence but in the opposite direction exists at the corresponding point on the other side. This unbalanced E.M.F., equal in amount but in opposite directions through the coils at each end of the short-circuited group, does not come into consideration from the present point of view, since it is only an E.M.F. in the same series direction through all the C_k coils, corresponding to a permanent curve of volts between brush and sectors inclined to the axis, with which we are concerned. Hence, the effect of any divergence between the ohmic resistance and the symmetrical field, when summed up between the limits of the brush edges, cancels out, and we are left with the inductive and cross volts only to cause ΔE.

The total difference of E.M.F. between sectors and brush at the two extreme edges is therefore at full load with brushes fixed in the geometrical centre

$$\Delta E_1 = C_k \left\{ \frac{2J(\rho + \Sigma . \rho')}{T} + 2wB_q Lv \times 10^{-8} \right\} \text{volts} \quad . \quad . \quad (196)$$

For purposes of comparison between different machines it will be sufficiently accurate to determine B_q for the actual centre between the poles from the approximate equation

$$B_q = \frac{1{\cdot}257\ mJZ/4p}{\xi c + Kl_g}$$

(2) But if the brushes may be fixed at the position corresponding to *half-load* or $J/2$, and it is assumed that ΔE then completely disappears with current $J/2$, and armature field-density $B_q/2$, the value of the initial reversing field $-B_o$ from the main excitation must be

$$-\left[\frac{J}{2wLv \times 10^{-8}}\left\{\frac{\rho + \Sigma . \rho'}{T} + R\left(\frac{t}{T} - \frac{1}{2}\right)\right\} + \frac{B_q}{2}\right]$$

This negative field now persists at no-load, although practically no reversing field is required. If B_q is calculated for full-load and for a point under the centre *of the brush*, then, analogously to the previous case, the average value of the incorrect E.M.F. is

$$-\left\{\frac{J . (\rho + \Sigma . \rho')}{T} + wB_q Lv \times 10^{-8}\right\}$$

since, when summed up between the extreme limits of the brush width, the inaccuracies due to the ohmic resistance cancel out. Hence

$$\Delta E_2 = -C_k \left\{\frac{J . (\rho + \Sigma . \rho')}{T} + wB_q . Lv \times 10^{-8}\right\} \text{volts} \quad (197)$$

On the other hand, at full-load the reversing field should be doubled, so that ΔE_2 has the same value, but is positive.

Thus the numerical value of ΔE_2 for constant field-excitation is approximately the same at full and at no-load. In practice, with a shunt-wound machine the degree of incorrectness is underestimated, since the negative reversing field is at no-load slightly increased by the absence of the back ampere-turns of half-load, and at full-load is weakened by the increase of the back ampere-turns; on the other hand, with compound-wound machines the conditions are considerably more favourable for the reverse reason.

It will be seen that the amount of ΔE is practically halved in case (2) as compared with the case of brushes fixed at the geometrical centre, although it must be remembered that, owing to the angle of lead in case (1), B_q is slightly larger than in the first case.

Of the two, ΔE_1 is the more readily calculated, since it does not

require a preliminary determination of B_q in the correct brush position for half-load. The term $\mathcal{L} + \Sigma \cdot \mathcal{M}$ of a drum coil is by equation (186)

$$= w^2\{l(\lambda_1 + \lambda_2) + 2l'\lambda'\} \times 10^{-9}$$

Hence if B_q is expressed in terms of the ampere-conductors per pole as $\dfrac{1{\cdot}257\, JCw/2p}{\xi c + Kl_g}$, and v in terms of the armature diameter in centimetres and revolutions per minute, *i.e.* as $\pi DN/60$, equation (196) becomes

$$\Delta E_1 = 2C_k w^2 J \left\{ \frac{l(\lambda_1 + \lambda_2) + 2l'\lambda'}{T} + \frac{4\pi^2 N}{60} \cdot \frac{D}{\xi c + Kl_g} \cdot \frac{CL}{2p} \right\} \times 10^{-9}$$

where all dimensions are in centimetres. Finally, with as many sets of brushes as there are poles, giving T its value as in equation (190), namely $\dfrac{b_1 - b_m + b\left(1 - \dfrac{a}{p}\right)}{v_c}$ seconds, where $v_c = \dfrac{\pi D_c N}{60}$, and taking $\dfrac{\pi N}{60}$ outside the $\{\ \}$ bracket,

$$\Delta E_1 = 0{\cdot}1047\, C_k w^2 JN \left[\frac{D_c}{b_1 - b_m + b\left(1 - \frac{a}{p}\right)} \times \{l(\lambda_1 + \lambda_2) + 2l'\lambda'\} + \frac{4\pi D}{\xi c + Kl_g} \times \frac{CL}{2p} \right] \times 10^{-9}$$

or, say

$$= C_k w^2 JN \left[\frac{D_c}{b_1 - b_m + b\left(1 - \frac{a}{p}\right)} \times \{l(\lambda_1 + \lambda_2) + 2l'\lambda'\} + \frac{4\pi D}{\xi c + Kl_g} \times \frac{CL}{2p} \right] \times 10^{-10} \text{ volts} \quad . \quad . \quad (198)$$

Since in each part of the expression there is a ratio of two lengths, $\dfrac{D_c}{b_1 - b_m + b\left(1 - \frac{a}{p}\right)}$ and $\dfrac{D}{\xi c + Kl_g}$ can be calculated in inches, and only L and $l(\lambda_1 + \lambda_2) + 2l'\lambda'$ need be found in C.G.S. units. The difference between L and l suggested by the use of different symbols is made for the reason mentioned at the end of § 24(*b*).

§ 33. The permissible value of ΔE, or the sparking limit of output.—With metallic brushes, say, of copper gauze, so feeble is the action of their contact-resistance that, roughly speaking, there must be a reversing field of value suited to each load; in other words, ΔE must very nearly $= 0$. This implies either that there

must be commutating poles present, or that the brushes must be shifted into a reversing field, and the want of exact balance must be confined within quite small limits, with a variation of, say, not more than 20 per cent., or 10 per cent. on either side of the correct load for the given brush position. Further, it implies that the armature core must almost necessarily be smooth, so that $S + \Sigma . a$ may not be increased by embedding the wires in iron.

But with carbon brushes, which are a practical necessity with slotted armatures, a much greater inaccuracy of adjustment may be allowed, and as a practical limit which will secure sparkless commutation under average conditions with carbon brushes of fairly high resistance may be given

$$\Delta E \leqq 4 \text{ to } 5 \text{ volts.}$$

We are thus met with a condition for sparklessness which, entirely apart from any question of heating, may limit the maximum current that can be passed through an armature, and therefore for a given speed of rotation and voltage may limit the output of the machine. Again it is seen from equation (198) how advantageous from the point of view of sparking is the multipolar machine, by reason primarily of the reduction of the inductive volts per section which it renders possible when the armature current is large. With the lap-wound armature and one-turn coils ($w = 1$), by the passage from a smaller to a larger number of poles, J can always be proportionately reduced until its value is reasonable, even though the product JZ or the ampere-conductors on the armature remain the same.

With carbon brushes and under the above limitation for ΔE, a fixed brush position becomes possible both with and without commutating poles, although with less ease in the latter case. With commutating poles, the want of balance between the reversing volts and the inductive volts should never exceed 4 to 5 volts (*cp.* § 42 and Figs. 374 5). Without commutating poles and with the brushes advanced half-way towards the correct position for full-load and fixed thereat for all loads, the condition

$$\Delta E_2 \leqq 4 \text{ to } 5 \text{ volts}$$

is obtainable.

If the brushes have in fact to be fixed in the geometrical centre at all loads, ΔE_1 should strictly not exceed the same amount, but under this more stringent case some greater latitude has usually to be allowed, and we have, say,

$$\Delta E_1 \leqq 6 \text{ to } 8 \text{ volts,}$$

preferably accompanied by the employment of harder brushes.

It has been stated that ΔE_1 is the more readily calculated quantity, so that finally it will be adopted as the criterion for the

non-commutating-pole machine, and equation (198) must not be found to yield more than 6 to 8 volts. Even when the brushes may be continuously shifted or may be set at the position for half-load, so that commutation need not be entirely "forced," the calculation of ΔE_1 serves as a practical guide to the good or bad qualities of the dynamo as regards sparklessness, since it is a measure of the *maximum voltage that the brushes will be called upon to correct, should they receive no adjustment to suit different loads. In fact, according to the circumstances of the case* ΔE_1 *may be given different values, although in every case the ideal should be to reduce it to the lowest possible amount.*

In the dynamo with commutating poles, ΔE_1 would only arise if for some reason the winding of the commutating poles became short-circuited and their excitation ceased; its value by equation (198) would then be exceeded owing to the presence of the iron commutating poles immediately above the armature poles and above the short-circuited coil-sides. Yet even with commutating poles and apart from the abnormal possibility above mentioned, the value of ΔE_1 may be adopted as a quantitative criterion for comparative purposes, if the second term of (198) dealing with the cross field is omitted when there are as many commutating as main poles and of equal axial length. If of shorter axial length, for L is to be substituted $\frac{L - L_r}{L}$, and if there are only half as many commutating as there are main poles $\frac{2L - L_r}{L_r}$ is to be substituted. The figures thus obtained are a measure of the density that the reversing field must reach.

In considering the limits laid down for ΔE_2 or ΔE_1, in relation to the maximum corrective action that may be expected from carbon brushes, it must be remembered that Δp is less, that it may at least partially be divided between the toe and heel of the brush, and further, that $q + \Sigma \cdot a$ has been calculated at its maximum possible value so as to be on the safe side.

§ 34. The separate factors influencing sparking. — Although equation (198) is only indirectly applicable as a criterion in the case of machines with commutating poles, yet in all cases from an examination of it a clear idea can be gained of the various factors upon which the sparkless running of a dynamo chiefly rests. The same essential relations may be expressed in many other ways, but when analysed they will always be found to resolve themselves into the combined effect of two fundamental factors, the first depending upon the self and mutual inductance of a section and the number simultaneously short-circuited, and the second upon the cross field or the magnetic effect of the armature ampere-conductors per pole so far as the short-circuited sections are not

covered by a commutating pole. The sum of their effects across the width of a brush must be so closely balanced by the voltage from the external field through which the coils are moving that the divergence between the two, ΔE, never exceeds a certain small number of volts. It remains to consider how far it is possible for the designer, within the limits imposed by the requirements of commercial economy, to influence favourably the values of the different items.

The disadvantage of a high peripheral speed, whether of armature coil or of commutator, is at once evident. Especially is this disadvantageous in the case of the commutator,[1] and herein lies the difficulty of the design of continuous-current dynamos for direct coupling to steam turbines, since the inductive voltage reaches such high values that special devices to secure more favourable conditions become imperative. There is, too, a limit to the output of kilowatts which can be satisfactorily reached with each voltage, although opinions may differ widely as to the exact point at which sparking sets a limit to the possible size of the machine.[2]

As regards the number of revolutions per minute, the designer has in almost all cases to accommodate his design to the requirements of the prime mover, so that N is virtually fixed. The quotient of the watts of output divided by the revolutions per minute is therefore the fundamental datum of the design. As will be explained in Chapter XXII, the given value of this very important ratio, even apart from any other considerations, necessitates a certain minimum value for the product of the square of the diameter and of the length of the armature core, *i.e.* of D^2L, in order to comply with usual heating conditions. Although the division of the product into its two factors is not thereby prescribed, the designer is now, generally speaking, enabled by reference to standard sizes and patterns to decide simultaneously the most suitable number of poles and type of winding in accordance with the principles of § 17, Chapter XII, and thence the separate dimensions D and L. These will be chosen so that J is neither unreasonably high nor unreasonably low, and so that the number of turns per coil or $w = Z/2C$, which is a most important factor of the whole expression, may, if possible, be reduced to 1 or retained at that value. With a simplex lap armature although by an increase in the number of poles Z is proportionally increased, the number of turns per section or $Z/2C$ can at least

[1] Rotary converters for 50-frequency and high voltages, with an ordinary commutator without shrink rings, may have to work with a peripheral velocity of commutator exceeding 5000 or even 8000 ft. per min. But such speeds have not in general to be met in continuous-current generators, unless driven directly by steam-turbines.

[2] *Cp.* S. Senstius, "Limitations in Direct-Current Machine Design," and the following discussion, *Trans. Amer. I.E.E.*, Vol. 24, p. 689; and Prof. W. Kummer, *Schweiz, Elek. Verein Bulletin* (1922), 13th year, No. 9.

theoretically be still maintained at the same value by proportionally increasing the number of commutator sectors. There are, however, practical limits both to the decrease of J and to the increase of C. On this account J in practice averages from 150–200, and seldom exceeds 300 amperes even in low-voltage machines, unless commutating poles are fitted, in which case it may be raised as high as 400–500 under favourable conditions. Next, constructional considerations from the size of the armature limit the greatest diameter that the commutator can conveniently have, and on the score of expense, of brush friction, and of commutator peripheral speed, the smallest convenient diameter will so far as possible be adhered to. But now it is evident that though it is always most desirable to bring w down to its limiting value of 1, when each section of the drum winding consists only of a single turn, this will not be attainable when $\pi D_c/C$, the pitch of a sector, becomes too small. As already stated in Chapter XII, § 17 (3), there is a minimum thickness of sector which permits of satisfactory connexion to the armature winding by a lug soldered into a saw-cut or riveted to the side of the sector. Hence if J is to start with, or is made, low, it may become necessary to pass to $w = 2$, or $= 4$, and so on. At each of these critical stages the designer must consider the possibility of slightly modifying the dimensions so as still to be able to retain the lower value of $w = 1$; or the effect of adopting a wave instead of a lap winding must be tried on the lines laid down in Chapter XII, § 17.

The width of the brush b_1 may be regarded as to some extent open to modification at will, but it must be one or other of a few standard sizes, and it must be such that the full or over-load current can be collected without overheating and without an unduly long and expensive commutator. Further, the introduction of the multiplier C_k in the criterion of sparking has the effect of limiting the possible use of very wide brushes.[1] Assuming any given commutator, speed and value of C, an increase in the width of the brushes increases the number of coils simultaneously short-circuited and also the time of commutation nearly in the same proportion, so that the ratio $\frac{C_k}{T}$ or $\frac{C_k}{b_1 + b_m}$ remains the same. On the other hand, $S + \Sigma . N$, or $l(\lambda_1 + \lambda_2) + 2l'\lambda'$, is increased proportionately when the additional short-circuited coils are in the same slot, but not proportionately when they are distributed over more than one slot in each interpolar region. But in either case the total inductive voltage across the width of a brush must be greater, and also the cross field voltage which increases with C_k. It thus results that b_1 practically becomes fixed.

[1] *Cp.* Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 362–4.

If then b_1 is fixed, and D_c is near its minimum value, but it is still open to the designer to reduce the value of w from, say, 2 to 1, or from 3 to 2 by increase of C, without the pitch of the sectors becoming mechanically too small, $C_k w$ remains constant, but $l(\lambda_1 + \lambda_2) + 2l'\lambda'$ increases owing to the increase in the value of j_a, j_b, j_c, so that there may be no great difference in the total value of $C_k w^2 \{l(\lambda_1 + \lambda_2) + 2l'\lambda'\}$. There would not then appear to be any advantage in adopting the higher value of C or lower value of w, and in this respect equation (198) fails to represent the true facts. The reason is that it is based on a uniform rate of change in all coils, but actually there may be considerable divergence from the supposed uniform rate. The $\mathcal{E}_{sz}$ of the coil with, say, only half the turns of another coil is then only one quarter as great, and its inductive effect upon exit from short-circuit with the additional current i_z' flowing in it is far less destructive. A reduction of w therefore always takes precedence as a means of reducing sparking, and such a condition must accompany the inferences that may be drawn from equation (198).

Next, if J and w have practicable values, but to accommodate the chosen number of sectors C, the diameter of the commutator must be increased above the minimum, it will be observed that although D_c is increased, C_k is reduced for a given brush width, and also $l(\lambda_1 + \lambda_2) + 2l'\lambda'$, so that the balance of advantage still lies with the larger number of sectors, until the size and cost of the commutator, its high peripheral speed, and friction loss become prohibitive.

Thus the control of the various sparking factors for a prescribed output in volts and amperes and a given speed is closely limited by various considerations of price and mechanical design. The attention of the designer must be concentrated on the value of $w = Z/2C$, and upon the reduction of $l(\lambda_1 + \lambda_2) + 2l'\lambda'$ by a careful disposition of the winding and choice of slot-pitch. Beyond this, at best, only a judicious compromise between many conflicting considerations remains open.

§ 35. Importance of a large number of sectors.—The final result of the examination of § 34, whether with or without commutating poles, is therefore to bring into especial prominence the value of w as the primary quantity which can be modified by the designer, and it has been shown that the extent to which it is advantageous to subdivide a given armature winding into a large number of small sections is only limited by the question of expense in manufacture and the difficulty of dealing with very thin commutator sectors. The armatures of closed-circuit machines for high pressures of from 500 to 1500 volts necessarily have a considerable number of turns per section, since they are wound with a large number of active conductors; hence, even though the current of

such machines may be comparatively small, special care is required to render them sparkless in working. The practical limits for the value of w as related to J have already been given in Chapter XII, § 17 (4).

In the case of armatures for large currents at low voltages and high speeds, the designer is often met with the difficulty of securing the minimum number of commutator sectors per pole which is advisable, and for which the limiting value has been set at 15. Especially with large bipolar machines and turbo-dynamos does this difficulty arise, since the total number of bars which is required may work out to less than 60; yet even with multipolar designs it may also occur. In such cases recourse will be made to multiplex windings; the commutation of the two or more subdivisions of the winding under each brush is not then exactly coincident, but one is always in advance of the other, so that some advantage is gained in the self and mutual inductance which will be somewhat less than that of the simple undivided loop. Yet against this advantage it must always be borne in mind that the time of commutation is reduced for the same width of brush as explained in § 27. The same reduction in the time of commutation is equally a disadvantage in the adoption of a multiplex wave-wound armature.

Where a duplex lap winding is to be recommended on account of the paucity of the number of active conductors and commutator bars per pole that otherwise results, an additional precaution for securing equal division of the current and sparkless running consists in the use of separate equalizing connexions for two independent windings, one set at each end of the armature, and the final interconnexion of the two sets of equalizing rings; the farther end of a bar should be at the same potential as the next commutator sector ahead of the one to which the bar is itself attached, and by the above device, due to Mr. F. Punga, this result is automatically attained at a number of points corresponding to the number of equalizing rings.[1]

A more drastic solution of the same problem is the adoption of a single-winding with a commutator at either end, the upper layers of bars being connected at each end to a sector; when the brushes are correctly placed, the unit which passes into and out of short-circuit is thereby reduced from a whole to a half loop, but the adjustment of the brush positions at the two ends calls for the greatest nicety to secure equal division of the current. Finally, by taking out commutator connexions at intermediate points along each bar through the air-ducts, each loop can be positively subdivided into sections (*cp.* Siemens' patent 11,471 (1904)). But care must be taken that such connectors do not themselves add a considerable amount of inductance (*cp.* Phœnix Dynamo Company's patent,

[1] *Journ. I.E.E.*, Vol. 39, p. 600.

11,701 (1907)), and the proper mechanical support of the leading-out wires always remains a difficulty.[1]

Above the minimum number of sectors per pole, say, 15, which is advisable to secure steadiness of voltage, and above the minimum which is necessary to bring the average voltage per sector or $\frac{V_b \cdot 2p}{C}$ below, say, 20 to 25 volts, so that there may be no flashing across from sector to sector over the intervening mica, no rational formula for C in terms of Z and J can be given which will supersede as a short-cut the longer calculation of the average inductive volts $(l + \Sigma . ll)2J/T$ or of ΔE, of which it forms the chief part.

If the armature be multipolar and parallel-connected, the number of slots and therefore also of sectors must be a multiple of the number of pole-pairs, to permit of equalizing cross-connexions joining points which should be at true equal potential.

But even when C has been provisionally decided upon, there remains the closely connected question of how many sectors or coils may be assigned to each slot in the toothed armature.

§ 36. The number of ampere-conductors and of sectors per slot.—In the toothed armature the concentration of more than two coil-sides in the same slot is, theoretically speaking, wrong, since with a greater number than two the spacial displacement of the sectors is not matched by an equal spacial displacement of the coils. The coils are not therefore precisely similarly circumstanced in their position relatively to the field when short-circuited.

Measured on the circumference of the armature, the maximum displacement of a coil-side within a slot from its correct position for truly uniform distribution corresponding to that of the commutator sectors is

$$\left(b \cdot \frac{D}{D_c} - r_s\right)\left(\frac{c-1}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (199)$$

the assumption being that the centre coil-side is taken as the correct standard, and that the brushes are adjusted to suit this coil-side. D and D_c are respectively the diameters of the armature and commutator, c is the number of sectors per slot or half the number of coil-sides per slot, and these are assumed to be arranged in two layers, and r_s is the distance between the centres of two adjacent coil-sides in the same slot and in the same layer.

In practice the use of a number of slots equal to the number of sectors usually involves too great a loss of space in insulation and too slender teeth. The wide tooth which results from grouping several coil-sides per layer in the same slot is stronger mechanically,

[1] See especially Dr. Pohl, *Journ. I.E.E.*, Vol. 40, p. 250; and Prof. Miles Walker's remarks, p. 256; and the latter's *Specification and Design of Dynamo-Electric Machinery*, p. 517.

and allows better ventilation through the core by air-ducts. The correspondingly wider slot has the incidental advantage that the slot-inductance of several coils simultaneously short-circuited in the same slot from the very fact of its width is not so much increased as might at first be expected. The practical advantages, therefore, of concentration outweigh the theoretical objections. But such concentration must not be pressed too far, since if certain limiting values are exceeded the hindermost sector of each slot, being the one that is most disadvantageously situated, becomes blackened or eaten away by sparking along its trailing edge. Indeed, this defect

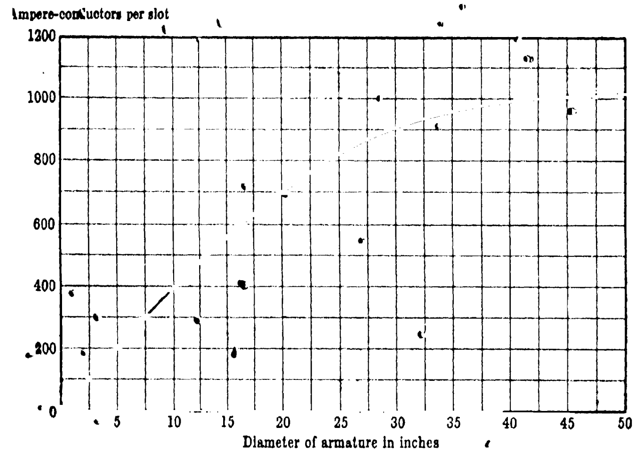


FIG. 373. Ampere-conductors per slot in relation to diameter of armature.

is not infrequent in dynamos in which the pressure of economical considerations has led to an undue concentration. The number of sections per slot must, in fact, be considered in relation to the current and number of turns in each coil and other conditions upon which the likelihood of sparking depends. Finally, as already mentioned in Chapter XIII, § 34, with straight-sided open slots, their width of opening should not much exceed ½" in order that the humming noise may not prove objectionable.

For ordinary voltages from 100 to 500 the number of ampere-conductors per unit area of slot remains very constant in generators of good modern design with toothed armatures, even when of widely different size. It ranges from 800 to 1100 per square inch of slot area, and on an average is 1000. The width of an open slot is practically limited by the necessity of avoiding eddy-currents in the pole-pieces even when laminated, and the permissible depth of

slot is limited by considerations of inductance, tooth-saturation (Chap. XIII, § 39), and heating. There is therefore a limit to the permissible size of slot, so that even in large multipolar machines its maximum cross-sectional area is about one square inch. In small machines the size of slot must necessarily be reduced, and the utilization of space is not so good. It thus results that the curve connecting the ampere-conductors per slot with the diameter of armature rises gradually, as shown in Fig. 373, and approaches a maximum of about 1000 ampere-conductors per slot.

Practice shows that when such values of the ampere-conductors per slot are accompanied by the condition that J with any given number of turns per coil does not exceed the limits laid down in Chapter XII, § 17, the difference between coils at the two edges of the slot will not be so marked as to cause sparking. The combination of the two conditions leads to the result that for a given diameter of armature with any particular value of J, and so of the given turns per coil, one, two, or three as the case may be, there is a *minimum* permissible number of sectors per slot. Thus an armature of 20″ diameter

with $J <$ 22·5 and 3 turns per coil, must have at least 5 sectors per slot.
,, $<$ 50, and 2 turns per coil ,, ,, 3 ,,
,, $>$ 50 up to 200, and 1 turn per coil ,, 2 ,,
An armature of 50″ diameter or over
with $J >$ 50 up to 200, and one turn per coil ,, 3 ,,

It further results from the two conditions that there is in practice a *maximum* possible number of sectors per slot in each case, which is given by the product of the minimum number and the number of turns per coil. It therefore coincides with the minimum number when J is $>$ 50 and there is only one turn per coil, but for small values of J it seldom is reached owing to the consequent size and expense of the commutator. Any number of sectors per slot larger than the minimum up to the maximum possible, being accompanied by the condition that the ampere-conductors per slot do not exceed the limits of Fig. 373, is to be regarded not as a concentration of coils into a slot, but rather as a finer subdivision of the winding, and so is to the advantage of the machine. In practice, two sections per slot is the rule for low voltages and high speeds, rising to three as the average, and even to four or five sections per slot in machines of high voltage and low speed.

The Non-Commutating-Pole Machine

§ 37. The limiting number of ampere-conductors per pole.— In the machine with commutating poles it is a necessary part of the design that the excitation of the commutating pole must more than counterbalance the armature ampere-conductors on the half

pole-pitch. But without commutating poles the main field excitation must be relied on to keep in check the cross field of the armature. Not only must the density of the main field component $-B_0$ at the brush position counterbalance B_q, but there must be left in reserve a definite reversing field B_{rc} at that spot. The accurate calculation of this is given by equation (147) in Chapter XIX, § 8 (*d*), and by the corresponding expression for the flux-density when y' is made equal to $c - c\chi$.

But without necessarily attempting any exact calculation of the distribution of the field throughout the interpolar gap, comparison between different designs or between different numbers of poles may be made by assuming that in every case the brushes are advanced as far as the edge of the leading pole-tip. Corresponding, then, to equation (174) for the density at the trailing pole-tip, we have for the density under the leading pole-tip

$$B_g' = 1{\cdot}257 \frac{\left\{AT_g + AT_t - \frac{JZ}{2p}\left(\frac{\beta}{2} + \frac{\lambda_e}{180^\circ}\right)\right\}}{Kl_g + \mathfrak{R}_t'}$$

Since λ_e is now assumed as $90^\circ(1-\beta)$ and $\mathfrak{R}_t'$ is practically negligible in comparison with Kl_g owing to the low density, this reduces to

$$B_g' = 1{\cdot}257 \frac{AT_g + AT_t - \frac{1}{2}\cdot\frac{JZ}{2p}}{Kl_g}$$

In order, then, that this may have some value in the required direction, it must be the case that

$$AT_g + AT_t > \frac{1}{2}\cdot\frac{JZ}{2p} \qquad \text{or} \qquad X_g + X_t > JZ/2p$$

and in practice

$$\frac{X_g + X_t}{JZ/2p} = \frac{X_g + X_t}{ac \cdot Y} = \frac{AT \text{ over double air-gap and teeth}}{\text{armature ampere-conductors per pole}} \text{ is } = 0{\cdot}92 \text{ to } 1{\cdot}1 \quad . \quad . \quad . \quad (176)$$

as already obtained from a different consideration in Chapter XIX, § 18, for machines with or without commutating poles.

The re-introduction of equation (176) as determined by considerations of sparking serves to emphasize its importance in the case of machines without commutating poles, and the advantage in this case of the multipolar over the bipolar. But for the same value of l_g and for a given value of JZ on an armature, the present requirement for sparklessness can always be met by suitably increasing the number of poles, which lessens Y while leaving ac unchanged.

The second item in the expression (198) has to do with the armature AT and is therefore closely related to the ratio

$$\frac{\text{ampere-turns over air-gaps and teeth}}{\text{armature ampere-conductors under a pole}}.$$

Hence a more or less accurate and similar result as to the sparking limit is reached if only the first item, *i.e.* the inductive voltage, is calculated, and this is accompanied by a further condition which makes it allowable to increase the limiting value for $(\gamma + \Sigma.\,w)2J/T$ as the ratio $\dfrac{X_g + X_t}{\text{ampere-conductors under pole-face}}$ becomes higher. Thus $C_k(\gamma + \Sigma.\,w)2J/T \leq 4$ might be laid down as a maximum permissible limit for complete sparklessness with a fixed brush position when accompanied by the secondary condition that the ampere-turns expended over the double air-gap and teeth should not be less than $1\frac{1}{2}$ times the ampere-conductors under a pole, *i.e.* when equations (175) and (176) are fulfilled. There is thus considerable room for practised judgment in choosing the right values for the two quantities to suit the degree of stringency in the terms of the specification to which the dynamo has to be built, or the nature of the work which it is to perform.

§ 38. The limiting number of ampere-conductors per unit length of circumference.—Although for a given JZ the maximum permissible number of ampere-conductors per pole as limited by the necessity for a reversing field need never be exceeded, equation (176) in the present connexion shows that for an armature with a given number of poles and therefore fixed value of Y, there is a maximum permissible value for ac, just as in Chapter XIX, § 16. The permissible number of ampere-conductors per pole is therefore but little else to the designer than a warning, and the quantity ac is the important factor. In small machines the heating limit is reached first, so that the actual values of ac may be only half of that fixed by sparking. In larger machines the two limits are reached more nearly simultaneously, so that values of ac from 400 to 650 per inch in medium sizes and of 800 in large sizes correspond very closely with the limits imposed at once by heating and sparking. Part of the art of designing non-commutating-pole machines consists in so choosing the number of poles, the length of air-gap, and the winding that the heating and sparking limits are so far as possible reached at the same output.

§ 39. The practical angle of lead.—Granting that by the above precautions a reversing field has been secured at the leading pole-tip of some such value as $B_g' = 1500$ to 2000 in the toothed armature with carbon brushes, the diameter of commutation does not usually require to be advanced so far as to make full use of the reversing field at the extreme pole-tip. Indeed, if a fixed brush position

is to be maintained at all loads, it must not be so far advanced owing to the steepness of the gradient of the field (*cp*. §. 18). The angle of lead λ_e will thus fall short of $90°\ (1-\beta)$, or when $\beta = 0{\cdot}735$ will be less than 24 electrical degrees. To predict the exact angle of lead required for any particular armature current would require not only that the distribution of the displaced field in the interpolar region should be accurately mapped out, but also that the curve of current-change in the short-circuited sections, as modified by the brush contact-resistance, should be determined. The process would therefore be complex and tedious, and its result at best only a mere approximation. In practice, when it becomes necessary to know $AT_b = \frac{JZ}{2p}\cdot\frac{\lambda_e}{180°}$ for the calculation of the field winding on the approximate method for full-load (*cp*. Chapter XIX, § 7 (*b*)), the designer falls back on the evidence of machines of similar type already built and tested. A safe allowance is $\lambda_e = 15$ electrical degrees, so that

$$AT_b = \frac{1}{12}\cdot\frac{JZ}{2p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (200)$$

Usually the angle of lead with carbon brushes fixed in position for all loads is not more than about 10 electrical degrees.

§ 40. Choice of pitch of winding and number of slots in machines without commutating poles. In the absence of commutating poles it is evident from § 25 that it is always advisable to adopt a sufficient degree of chord winding so that the two layers of coil-sides short-circuited in each zone do not overlap greatly. At the same time, this shortening of the chord cannot be carried very far without bringing the band of short-circuited coil-sides too near the pole-tips.

The total width of the band from edge to edge, including any intervening slots not filled with short-circuited coil-sides, is in each interpolar zone $\frac{S}{2p} - y_n^1 + \frac{1}{c}\cdot\frac{(b_1 - b_m)}{b}$ where c is the number of sectors per slot, and the coil-sides are arranged in two layers. If possible, this expression should not exceed 70 per cent. of the number of slots between the pole-tips or $\frac{S}{2p}(1-\beta)$; or with a further allowance for the different commutating positions of the short-circuited coils when c is large, say[1]

$$y_n^1 - \frac{1}{c}\cdot\frac{b_1 - b_m}{b} + \frac{c-1}{c} \geq \frac{S}{2p}\,(0{\cdot}7\beta + 0{\cdot}3) \quad . \quad . \quad . \quad (201)$$

A decided check is therefore placed upon the possibility of shortening y_n^1 considerably. Generally speaking, y_n^1 should fall short of the pole-pitch by one slot. As soon as the coil-sides short-circuited at

[1] Niethammer, *Elektrische Maschinen, Apparate und Anlagen*, Vol. 1, p. 158.

adjacent brushes fall in different slots, there is no further reduction obtainable in the slot inductance, so that when $S/2p$ is fractional, and the remainder exceeds $\frac{1}{2}$, there is little advantage gained by shortening the pitch by more than one slot.

While from the point of view of economy in manufacture a very large number of slots per pole is objectionable, owing to the loss of valuable space in insulation and the reduction in the area of iron at the roots of the teeth through their taper in small armatures, there is, on the other hand, a limit to the minimum number of slots per pole. Apart from considerations connected with the number of commutator sectors per slot, a very small number of slots is open to the objection that the possible choice of the back-pitch for a coil reckoned in slots becomes greatly restricted. In order that the span of the short-circuited coil should not approach too closely to the polar arc by equation (201), it is advisable that y_B^1 should not be less than, say, 89 per cent. of the pole-pitch with usual widths of pole-face; while, on the other hand, in order to spread out the short-circuited coils in several slots, y_B^1 should fall short of the pole-pitch. It therefore usually falls between the limits of 89 and 93 per cent., and the number of slots must not be so far reduced that this condition becomes difficult of attainment.

The Commutating-Pole Machine

§ 41. The ampere-turns of commutating poles.—The much greater freedom which the designer obtains by the use of commutating poles is obvious; he is thereby rendered independent of the main field excitation, so that when rightly designed they practically remove the commutating difficulty and reduce the output limit mainly to that from heating only. That is, ac is fixed chiefly by heating, although the effect upon the voltage between sectors from distortion of the field (Chapter XIX, § 18) must still be borne in mind.

Corresponding to one commutating pole

$$AT_{gr} = \frac{1}{2} \cdot \frac{JZ}{2p} + 0{\cdot}8B_{gr} \,.\, K_r \, l_{gr}$$

but though it is a certain reversing density that is required for commutating purposes, the total reversing flux ϕ_r and the leakage flux ϕ_{lr} to be added thereto must be known in order to find the density in the commutating pole itself.

Half of the gap between the adjacent edges of a main and a commutating pole is usually five or more times the direct air-gap between commutating pole-face and armature. Taking then the mean of the values of K_1 and K_2 in Figs. 253–4 corresponding to the ratio $c/l_{gr} = 5$ in accordance with Chapter XIX, § 13 (*end*), $\frac{1}{2}(K_1 + K_2)$ = about 2·7, or the joint width of the two equivalent

strips, one along *each* side of the commutating pole filled with lines at the normal density over the pole-face, is approximately a constant of value 2·7 times the air-gap. The total area is then

$$a_{gr} = (w_c + 2{\cdot}7\, l_{gr})\,(l_{rr} + K_2 l_{gr})$$

and

$$\phi_r = B_{gr} \, . \, a_{gr}.$$

For B_{gr} will then be substituted the values of B_{rc} from equations (193 5) with the variable resistance term $R\left(\frac{t}{T} - \frac{1}{2}\right)$ omitted. In place of the more accurate expression of equation (173), it often suffices in practice to assume without further detailed calculation

$$AT_r = 1{\cdot}25 \text{ to } 1{\cdot}3\, JZ/4p \quad . \quad . \quad . \quad . \quad . \quad (202)$$

§ 42. The importance of the leakage and saturation of the commutating pole.—Given the amount of the total flux, both useful and stray leakage, in the commutating pole with the main poles unexcited, then, when the latter are excited, on the one side the magnetic potential of the pole which is of the same sign reduces the flux, while on the other side the magnetic potential of the pole which is of opposite polarity increases it. Hence the excitation of the main poles under given conditions of ampere-turns on the commutating pole and armature does not greatly affect the useful flux of the commutating pole, although it is reduced in amount owing to the influence of unequal saturation in the two neighbouring sections of the yoke, as described in Chapter XIX, § 13.

Owing to the opposing effect of the armature ampere-turns as causing a difference of magnetic potential between armature and commutating pole-shoe which checks the passage of the useful flux, the stray flux of the commutating pole often greatly exceeds the useful flux. To improve this proportion, a very short air-gap is advantageous, but on the other hand this renders the commutating field very fluctuating according to whether a slot or a tooth is situated centrally under the pole-shoe (*cp.* Fig. 376), and this fluctuation reacts unfavourably on the commutation; further, it may cause overheating of the pole-shoe (particularly if solid) by eddy currents.

The degree of saturation of the iron of the commutating pole is a question of the greatest importance, especially in machines subjected to heavy over-loads, since owing to the leakage flux the proportionality between the reversing field and the armature current to be commuted can only hold so long as the commutating pole can be regarded as of constant reluctance, *i.e.* from no-load up to a certain limit of load. Owing to this effect of the leakage, the proportion of the flank and side surfaces of the commutating pole in relation to its sectional area requires careful consideration. To secure the maximum sectional area with minimum surface, it may thus become advisable to employ pole-faces shortened so that their

axial length is less than that of the main poles or of the armature ($L_r < L$) and circular cores.

The total and the useful fluxes can be measured on a machine that has been built without the use of a ballistic galvanometer by a convenient method described by H. E. Stokes.[1] A few turns of flexible wire of large area are wound round the commutating pole at its junction with the yoke to measure the total flux, and others round the pole-shoe to measure the useful reversing flux, and in each case are connected to a milli-voltmeter. The armature is fixed and the shunt coils are normally excited. Current is then supplied to armature, series, and commutating-pole winding from a booster capable of giving 3 or 4 times the full-load current of the machine under test, and having a rheostat in its field which is separately excited. The current supplied is varied by means of the rheostat from zero up to the maximum that can safely be reached and back again. The change of the flux through the exploring coil will deflect the milli-voltmeter, and the rate of variation of the current must be such that steady voltage readings can be taken, say, every 3 seconds with simultaneous readings of current and time. The change of flux can then be calculated from the volts and time, and when integrated, a curve can be plotted connecting flux with exciting ampere-turns (Fig. 374).

Typical curves of the flux-density in the commutating pole, and of the reversing flux-density in the air-gap under the commutating pole-face are shown as B_{mr} and B_{gr} in Fig. 374, from which it will be seen that the proportionality between the ordinates of the two curves which holds so long as the iron may be regarded as of constant reluctance is gradually lost as the current is increased. As the leakage flux increases, its passage with the useful flux through the iron absorbs more and more of the total ampere-turns on the commutating pole. The useful flux increases to a maximum and then decreases until at a certain load the surplus of the total commutating ampere-turns over the ampere-turns expended on the iron is exactly equal to the armature ampere-turns as acting on the commutating pole air-gap; no reversing flux can then flow, and the point x is reached. Beyond this load the difference between the total ampere-turns and the ampere-turns expended in driving the leakage flux through the iron is less than the armature ampere-turns, and the flux in the commutating air-gap becomes reversed in direction.

The case is made clear by Fig. 375. Both the commutating and the armature ampere-turns increase proportionately with the load current as shown by the inclined straight lines OAA' and OBB'.

[1] "Commutating Pole Saturation in D.-C. Machines," *Trans. Amer. I.E.E.*, July, 1913, Vol. 32, p. 1327, which has been freely drawn upon in the present section, and to which the reader is referred for further details of the method.

Let the difference between the straight line OAA' and the curve OCC' represent the ampere-turns required over the iron, rising at first proportionately with the total flux, and later increasing more rapidly; then the shaded difference between the curve OCC' and the straight line, OBB' gives the effective ampere-turns available to drive the useful reversing flux through the commutating-pole air-gap. It will be seen that these are reduced to zero at the point X and current x.

If the reversing flux-density OB' in Fig. 374 were correct to balance the inductive volts of the short-circuited sections with increasing current, then it will be seen that for any load above 600 amperes unbalanced volts arise, and their amount at any load can be judged by such a diagram as that of Fig. 375.

In order easily to plot such a curve when once the value of the commutating leakage permeance $\mathfrak{P}_{lr}$ or its reciprocal, the reluctance $\mathfrak{R}_{lr}$ has been calculated, it is convenient to express the useful reversing flux ϕ_r or its density in the air-gap B_{gr} in terms of the total reversing and leakage flux $\Phi_{mr} = \phi_r + \phi_{lr}$ or of their density in the commutating pole B_{mr}.

Let AT_r = the total commutating-pole ampere-turns $= TqJ$, where T are the turns per pole and it is assumed that the whole of the armature current is taken round each commutating pole.

Let AT_i = the ampere-turns expended over the iron of the commutating pole $= f'(B_{mr}) \cdot l_{cp}$

$$\text{and let } k = \frac{\text{total ampere-turns per comm. pole}}{\text{total } AT \text{ per comm. pole} - \text{armature } AT \text{ per pole}}$$

$$= \frac{AT_r}{AT_r - JZ/4p} = \frac{TqJ}{TqJ - JZ/4p} = \frac{Tq}{Tq - Z/4p}$$

$$\text{Since } \phi_{lr} = \Phi_{mr} - \phi_r = \frac{1{\cdot}257\,(AT_r - AT_i)}{\mathfrak{R}_{lr}}$$

$$\phi_r \cdot \mathfrak{R}_{lr} = \Phi_{mr} \cdot \mathfrak{R}_{lr} - 1{\cdot}257\,(AT_r - AT_i) \quad . \quad . \quad . \quad . \quad (203)$$

Also approximately by equation (173)

$$\phi_r \cdot \mathfrak{R}_{gr} = 1{\cdot}257\left(AT_r - \frac{JZ}{4p} - AT_i\right) = 1{\cdot}257(AT_r/k - AT_i)$$

$$\phi_r \cdot k \cdot \mathfrak{R}_{gr} = 1{\cdot}257\,(AT_r - k\,AT_i) \quad . \quad . \quad . \quad . \quad (204)$$

Adding (203) and (204) together and substituting $B_{gr} \cdot a_{gr}$ for ϕ_r

$$B_{gr}(a_{gr} \cdot \mathfrak{R}_{lr} + k \cdot K_r l_{gr}) = \Phi_{mr} \cdot \mathfrak{R}_{lr} - 1{\cdot}257\,AT_i\,(k-1)$$

$$B_{gr} = \frac{B_{mr} \cdot a_{cp} \cdot \mathfrak{R}_{lr} - 1{\cdot}257 f'(B_{mr}) l_{cp} \times \dfrac{Z/4p}{Tq - Z/4p}}{a_{gr} \cdot \mathfrak{R}_{lr} + K_r l_{gr} \cdot \dfrac{Tq}{Tq - Z/4p}} \quad . \quad (205)$$

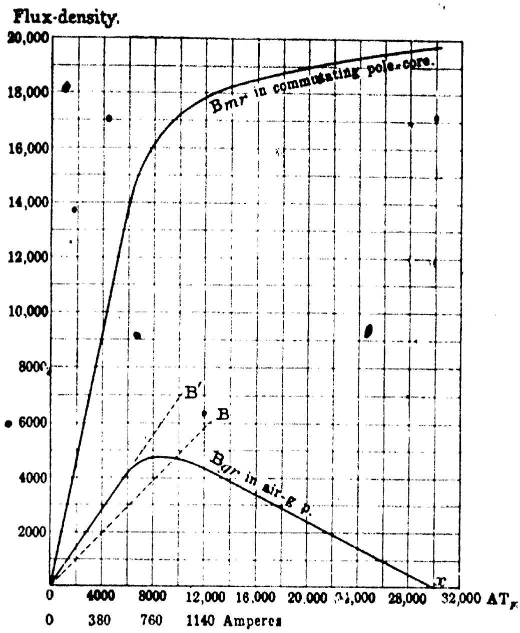


FIG. 374. Flux-density and excitation of commutating poles.

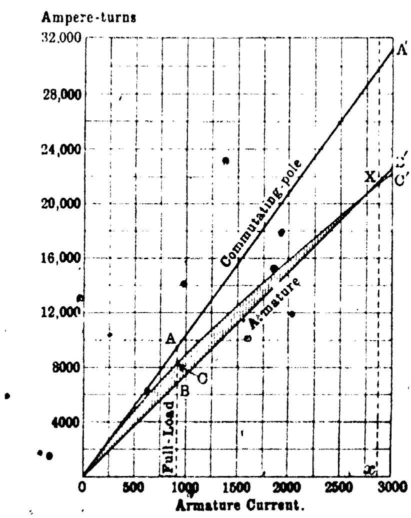


FIG. 375.—Ampere-turns and load current with commutating poles.

Hence by giving different values to B_{mr}, the corresponding values of B_{gr} can be found, and for each value of B_{mr}

$$AT_r = 0{\cdot}8(B_{mr} \,.\, a_{cp} - B_{gr} \,.\, a_{gr})\mathcal{R}_{lr} + f'(B_{mr})l_{cp}$$

is found and can be expressed in terms of B_{mr} and $f'(B_{mr})$.

In order that the useful reversing flux may rise in proportion to the inductive volts over a wide range,[1] it is necessary that $\mathcal{R}_{lr}$ should be as high as possible in relation to $\mathcal{R}_{gr}$. A high ratio of commutating-pole ampere-turns to armature ampere-turns, *i.e.* a high value of $\dfrac{Tq}{Z/4p}$, is also of advantage, but less so than a high value of $\mathcal{R}_{lr}/\mathcal{R}_{gr}$. The introduction of a second air-gap by a brass liner, say $\frac{3}{4}$ in. thick, between the foot of the commutating pole and its seating on the yoke, increases the ratio of commutating to armature turns and also incidentally reduces the leakage slightly[2]; but such "high-reluctance" commutating poles are more feasible on rotary converters than on continuous-current generators.

§ 43. Example with only half as many commutating poles as main poles.—The curves of Figs. 374 5 relate to a 500-kW. machine with only half as many commutating poles as main poles, and it will be seen how much useful information may be obtained from such curves. The machine is chosen for purposes of illustration owing to the greater liability to saturation in such a design in which each commutating pole has to do double duty. The leading dimensions are given in Fig. 273, and an estimate upon the lines of this figure, with the several sections of the permeances reduced in proportion to the ampere-turns acting upon them and with the addition of flank permeance, shows that the total leakage permeance regarded as in parallel with the air-gap and acted upon by $(AT_r - AT_i)$ may fairly be represented by $\mathcal{P}_{lr} = 79{\cdot}5$, or $\mathcal{R}_{lr} = 0{\cdot}0126$. In other words $\dfrac{1{\cdot}257\,(AT_r - AT_i)}{\mathcal{R}_{lr}}$ will give a number of leakage lines ϕ_{lr} which when added to ϕ_r yield an average flux-density in the pole-core determining with sufficient accuracy the iron ampere-turns. From the data of the machine, $a_{cp} = 108{\cdot}5$, $l_{cp} = 30{\cdot}5$, $K_r l_{gr} = 1{\cdot}16 \times 0{\cdot}381 = 0{\cdot}442$, $a_{gr} = 212$,

and
$$k = \frac{21 \times 4}{(21 \times 4) - 60} = 3{\cdot}5.$$

[1] For other means for securing the same result by displacing half of the brushes on each brush arm relatively to the other half and supplying current to the commutating pole only from the forward half, or by an auxiliary shunt winding on the commutating pole, see Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 346 352.

[2] Miles Walker, *loc. cit.*

Thence

$$B_{gr} = \frac{B_{mr} \times 108{\cdot}5 \times 0{\cdot}0126 - 1{\cdot}257 f'(B_{mr}) \times 30{\cdot}5 \times 2{\cdot}5}{0{\cdot}0126 \times 212 + 3{\cdot}5 \times 0{\cdot}442},$$

$$= 0{\cdot}324 B_{mr} - 22{\cdot}75 f'(B_{mr})$$

and using this value for B_{gr}

$$AT_r = 0{\cdot}8\,(B_{mr} \times 108{\cdot}5 - B_{gr} \times 212) \times 0{\cdot}0126 + 30{\cdot}5 f'(B_{mr})$$

$$= 0{\cdot}402\, B_{mr} + 79{\cdot}1\, f'(B_{mr}).$$

Assuming the values of the second column in Table XIII for a wrought-iron forging, the values of the eighth column are thus found and are plotted in Figs. 374–5.

TABLE XIII

B_{mr}	$f'(B_{mr})$	$0{\cdot}324 B_{mr}$	$22{\cdot}75 \cdot f'(B_{mr})$	B_{gr}	$0{\cdot}402 B_{mr}$	$79{\cdot}1 \cdot f'(B_{mr})$	AT_r
10,000	4	3,240	91	3,149	4,020	316	4,336
14,000	8·2	4,540	186	4,354	5,630	649	6,279
15,000	11	4,860	250	4,610	6,040	870	6,910
16,000	19·65	5,190	448	4,742	6,440	1,555	7,995
17,000	35	5,506	796	4,710	6,830	2,770	9,600
17,500	51·3	5,665	1,165	4,500	7,045	4,060	11,105
18,000	64	5,830	1,455	4,375	7,245	5,060	12,305
18,500	120	6,000	2,730	3,270	7,450	9,500	16,950
19,000	185	6,150	4,210	1,940	7,650	14,610	22,260
19,750	281	6,400	6,400	0	7,950	22,250	30,200

For very high saturations, the figures can only be regarded as approximations, but they well show the nature of the action as the reversing field declines to zero. The curves further show that the design fails to secure exact proportionality between the inductive volts and the reversing field within the working range of the armature current, due to saturation of the commutating pole-core. The correct reversing density would follow the line OB in Fig. 374, and the line OB' should fall on OB. The correct field is obtained at full-current, but for lesser values of the current there is over-commutation and at all over-loads there is under-commutation. This result is hardly to be avoided with only half as many commutating poles as there are main poles, and on this account the design has been chosen to illustrate the case. Actually the evil has been above over-estimated, since, as shown in Fig. 273, the thickness of the commutating pole can and should be increased towards the root, and the saturation and iron ampere-turns can be thereby reduced. The amount of the inaccuracy then left often does not warrant the expense of doubling the number of poles, even though each carries many less ampere-turns. An alternative design with twice as many commutating poles would call for $B_{gr} = 2330$ at full-load and only

about 4000 AT_r per pole, but on 8 poles this would amount to 32,000 in all.'

§ 44. Range of excitation of commutating poles.—*The experiments of J. Rezelman with commutating poles show that the ratio* of the maximum to the minimum excitation when sparks just begin to appear in both cases is much greater at low speeds than at high speeds. This may be explained upon the hypothesis that the brushes by their corrective action can make up for any over- or under-excitation when the inductive volts are themselves low. Further, as might be expected, at all speeds the range of excitation without sparking is much less in the case of copper and carbon brushes of high conductivity than in the case of high-resistance brushes (in a particular machine 1·45 as against 1·15), so that the excitation for the former must be much more nicely adjusted. Lastly, the minimum excitation is almost independent of the speed, and the possible range is almost entirely due to the over-excitation which becomes permissible especially at low speeds.

§ 45. The proportions, etc., of commutating poles.—The commutating pole must be mechanically strong and well supported in order to prevent its being set into oscillation by the varying drag of the armature teeth as they pass under it. When it falls on the horizontal division of the magnet frame, its seating can be arranged eccentrically, as in Fig. 273.

The shorter the air-gap, the greater the pulsation of the flux due to the varying position of the slot-opening in relation to the pole. Fig. 376 shows the diminution of the forked shape of the flux-density due to a slot centrally under the pole as the air-gap is increased.

The width of the commutating pole in the direction of rotation must be at least equal to the tooth-pitch in order that the reversing field may not vary very greatly during the passage of a slot under it. As a second condition, if the slots contain several coil-sides in each layer, in order to keep each coil-side under the pole during the whole of the period of commutation, the width must with diametric winding be equal to the peripheral speed of the armature multiplied by the time of commutation *plus* the amount by which the hindermost coil-side of a slot is displaced from its position for true uniformity as compared with the foremost coil-side. Hence by addition of twice the expression of (199) to $T \times v$, the value of T being as in equation (190), the width of the commutating pole-face would be

$$\left\{b_1 + b\left(c - \frac{a}{p}\right)\right\}\frac{v}{v_c} - r_s\,(c-1) = \left(b_1 - \frac{a}{p}\cdot b\right)\frac{D}{D_c} + t_1 + r_s\,(c-1)\,. \quad (205)$$

which usually averages about 1·66 to 1·75 times the tooth-pitch t_1. If the winding be long-chord, an additional tooth-pitch should be

added, but considerations of leakage usually forbid such an addition in full. Or the pole-shoes can be set aslant to the axis of the armature core, so as to increase the time of their action without increasing the leakage. But in either case it must be remembered that the fringe from the sides has itself considerable effect in extending the time of strong reversing action. In order to accommodate the commutating poles without bringing them too close to the main pole-shoes, the ratio of the pole-arc to the pole-pitch is usually not

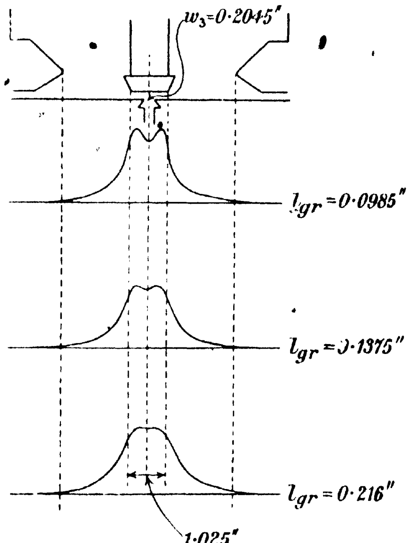


FIG. 376.—Influence of air-gap on shape of commutating flux-curve. (After Rezelman.)

more than 0·66 or even less, and there should be at least $3\frac{1}{2}$ to 4 slots in the zone between the main poles. Strictly speaking, in order to secure exact instantaneous balancing of the combined ohmic and inductive voltage in the short-circuited coil, the reversing field in a generator should rise in density from one side to the other by an amount proportional to $R = r + 2r_c \,.\, b/b_1$; but such refinements are not of value in practice.

With a commutating pole-shoe of breadth $2t_1$, the edges may with advantage be sharply bevelled off (Fig. 377) as recommended by J. Rezelman.

One other point of importance must also here again be mentioned.

In order to keep the circumferential breadth of the commutating pole within practical limits, and to minimize leakage between commutating pole-tip and main pole-tip, the armature winding should, as stated in § 26, be diametric or more nearly concentrated than would otherwise be advisable (*see also* p. 167 for " split coils " with $y_{B}{}^{1}$ variable, used in commutating-pole machines); the two sides of a short-circuited coil will then be acted upon by almost equal E.M.F.'s from the reversing fields in which they are situated.

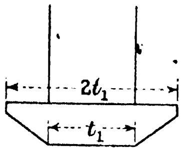


FIG. 377. Shape of commutating pole.

With fairly wide commutating poles and such conditions that the brushes admit of some shifting backwards without sparking, their position may be so adjusted as to produce an appreciable compounding effect. Even when this cannot be secured, the regulation of a shunt-wound generator with commutating poles is usually good, and better than that of the machine without such poles.

While the copper on the field winding proper is reduced by commutating poles owing to the shorter air-gaps that may then become admissible, the efficiency of the machine is but little affected. But owing to the close proximity of the commutating and main coils, the ventilation of the field-magnet system is to some extent lessened as compared with the dynamo of the same size without commutating poles, and this consideration must be duly allowed for in the design. Especially is it difficult to secure a low rise of temperature on the commutating coils at full-load or over-loads, and on this account it is common to wind the commutating poles with bare strip on edge (*cp.* Fig. 282). With large currents at low voltages, and on turbo-generators up to 250 volts, the commutating coil may be formed of a bare copper cylinder divided spirally to form a helix of a few turns.[1]

The air-gap of the commutating poles on large machines with toothed armatures should not be less than $\frac{1}{8}''$ to avoid undue heating of their polar surface by eddy currents.

Experimentally the correct ampere-turns for the commutating poles are best obtained by separately exciting them and altering the excitation until the voltage read between two points touching on the commutator in line with the edges of the brushes, and therefore ΔE_{r} is a minimum or as nearly zero as possible both for light-load and full-load armature currents.

[1] For illustrations and other details of machines with commutating poles, see Miles Walker, *The Specification and Design of Dynamo-electric Machinery*, Chap. XVIII; and Page and Hiss, " Direct-Current Design as influenced by Interpoles," *Journ. I.E.E.*, Vol. 39, p. 570.

In order to adjust the winding of the commutating poles to the best amount for sparkless and cool running, a diverter may be used, formed of coils on an iron core, and so designed as to have the same time constant as the commutating coils; otherwise, with a non-inductive resistance an undue proportion of the current will be shunted during rapid changes of the load. When the fluctuations of load are very rapid, as in traction work, a difficulty sometimes arises from the inability of the commutating field to follow with sufficient rapidity the change of the armature current which is to be commuted. The diverter can then be adjusted so as to shunt a greater proportion of the current momentarily through the commutating poles and thereby to accelerate the change of the commutating field upon sudden increase of the load.[1] But apart from such use, it is best for the winding to be correct so as to dispense with any need for a diverter.

§ 46. Experimental comparison of inductance of machines.—The apparent inductance of a section of the armature winding in the centre of the interpolar zone of a non-commutating-pole machine with the field-magnet circuit closed can be approximately measured by passing an alternating current of known value and frequency through it by means of two narrow brushes placed on the commutator sectors which terminate the coil, and measuring the voltage between the brushes $E_s = 2\pi f \, l_s I$ (the resistance being negligible). The rest of the armature winding is then in parallel with the section under consideration, but if the frequency of the current is high, the great inductance and the higher resistance of the longer path allow only a negligible proportion of the total current to flow through it.

This method may be extended by including between the brushes as many sectors $\left(\frac{b_1 - b_m}{b}\right)_+$ as are short-circuited at a brush at a time, when $\frac{E}{C_k . 2\pi f I} = l + \Sigma . M$.

Comparative figures may similarly be obtained for various machines with and without commutating poles when the field-magnet is removed as mentioned in § 24 (*b*) and according to Karl Pichelmayer[2] the results are borne out by measurements of the reversing field actually required. There is, however, in either case the theoretical objection that the frequency during the test is, say, only 50, while to be strictly comparable with that of commutation it should be perhaps 400 or 500, so that the damping is then different.[3]

Another method of measuring the apparent inductance is to place narrow brushes on sectors of the commutator at their normal positions in regular work, and to pass an alternating current through the q paths of the armature winding in parallel; the alternating voltage is then measured between the sectors adjacent to the brush, whence $l_s = \frac{qE}{2\pi f I}$ since only $\frac{1}{q}$ of the current passes through a section.[4] The voltage from the sector on which a brush rests across $\left(\frac{b_1 - b_m}{2b}\right)_+$ coils on one side of the brush should be added to that

[1] *Cp.* R. Pohl, *Journ. I.E.E.*, Vol. 40, p. 249; W. Hoult, *ibid.*, Vol. 40, p. 630; and Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 345.

[2] *Electrician*, Vol. 70, p. 973, abstracted from *E.T.Z.*, Vol. 33, p. 1100. *Cp.* also J. Rezelman, *Recherches sur les Phénomènes de la Commutation.*

[3] *Cp.* Niethammer, *E.u.M.*, Vol. 30, p. 55 (21 Jan., 1912).

[4] A. Mauduit, p. 276.

across $\left(\frac{b_1 - b_m}{2b}\right)_+$ coils on the other side of the brush to find the total effect from the simultaneously short-circuited sections. The results so obtained measure not only $'/ + \Sigma . //$, but also the voltage caused by rise and fall of the cross field due to the remaining sections of the armature which are not short-circuited when in normal work. They are therefore higher than those obtained by the first method in virtue of the cross field effect, and they measure the total inductance from the flux which has to be reversed through a coil during short-circuit as due to all the ampere-turns of the armature. If the brushes are on the line of symmetry, the voltage across similar sections on either side of the brush are of course alike, but when the brushes are displaced from the line of symmetry they differ, and in a machine without commutating poles they are naturally higher in the sections brought nearer to the pole-tips on the one side of the brush than in those on the other side which have been withdrawn from the pole-tips.

§ 47. Experimental determination of short-circuit current.—If an armature coil is severed at some spot and the two free ends are connected to a pair of slip-rings, and if upon these rings rest brushes which are short-circuited by a standard low resistance of known amount, the current in the coil can be traced by the oscillograph through a complete revolution, *i.e.* not only when the coil is under a pole but also in the brief periods when it is short-circuited by the brushes. The armature circuit still remains closed, and the insertion of the low resistance at the one point hardly affects the conditions. Potential leads are taken from the ends of the low resistance to the oscillograph, and the current in the coil can thus be measured. By special arrangements the horizontal scale can be increased, and the curve of current-change during the period of short circuit be extended, so as to enable the whole process to be carefully watched and recorded. Curves taken by this method[1] show that the change of the short-circuit current is often extremely irregular owing to obscure secondary causes such as the exact bedding of the brush surface, yet they fully bear out all the conclusions that had been previously drawn on more theoretical grounds. When the brush position in a dynamo is advanced into too strong a reversing field, or is moved backwards into a strong field towards the trailing pole, the heavy current in the new or in the old direction is shown in the curves at the end or at the beginning of short-circuit by sharply pointed peaks which fluctuate violently when excessive sparking takes place. Even when no sparking takes place, if the brushes are too far forwards or backwards, considerable pulsations are set up in the magnetic field, the excess current in the short-circuited coils causing the value of the direct magnetizing turns of the armature and their effect on the field to pulsate with the frequency of commutation. The main field through the entire magnetic circuit of yoke and poles is thus set into oscillation, which appears as a ripple in the wave of E.M.F. or current in a coil, while it is passing under the poles, especially towards the pole-tips. This shows that under such circumstances it is only approximately true to regard the apparent inductance of the coil as due solely to the field within the interpolar region. An excessive short circuit current can, in fact, even affect the voltage given by a machine for the same number of ampere-turns on the field, and after allowance has been made for the actual value of the direct magnetizing turns (other than those of the short-circuited coils) due to the angle of lead or trail.[2]

[1] *See* especially *Journ. I.E.E.*, Vol. 38, pp 548, 557 (Dr. W. M. Thornton); p. 1023 (Dr. D. K. Morris and J. K. Catterson-Smith); Vol. 35, p. 430 (J. K. Catterson-Smith); Vol 38, p. 176 (Prof. F. G. Baily and W. S. H. Cleghorne), where oscillograms are given of the current flowing through a sector into the brush, *i.e.* i_1 or i_2; and C. Shenfer, *Journ. Amer. I.E.E.* Vol. 40, p. 843.

[2] *Cp.* "Ueber Magnetische Wirkungen der Kurzschlussströme in Gleichstromankern," by Dr. R. Pohl, Vol. 6, *Sammlung elektrotechnischer Vorträge* (Stuttgart, Ferdinand Enke (1905)). The similar effect with commutating poles is especially noticeable in the machine investigated by Prof. F. G. Baily and Mr. Cleghorne, *Journ. I.E.E.*, Vol. 38, p. 171 ff.

§ **48. Brushes and brush-holders.**—The width of each brush along the axis of the commutator is usually from $\frac{3}{4}''$ to $2''$, the latter dimension being seldom exceeded, since it then becomes troublesome to maintain proper contact along its entire bearing surface. Hence, to carry any considerable current, two or more brushes are mounted in line on each brush-spindle, forming in effect one wide brush. This arrangement also renders it possible to adjust each brush separately, or even to remove one temporarily, without interrupting

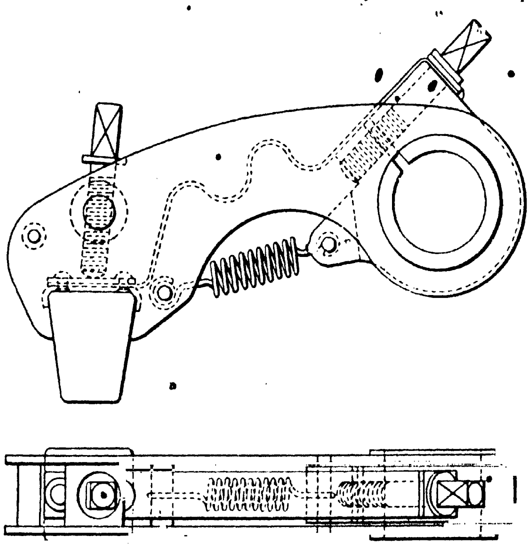


FIG. 378. Carbon brush-holder of hammer type.

the current; and this advantage is so great that every dynamo which is more than a toy is invariably furnished with at least two brushes on each arm, each brush being of such width that, if one be removed, the other can temporarily carry the current of both.

Carbon brushes, in order to obtain sufficient contact-surface without unduly increasing the length of the commutator, are usually from $\frac{1}{2}''$ to $1''$ thick in the direction of rotation. They thus cover more than one sector, as a general rule two or three, and in cases of very narrow sectors as many as four; but when this is the case a hard variety of carbon is to be recommended rather than a soft graphitic quality. As the conductivity approaches more and more nearly to that of a metal brush, it must be given a somewhat analogous width of, say, $1\frac{1}{2}$ sectors.

Carbon brush-holders may be classified under one or other of two

leading types. In the first or pivoted "hammer" type the more or less wedge-shaped block of carbon is fixed rigidly within its box, which forms the farther end of a pair of stamped or cast brass or aluminium cheeks; these latter are pivoted on the brush spindle so as to be free to turn round it, were it not for the constraining action of the pressure spring (Fig. 378). In the second type, which is best for all medium and high speeds, the brush is a rectangular slab, free to slide radially up or down in a guiding box, but pressed down by a helical, clock, or other spring (Figs. 379–381). In both

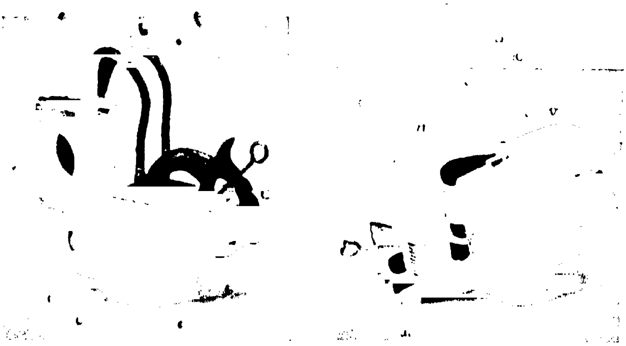


FIG. 379.—Brush box, with sliding carbon, for brush spindle of small machine. (W. H. Allen, Sons & Co., Ltd.)

cases the carbon brush is nearly radial, although it may have a slight rake (about 10° from the radial line) in the direction of rotation, which reduces the tendency to "chattering." In the second type the carbons require some attention, so that they may not become set fast in their boxes through dust and dirt; but, on the other hand, they must not be too loose in fit, whereby they tend to take up different positions in the boxes according to the speed when this is variable, with consequent disturbance to their bearing surface. A clearance of about 6 mils circumferentially and 10 mils axially is usually sufficient. The brush box should be fairly deep with perfectly smooth sides unbroken by slots or holes. It must be solid and substantial, and preferably not a sheet-metal stamping liable to become twisted, but cast. The lesser inertia of the sliding carbon as compared with that of box and carbon is a feature greatly in favour of the second type; owing to it the pressure-spring can cause the brush to follow up any local unevenness in the commutator or departure from a perfectly circular track much more quickly. In Figs. 380*a* and *b* the brush boxes can be brought nearer to the commutator surface by being

slid along a lower set of grooves on the rack-work; wear of the commutator can thus be followed up, and still the correct radial setting of the brushes is strictly maintained. Another type is shown in Fig. 381. Here a flat spring exerts not only a radial pressure

Fig. 380, *a* and *b*.—Set of brush boxes for larger machine, and component part (W. H. Allen, Sons & Co., Ltd.)

on the brush, but also a front-to-back pressure which keeps the trailing edge of the brush firmly against the side of the brush box and prevents the brush tilting. The spring, which can be moved to take up brush wear, can be lifted up and remain so for inspection of the carbon, while behind it in the working position are the two pigtails from brush to bracket. Each set of carbons is arranged in a joint brush box with thin division plates between them.

In all cases a good electrical connexion directly between the

carbon and the fixed box or brush spindle is of vital importance. In the hammer type the brush may be wedged or drawn tight up into its box, with an interposed layer of copper gauze to form a good contact between the two; the box is then joined by a flexible copper connector to the central part of the brush-holder, which is either clamped or screwed to the brush spindle. In the sliding type the brush is itself drilled with a hole into which is tightly wedged a flanged copper tube under one or both ends of which is spread the end of the flexible, the unflanged end of the tube being

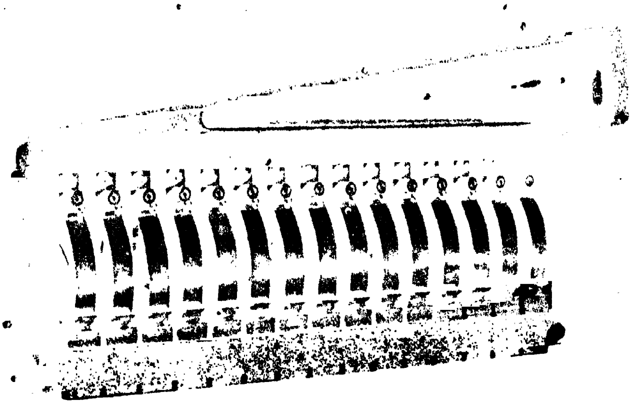


FIG. 381.—Brush bracket with springs down. (The British Thomson-Houston Co., Ltd.)

then spun over a copper washer (Le Carbone); or the end of the flexible may be splayed out into strands which are worked up into the substance of the brush during its manufacture. In the "Battersea" connexion of the Morgan Crucible Co., the splayed-out ends, after insertion in a hole in the carbon, are firmly held by metallic powder compressed round them. Metal plates on the top of the brush are also used, sometimes of leaf metal bent over so as to form in effect an additional spring, but in any case the use of solder is best avoided. The flexible copper connector is formed up into a twisted pigtail with enough slack to allow of the carbon being withdrawn from the box for examination. Brushes coppered at the top are frequently used, but except when current is to be passed into some attachment fitting on to the carbon, the coppering is of doubtful advantage, since current passing by it

into the box tends to eat away the metal, and the need for it is removed by a thoroughly good flexible connexion. No current should pass through the pressure-spring or pressure-finger, and to prevent this the brush top is sometimes covered with an insulating cap.[1]

§ 49. Causes of local sparking on particular sectors. The pressure of the brush-tips on the commutator may be adjusted by altering the tension or pressure of the "hold-on" spring. "Jumping" of the brushes, due to vibration of the machine when running, must be carefully avoided, since it will give rise to sparking, and on this account a substantial brush-carrier with strong but light brush-holders, capable of being firmly fastened, is an essential part of a well-designed and well-built dynamo. The brushes should then bear lightly and evenly on the commutator. Any pressure beyond this should be avoided, since it will cause increased friction and wear. Occasionally, one or two sectors in a commutator wear down below the general cylindrical surface of the rest, and form what is known as a *flat*; as the brushes pass over the faulty spot the circuit is momentarily broken and sparking occurs, which rapidly increases the evil. The development of a flat is often attributed to inequality in the wear-resisting properties of the sectors, but it is almost always due solely to sparking. Owing to a want of uniformity in the spacing of the winding on the armature surface, a particular section may be short-circuited when in an incorrect position; its passage under the brushes is then accompanied by sparking, and the sector to which it is attached becomes worn. With carbon brushes it is especially important to employ a soft quality of mica having approximately the same rate of wear as that of the metal sectors.[2] Any recessing of the mica strips should be very slight and carefully done, the edges of the shallow grooves being slightly bevelled to prevent catching of the brush on them. Oil and dirt must never be allowed to collect in the grooves, since they carbonize and the former may have a solvent action on the adhesive material employed as cement in building up the mica plates. As an alternative to recessing the mica, brushes containing a small admixture of abrasive material are occasionally used in extreme cases of hard mica strips which tend to stand "high" in wear.

"Copper picking" is one of the most troublesome ills to which carbon brushes are liable, and difficult to cure; the conditions giving rise to it are but little understood, yet it would appear to be simply due to very small sparks carrying over particles of copper in the direction of the current from commutator to brush

[1] For many practical details in connexion with brush boxes and brush gear, see Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 308–319.

[2] *See* Chapter XIII. § 30.

(positive of a generator), and under some circumstances these adhere to the brush, plating its working face, even though no acid or moisture to cause electrodeposition is present.

If an armature wire is broken or its connexion to the commutator becomes loose, violent sparking may be set up, and the faulty coil may then be located by running the machine until one sector becomes pitted by the sparks. In a lap-wound armature the fault lies in the coil behind the marked sector against the direction of rotation; in a wave-connected armature with as many sets of

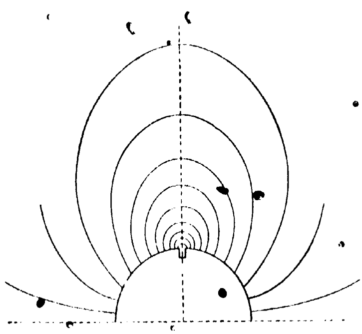


FIG. 382. Paths of flux from surface of 2 pole armature

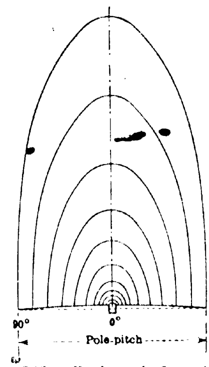


FIG. 383. Paths of flux from pole pitch with many poles.

brushes as there are poles, it may be in either the coil behind or in that ahead of the marked sector. A complete break may be identified by the greenish colour and snapping sound of the sparks.

The length of the commutator should be such that the sets of brushes can be relatively staggered enough to overlap one another, so as to distribute the wear; otherwise if there is any sideways movement of the armature, sparking may be set up by the brushes striking against the sides of the ridges formed when the brushes are exactly in line. With four or more sets of brushes, they are best staggered in pairs, so that a positive and a negative brush sweep over the same path.

NOTE TO CHAPTER XX

THE SURFACE-OF-CORE AND END-CONNEXION INDUCTANCE OF CONTINUOUS-CURRENT ARMATURES

I. The surface-of-the-core permeance of an armature in air.—(i) *The permeance within the polar arc.* With a pair of diametric slots on the convex surface of a 2-pole armature in air, the paths of the flux embracing one slot are roughly as shown by the half armature of Fig. 382; with many poles, so that the pole-pitch becomes nearly flat, the paths resembling ellipses become narrowed but still spread out widely into the surrounding space (Fig. 383). In both cases the paths are very nearly semicircles near to the

slot opening, and at the actual opening are bounded by the semicircular zone which in Chapter XX, § 24 (a''), has been added to the slot inductance proper.

At the outset it may be remarked that for the same number of poles, the permeance per cm. length due to the surface of the core is very nearly independent of the diameter of the armature, so that calculations from a single

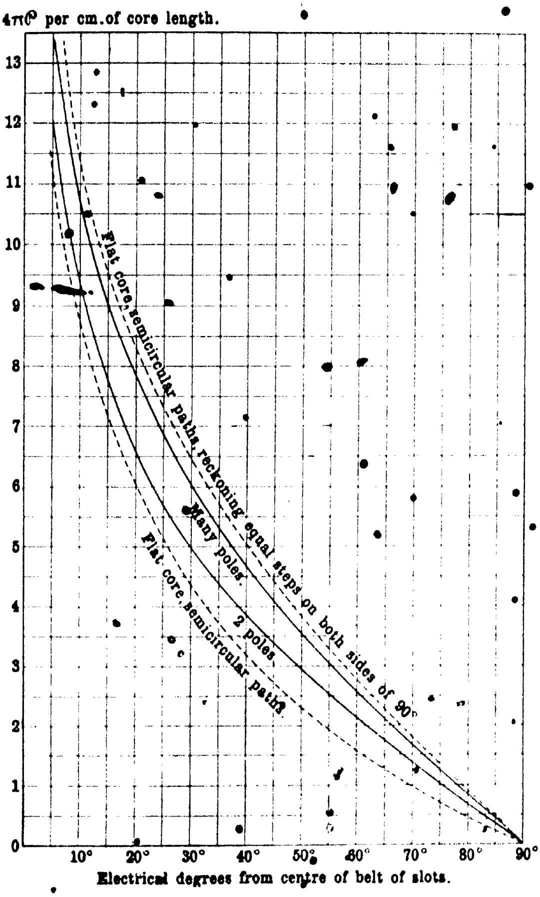


FIG. 384.—Integrated permeance × 4π.

diagram are practically applicable to all diameters. The simple reason is that as the surfaces from which the flux springs increase in width with increasing diameter, so also do the lengths of path increase nearly in the same proportion.

Starting from the mid-point between two diametric coil sides, *i.e.* at 90 *electrical* degrees from the centre of the considered coil-side, the permeance can be summed up as we approach the coil side even when the width of opening of the slot is unknown. This has been done graphically to a fair degree

of approximation from diagrams similar to Figs. 382–3 for the two cases of $p = 1$, and of a core with so many poles that the pole-pitch becomes practically flat. The results multiplied by 4π, *i.e.* $4\pi\mathcal{P}$ are given in the two full-line curves of Fig. 384, and these show that the number of poles makes comparatively little difference. In fact, with a 4-pole machine a great step is made towards the case of a flat core.

The proportionate values of the flux-density in the two cases are indicated in the full lines 2 and 3 of Fig. 385. Naturally the greater curvature of the 2-pole case leads to its flux-density falling to the lowest value which it takes near 90°, the longer length of the paths not being fully compensated by their gradually increasing area. But in general the flux density on the convex surface tends to become nearly uniform at some distance from a single slot, and the permeance exceeds that of the same region on a flat core with lines assumed to be semicircular. The results being plotted on a base of electrical degrees, it becomes possible to make ready comparison of any

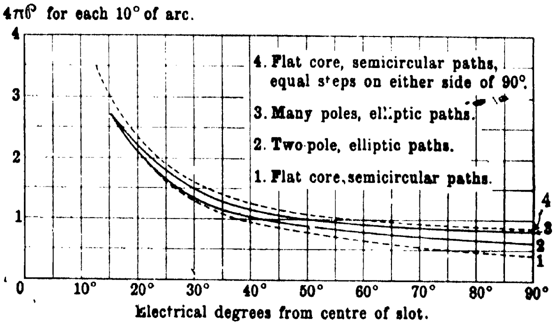


FIG. 385. Integrated permeance for each 10° of arc.

number of poles with the 2-pole case, and further to compare the case of an infinite number of poles or a flat core, both with elliptic and with semicircular paths.

If the permeance of a flat core with semicircular paths is integrated only up to 90 electrical degrees, *i.e.* over half the pole-pitch on either side of the central slot (*cp.* Fig. 386), the lowest curves of Figs. 384–5 are obtained. The density is seen from Fig. 385 to fall off too rapidly, and this is borne out by experiment.[1] But in the ordinary formula for the inductance of two parallel wires in air or on a flat core, the permeance in relation to one wire is integrated from the one wire to the other wire. When, however, the actual resultant distribution of the flux is drawn, and assumed as the basis of calculation as in Figs. 382–3, it would be incorrect to proceed forwards with the integration past the horizontal diameter of Fig. 382 or past the dividing line of the pole-pitch of Fig. 383. It is then only necessary to integrate the permeance backwards from 90°; *i.e.* from the centre point between the two coil-sides, and the magnetic effect of the second coil-side appears in the resultant paths and the areas of the tubes. With semicircular lines, as in Fig. 386, the paths followed by the fictitious separate fluxes attributed to each coil-side intersect; in actual fact the flux close to the centre between the two wires continues onwards nearly in a straight line and the actual paths are nowhere truly circular. In order then to compare the results obtained on the semicircular assumption with those actually holding, some account must be taken of the semicircular flux beyond the pole-pitch. But the whole of the flux from 90° onwards to 180° must not be directly added to each of the steps backwards in the lower curve of Fig. 384; for we are not at present making any assumption as to the arc covered by the current-carrying coil-sides,

[1] J. Rezelman, *Recherches sur les Phénomènes de la Commutation*, pp. 65–7.

and it is therefore not known when the opposite coil-side is reached and the M.M.F. of the first coil-side begins to be neutralized and finally to vanish. A just comparison is, however, made if in Fig. 386 for every step of, say, 10 degrees backward from 90° towards 0°, an equal step forward from 90° toward 180° is made. When this is done, the upper dotted curves of Figs. 384–5 are obtained, and it will be seen how nearly the two treatments coincide in the case of a flat core or indeed for any number of poles from four upwards.

All the curves of Fig. 384 approach infinity at their upper end, but it must be remembered that with only a single slot the upper limit is fixed by half the angle corresponding to the width w_3 of the slot opening, or again if a winding spread over several slots or over some arc were being considered, the difference of magnetic potential acting between the ends of the tubes continually diminishes as the centre line of the coil side is approached and at the centre becomes zero, so that the flux and the inductance always remains finite.

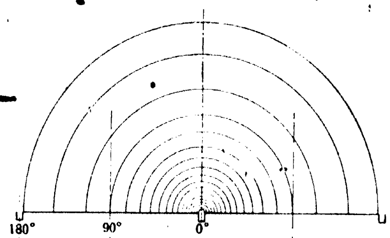


FIG. 386. Calculation of permeance with flat core and semicircular paths.

It will be observed that, for a normal polar arc = 0·7 of the pole-pitch, the integrated permeance up to 13½° is fairly represented by $4\pi s \times 9{\cdot}5$ for any number of poles when the armature is in air. To this must further be added some allowance for the lines spreading out laterally on each side of the core,[1] so that the density of the flux on the surface of the core at its edges is greater than in the centre.

Finally, therefore, the permeance in air within the arc normally covered by the pole-faces may with considerable accuracy be said to be of the order $4\pi s \times 10$, and this value will be here adopted. The reason for the division of the surface into the two portions covered and not covered by the poles has been already explained in Chapter XX, § 24 (*b*).

(ii) We now pass to the consideration of the *equivalent permeance for flux within the interpolar arc*. When a magnetizing coil has its turns divided into groups and lodged in slots, the general outline of the actual course which the flux lines within the span of the magnetizing slots follow is roughly indicated in Fig. 387 (*c*), from which it will be seen that in the slots on either side of the central slot with an uneven number, or of the central pair with an even number, the flux crosses the slots in a more or less slanting direction (*cp.* Fig. 309).

Now in the previous calculation of the slot inductance (Chapter XX, § 24 (*a*)), each slot has been credited with its own local system of flux, which passing transversely across the slot or its mouth would yield in each tooth radial bands in opposite directions, as shown in Fig. 387 (*a*). With two slots only, the exact centre line of the intervening tooth can carry no flux, since from considerations of symmetry the M.M.F. of the one slot acting, say, outwards is balanced by that of the other slot acting inwards. The effect of the second slot decreases as we proceed away from the centre line of the intervening tooth to the wall of the first slot, and *vice versa*, so that taking

[1] J. Rezelman, *loc. cit.*, p. 15.

any section across the tooth the local flux-density is a maximum at the wall and thence decreases up to the centre line of the tooth where it changes direction. The same applies equally to every pair of adjacent slots, so far as any local system of lines peculiar to each slot is concerned (Fig. 387 (*a*)).

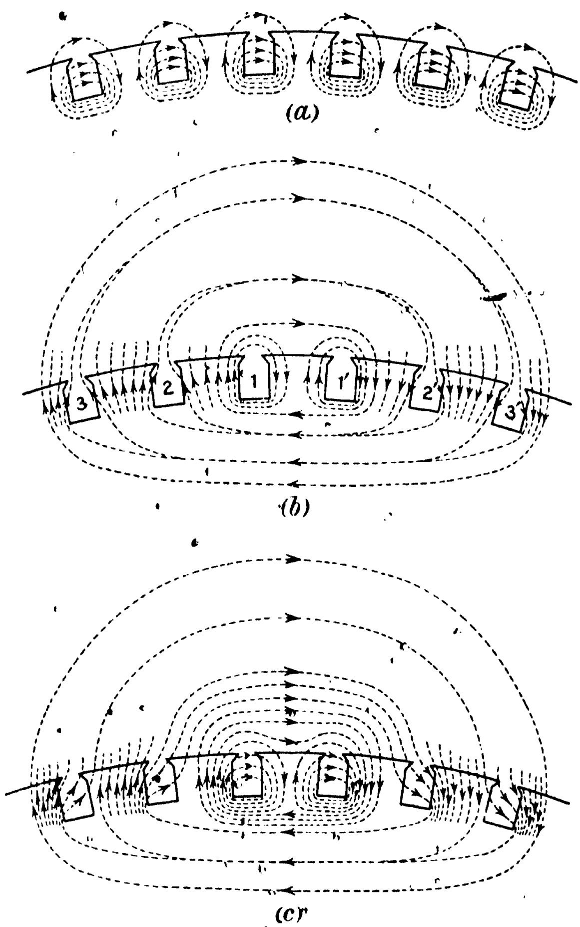


FIG. 387.—Paths of flux (*a*) corresponding to slot inductance, (*b*) in bands linked with one or more slots, (*c*) resultant.

Now the transversal component of the actual lines of Fig. 387 (*c*) crossing the slots is already taken into account in the slot inductance previously considered. In calculating therefore the surface-of-the-core inductance, the system of Fig. 387 (*a*) only requires to be supplemented by a *symmetrical* system of bands of flux common to groups of slots or to the slots as a whole

(Fig. 387 (*b*)). The action may be roughly explained by saying that a portion of the joint flux due to the M.M.F. of the two central slots 1 and 1′ is carried onwards across slots 2 and 2′ by the M.M.F. of these latter; this joint flux must not therefore be treated as partly linked with slots 2 and 2′, since these linkages have already been taken into account in the slot inductance. Similarly, a part of the joint flux due to the M.M.F. of the wires in slots 2, 1, 1′, 2′ is carried onwards across the slots 3 and 3′, and so on. It will be seen that the separate fluxes of Fig. 387 (*a*) and (*b*) at that edge of each tooth which is nearer to the centre are additive, but that at the edge of each tooth which is farther from the centre they are in opposite directions. The actual result is that the flux-density across a section of each tooth decreases as we proceed from the inner wall nearer to the centre to the outer wall further away, the flux being as it were driven across the slot into the next tooth. Or the same effect may be described by saying that the flux embracing the outer slots is drawn inwards across the slots. Whether we say that the actual flux is expanded further outwards from the centre or contracted inwards depends entirely upon the point of view adopted, and each is equally true.

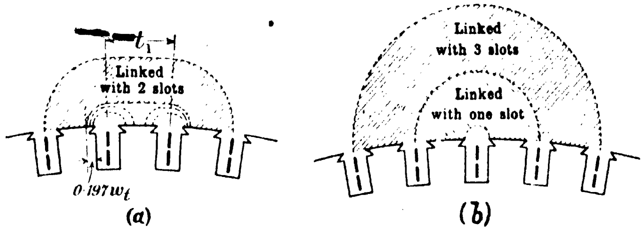


FIG. 388.—Calculation of inductance from lines linked with one or more slots, (*a*) even, (*b*) uneven in number.

In the case of an even number of magnetizing slots, each half of the central tooth continues to carry flux in opposite directions, and in the account given above it is assumed that this is the only tooth of which this is true. It is, however, possible that if the local flux is very strong as compared with the joint flux, the M.M.F. of other slots may be able to establish partially its own local system as an actuality, but in this case it will only be within some contracted area, and the local flux will be driven into a narrower and narrower strip down the edge of each tooth further from the centre as the joint M.M.F. of the ampere-turns increases with increasing numbers of slots embraced by the symmetrical flux.

This possibility is, however, greatest when the armature is in air, since then the symmetrical field is weakest and the length of path in air increases somewhat as the ampere-conductors increase so that the density of the symmetrical field does not much increase.

An exact determination of the distribution and magnitude of the local fluxes linked with certain slots out of the total group would be a mathematical problem of great complexity, even when the iron is neglected as being by comparison infinitely permeable. Its solution would also only be true for the particular machine possessing the assumed relative dimensions of slot and tooth.

In order, then, to calculate the bands of symmetrical flux and to use up all the available air-space, it will be best to imagine the ampere-turns of each slot concentrated into a line down the centre of the slot. With an uneven number of slots (Fig. 388 (*b*)), the central slot is embraced by its own semi-circular flux at its mouth, and the remaining bands will also be assumed to be semicircular. With an even number of slots (Fig. 388 (*a*)) the central pair have their own small local flux, and it will be better as an approximation to assume paths for the joint bands formed by quadrants struck from the

centre of the slots nearest the middle and joined by straight lines of length t_1. The extension of the symmetrical flux over the slot openings is not far from correct since, in proportion to the total length of path in the air, only a short distance is required to complete the path into the inner edges of the slot opening.

With *two slots* equally filled with conductors (*cp.* Fig. 388 (*a*)), each slot will be capped by its own local flux; this may be assumed to follow semicircular paths on the outer sides, and thence to spread out with reduced density until it dies away to zero on the centre line of the middle tooth under the action of the opposing M.M.F. of the second slot. At a certain point along the tooth outside the pair of slots, the flux linked with both slots begins, the dividing line occurring when the M.M.F. of one slot divided by the reluctance of its path is equal to the M.M.F. of two slots divided by the reluctance of the alternative path. Let x be the fraction of the tooth width w_{t1} at which the boundary line falls; then the fraction x is given by the relation

$$\frac{1}{\pi(x w_{t1} + 0{\cdot}5 w_3)} = \frac{2}{\pi(x w_{t1} + 0{\cdot}5 w_3) + t_1}$$

$$x = \frac{1}{\pi} - \left(\frac{1}{2} - \frac{1}{\pi}\right)\frac{w_3}{w_{t1}}$$

or in terms of the tooth-pitch,

$$x\, w_{t1} = \frac{w_{t1}}{\pi} + \frac{w_3}{\pi} - \frac{1}{2} w_3 = \frac{t_1}{\pi} - \frac{1}{2} w_3$$

so that the local flux linked only with one slot always ends at a distance of $\left(\frac{t_1}{\pi} - \frac{w_3}{2}\right) + \frac{w_3}{2} + \frac{t_1}{2} = t_1\left(\frac{1}{\pi} + \frac{1}{2}\right) = 0{\cdot}818 t_1$ from the centre line in the case of a flat core. Owing to the convex curvature of the armature, the real value of x is somewhat greater, but the difference is negligible.

The permeance of the local flux is then

$$\frac{2{\cdot}3}{\pi} \log \frac{\frac{1}{2} w_3 + \left(\frac{t_1}{\pi} - \frac{1}{2} w_3\right)}{\frac{1}{2} w_3} = \frac{2{\cdot}3}{\pi} \log \left(\frac{2}{\pi} \cdot \frac{t_1}{w_3}\right)$$

With $t_1 = 2{\cdot}5\, w_3$, this is equal to $\frac{2{\cdot}3}{\pi} \times 0{\cdot}202$, and the coefficient in relation to j_b is

$$\tfrac{1}{2} \times 9{\cdot}2 \times 0{\cdot}202 = 0{\cdot}93$$

The permeance of the joint flux up to the pole-tips is

$$\frac{2{\cdot}3}{\pi} \log \frac{\pi\left(\frac{\pi D}{2p} \times 0{\cdot}15 - \frac{1}{2} t_1\right) + t_1}{\pi\left\{\left(\frac{t_1}{\pi} - \frac{1}{2} w_3\right) + \frac{1}{2} w_3\right\} + t_1} = \frac{2{\cdot}3}{\pi} \log \left(\frac{D}{2p t_1} \times 0{\cdot}74 - \frac{\pi}{4} + \frac{1}{2}\right)$$

$$= \frac{2{\cdot}3}{\pi} \log \left(\frac{D}{2p t_1} \times 0{\cdot}74 - 0{\cdot}285\right)$$

and if $t_1 = 2{\cdot}5\, w_3$,

$$= \frac{2{\cdot}3}{\pi} \log \left(\frac{D}{2p \times w_3} \times 0{\cdot}296 - 0{\cdot}285\right)$$

Adding 10 for the region normally under the poles, the coefficient for the total equivalent permeance is

$$b = 9{\cdot}2 \log \left(\frac{D}{2p \times w_3} \times 0{\cdot}296 - 0{\cdot}285\right) + 0{\cdot}93 + 10$$

The curve for $\frac{f}{j_b} = 0{\cdot}5$ in Fig. 360 is thus obtained.

Trial with $w_t = w_3$ and $t_1 = 2w_3$ shows that but little difference is produced by change in the ratio of $w_3 : w_t$; the local flux is reduced, but the joint flux is increased, and on the whole there is a slight increase.

When the short-circuited coil-sides are not equally divided between the two slots, let f = the number of coil-sides short-circuited in that slot which has the greater number, and let g = the smaller number of the other slot. The local flux of the f group naturally exceeds that of the g group and demands more width on the tooth-crown on either side of the f slot. The centre of gravity as it were of the total flux is thus shifted towards the f group. It may, however, with sufficient accuracy be assumed that the dividing line between the local fluxes and the joint flux is simply shifted, so that the mean value $\frac{1}{2}(x_1 w_{t1} + x_2 w_{t1}) = x w_{t1}$ for equally filled slots. The lesser arc and permeance on the f side between the dividing line and the pole-tip will further be practically balanced by the greater arc and permeance on the g side up

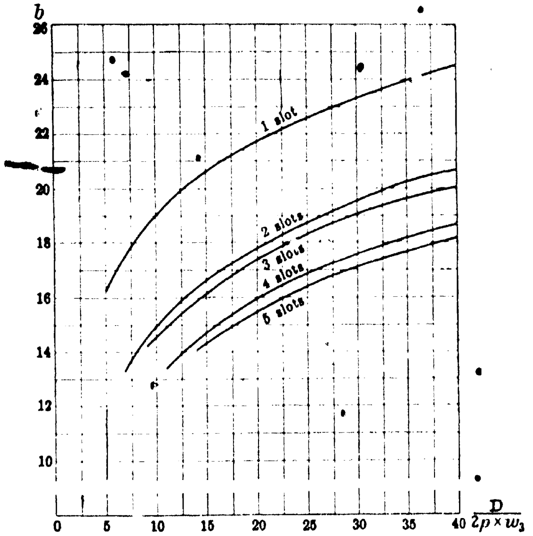


FIG. 389.—Calculation of permeance for centremost slot out of a number.

to the other pole-tip. On such approximate assumptions the second and fourth curves of Fig. 360 are obtained, and analogously those for 3 and 4 slots. (Figs. 361, 363.)

The curves for one slot and for the central or the slot nearest to the centre out of 2, 3, 4, 5 slots, in each case equally filled, are collected in Fig. 389, from which it will be seen that the curves for 2 and 4 slots are but little higher than those for 3 and 5 slots respectively. This might be expected from the comparatively small amount of flux between the central pair of an even number of slots, and the fact that with an even number no one slot is linked with all the flux.

If the back pitch $y_B{}^1$ reckoned in slots is made $(y_B - 1)/u$ as recommended in Chapter XI, § 12 (equation 49), the advantage is gained that the c coils corresponding to a slot can be taped up into a joint composite coil for insertion into the slots as a whole. But the divergence of $y_B{}^1$ from $S/2p$ may then be appreciable (especially in the wave-connected armature with $S/2p$ fractional), and consequently the spread of the short-circuited coil-sides in the commutating zone calls for a proportionately wide commutating pole-shoe (*cp.* Fig. 364). In such cases, in order to lessen the necessary width

of the commutating pole-face, it may become advisable to adopt two values for the back slot-pitch, *e.g.* with 6 coil-sides per slot, two of the c coils to have $y_B^1 = (y_B - 1)/u$ and the remaining third coil to have the longer back slot-pitch of $y_B^1 + 1$. The composite coil is thus "split," and the two divisions of the one coil-side must be separately taped and inserted separately as an upper layer in two slots. But the average pitch of the three is now $y_B^1 + \frac{1}{3}$, or in general it is $y_B^1 +$ *a fraction*, which may be more nearly equal to $S/2p$ and closely approach the diametric case.

The effect of such "split" coils is to modify such diagrams as those given in Fig. 364 for a back slot-pitch, which is the nearest to the pole-pitch; intermediate cases are obtained, and both the slot and core-surface permeances are affected. While beneficial from the standpoint of commutating-pole width, it destroys complete similarity between all the component coils. On the other hand, it must be remembered that so far as commutation is concerned, the last coil in a slot to be commuted is already under conditions different from those of its leading neighbours in the same slot. The advantage or otherwise of the arrangement therefore depends on the circumstances of the case.

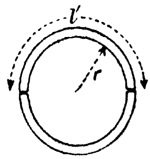


FIG. 390.

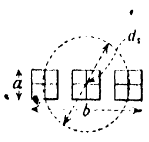


FIG. 391.

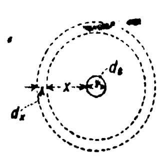


FIG. 392.

II. The end-connexion permeance.—The inductance of the *end-connexions* of a coil of a barrel-wound armature is in strictness not simply proportional to their length, since it depends upon the shape of the coil and the area of the path which is traversed by the lines linked with the ends. In the case of a circular coil entirely in air or half embedded in iron throughout its length, so that every centimetre length is exactly similarly circumstanced, the area corresponding to a centimetre length of the periphery is a wedge-shaped sector; since the density of the lines decreases towards the centre, a square centimetre near the periphery of the ring is of more account than one near the centre; hence, as the diameter and length of a turn are increased, the lines per centimetre length of the periphery and per C.G.S. current-turn rise very slowly, and become fairly constant. When surrounded entirely by air, this point at which the curve of lines per centimetre length becomes nearly flat is reached when a diameter of 50 centimetres is exceeded, and a figure of some 8 to 10 lines per centimetre length is reached. With a rectangular coil free in air the same effect is present; the maximum number of lines per centimetre length is necessarily obtained when the coil is square, but the reduction as the coil is narrowed is not very marked until one pair of parallel sides is less than 20 centimetres apart.[1] The V-shaped end-connexions of a barrel-wound coil, each of length l' centimetres, if grouped together (Fig. 390), approach fairly closely to the case of a circle in air, and may be replaced thereby if we ignore the influence of the proximity of the iron core; in a bar-wound armature with involute end-connectors which lie more closely to the core, a somewhat higher inductance would be reached.

It is therefore practically legitimate to treat the end-connectors as linked with a certain flux per centimetre length, and the inductance of the ends of a drum-coil when barrel-wound may be approximated as follows. At each end the end-connexions of j_c coils run side by side, where $j_c = \left(\frac{b_1 - b_m}{b}\right)_+$ the

[1] H. M. Hobart, "Modern Commutating Dynamo Machinery," *Journ. I.E.E.*, Vol. 31, pp. 185 ff.

number of sections short-circuited simultaneously at one brush, and between the considered coil and the remainder there is mutual inductance. Let d_s = the diameter of a circle whose periphery is equal to that of the group of j_c coil-ends, inclusive of insulation and any air-gaps which there may be between them (Fig. 391, where, *e.g.* the periphery of the dotted circle is equal to the periphery of the rectangular packet of three coil-ends, each containing four wires). The circle of Fig. 390 equivalent to the two V-shaped sets of a barrel armature has a radius of $r = l'/\pi$. The permeance of a cylinder of air round the coil-ends, of width dx (Fig. 392), and extending along half the circle corresponding to the connexions at one end of the armature, is $\frac{l'dx}{2\pi x}$, if we neglect its gradual contraction towards the centre of the circle; this is acted upon by a M.M.F. of $4\pi w j_c$. An element of the inductance is therefore $d\lambda = \frac{4\pi w^2 j_c l' dx}{2\pi x} = 2w^2 j_c l' \frac{dx}{x}$, and the integral of dx/x between the limits of the radius r_0 of the complete equivalent circle and the radius $d_s/2$ of the small circle representing the section of the coils is

$$\int_{\frac{d_s}{2}}^{\frac{l'}{\pi}} \frac{dx}{x} = \log_e \frac{l'}{\pi} - \log_e \frac{d_s}{2} = 2\cdot3\left[\log_{10}\frac{l'}{d_s} - \log_{10}\frac{\pi}{2}\right] = 2\cdot3 \log_{10}\frac{l'}{d_s} - 0\cdot45$$

If a and b are the two dimensions of the packet, *i.e.* its height and width (Fig. 391) so that its periphery is $2(a+b) = \pi d_s$, it is simpler to retain these terms, and we have

$$2\cdot3\left(\log\frac{l'}{d_s} - \log\frac{\pi}{2}\right) = 2\cdot3 \log\frac{l'}{a+b}$$

The self and mutual inductance of one end of length l' would therefore be

$$w^2 l' \, 2j_c \left(2\cdot3 \log\frac{l'}{a+b}\right) \times 10^{-9}$$

$$= w^2 l' \, 4\cdot6 \left(\frac{b_1 - b_m}{b} + 1\right) \times \log\frac{l'}{a+b} \times 10^{-9} \text{ henrys}$$

and the value $\lambda' = e_c j_c$ for insertion in (187) would be

$$\lambda' = 4\cdot6\left(\frac{b_1 - b_m}{b} + 1\right) \times \log\frac{l'}{(a+b)}$$

This slightly exceeds the true value even for a circle, and more exactly if the V-shaped ends are grouped together to form a square (Fig. 393) instead of a circle the logarithmic term becomes $\log\frac{l'}{1\cdot16(a+b)}$. An exact expression for the circular approximation would in fact give the maximum for a given value of l', and the more pointed the V, the less the inductance; but the above expression is sufficiently near to the truth.

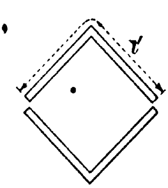


FIG. 393.—Length for calculation of permeance of V end-connexion.

But though a single end-connexion or a small group of j_c end-connexions by themselves would be encircled by the above flux, in their actual position on a barrel-wound armature in two layers their case is quite different. Just before commutation the current-sheets on either side of them in the same layer are in opposite directions, and may be allowed to cancel one another on each side of the end-connexion considered, but in addition the half end-connexion in the upper layer finds itself crossing nearly at right angles a current-sheet in one direction, and the half end-connexion in the lower layer finds itself below a current-sheet crossing it at right angles in the opposite direction. If, therefore, both layers are cut through at various points down the V, and the direction of the axial component of

the current[1] is marked, the general effect is that one leg is situated above currents in the opposite direction, and this leg changes during commutation (Fig. 394). The same is also true of the circumferential components. The consequence is that the end-connexion does not start with the full value of the flux as above calculated, nor does it end with the full value reversed. As an approximate allowance the something over half will be taken, and

$$\lambda' = \left(\frac{b_1 - b_m}{b}\right)_+ \times 3 \log \frac{l'}{(a+b)} \quad . \quad . \quad . \quad . \quad (189)$$

has been assumed in Chapter XX, § 24 (c).

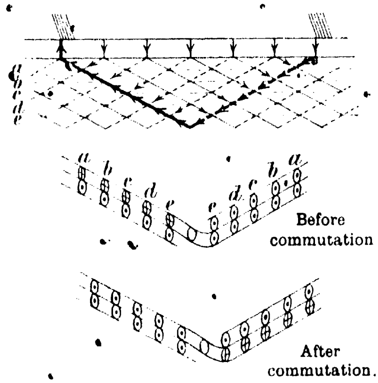


FIG. 394. Currents in double-layer end-connexions in relation to short-circuited loop.

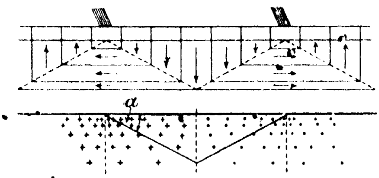


FIG. 395.—Total field from double-layer end-connexions.

The same result must follow with perfect linear commutation if the end-connexions are assumed to move through the actual armature end-field fixed in space (cp. Chapter XX, § 31). In the actual end-connexions sheets of current cross one another in directions nearly at right angles in the upper and lower layers respectively. Their resultant magnetic effect at any spot is therefore that due to an equivalent single sheet of current which is the resultant of the two. When the end-connexions of a pole-pair are thus analysed they resolve themselves into a pair of triangular current-sheets in which the magnetizing current flows axially, and a pair of similar triangular sheets in which the current flows circumferentially. Joining up the strips of current, the field as shown

[1] For "axial" and "circumferential" component currents in the end-connexions, cp. Chap. XIX, § 2.

by the dots and crosses in Fig. 395 is found to change its direction at the centre between two commutation zones, to be a maximum at each commutation zone, and to decrease in density towards the outer edge of the winding. The end-connexions of diametric loops in the middle of their commutation form the boundaries of the triangular current-sheets, and with linear commutation these latter remain perfectly fixed in space. If therefore B_e be the maximum value of the flux-density near to the armature and this falls off at a uniform rate to zero at the apex of the V, the E.M.F. induced in one end-connexion of a coil is $2\left(w\frac{B_e}{2}\times\frac{l'}{2}\sin\alpha\times v\right)\times 10^{-8}$

The density B_e is roughly $\dfrac{4\pi\dfrac{JZ}{2p}\sin\alpha}{\dfrac{\pi D}{2p}\times\pi} = \dfrac{4JZ\sin\alpha}{\pi D}$

and thence by substitution, the E.M.F. of the end is

$$w^2\frac{2f}{T}l'\times 20\sin^2\alpha\times\frac{b_1-b_m}{b}\times 10^{-9}$$

so that $\lambda' = 20\dfrac{b_1-b_m}{b}\sin^2\alpha$, and if $\alpha = 30^\circ$, $= 5\dfrac{b_1-b_m}{b}$, which is of the right order of figures.

Owing to the mutual inductance from coil-sides short-circuited in the same slots but in another layer, the simple addition of the axial width of the air-ducts $n_d w_d$ to the length l' of the end-connexion, as recommended on p. 111, is not strictly correct and slightly under-estimates the effect; cf. F. Unger, *E.u.M.*, Vol. 36, p. 161, and *E.T.Z.*, Vol. 41, p. 627.

CHAPTER XXI

THE HEATING OF DYNAMOS

§ 1. Rise of temperature in dynamo at work.—No subject is of greater importance, alike to the designer and the user, than the question of the heating of dynamos. The continuous generation of heat in the armature and magnet-windings of all dynamos, so long as they are at work, is a necessary consequence of the passage of the current through their coils, and the appearance of this heat implies that a corresponding amount of energy is "lost," in so far as no useful work is derived from it. All that can be done from the point of view of economy is to minimize the amount of the heat which is thus generated, so as to obtain a reasonably high efficiency, suited to the circumstances of any given case. Apart, however, from the question of the amount of heat produced every second, or its rate of generation in watts, there is the further and equally important question of the temperature to which any part of the dynamo is thereby raised. Whether it be the field magnet coils or the armature which is the source of heat in question, when the machine is set to work the temperature of their mass gradually and continuously rises above the temperature of the surrounding air, until, finally, the rate at which the heat is generated is balanced by the rate at which it is carried off by radiation, convection, and conduction. Evidently the *rise* of temperature depends essentially upon the amount of cooling surface provided and its actual effectiveness in dissipating heat, and, this being so, it follows that it may be regulated so as not to exceed a certain maximum, if the amount of cooling surface be duly proportioned to the watts expended. Careful consideration of the matter is necessary for the following reasons. A large range of temperature is disadvantageous to the working of a dynamo through its effect on the regulation of its voltage. A high temperature is detrimental through its effect on the efficiency of the machine; and still more so on account of its effect on the durability of the insulation. Each of the three reasons will now in turn be considered, the last being of the greatest importance.

§ 2. Disadvantage of large temperature range. Lesser constancy of voltage.—Owing to the rise in temperature of a dynamo when at work, the separately-excited or shunt-wound machine requires the P.D. applied to its exciting coils to be raised if the same number of ampere-turns is to be maintained when the field-winding is hot as when it is cold; while, if the terminal voltage of the dynamo is to be kept constant, its internal E.M.F. must be increased in order to compensate for the increased loss of volts over the heated armature coils, and this necessitates either a further increase in the exciting P.D. or a higher speed of rotation. Usually, therefore,

allowance must be made for the heating of the machine in the design of its adjustable field rheostat.

Similarly, the self-regulation of the compound-wound machine for constant potential is injuriously affected by the differences in the resistances of its shunt, series, and armature coils when hot and when cold; if correctly compounded when cold, the constancy of the potential must necessarily be inferior when it is hot, or *vice versa*. In fact, in designing compound-wound machines it is especially important that the rise of temperature of the field-magnet winding be not so great as to affect seriously the compounding action of the two sets of coils, and it should preferably be limited at the most to about 30° C. or 55° F. on the surface.

§ 3. Disadvantages of high temperatures. Increase of electrical resistance.—In the next place, the higher the temperature of any portion of the electrical circuit of a dynamo, the greater is the loss of volts and also of energy due to the passage of a given current through it. The limits of temperature within which dynamos are worked under average conditions may be taken as 20° C. and 60° C., or, say, 70° F. and 140° F., the former corresponding to an average value for the temperature of surrounding air in the engine or dynamo room, and the latter to an ultimate temperature which it is usual for the coils to attain when the dynamo is worked continuously, or for many hours together, at its normal output.

Between 0° C. and 100° C. the resistivity of annealed copper increases uniformly with the temperature, *i.e.* follows a straight-line law. If this law still held strictly at lower temperatures, the resistivity would be zero at some low temperature, just as the volume of a perfect gas would be zero at the absolute zero of temperature. Taking the International Electrotechnical Commission's (1913) figure of 0·00427 per 1° C. for the temperature coefficient (at constant mass) of 100 per cent. conductivity copper at 0° C., the inferred zero of resistivity will occur at

$$-\frac{1}{0{\cdot}00427} = -234{\cdot}5^\circ\ \text{C.}^{1} = -390^\circ\ \text{F.}$$

We then have

$$\frac{R_{T^\circ+t^\circ}}{R_{T^\circ}} = \frac{(T^\circ + t^\circ) + 234{\cdot}5}{T^\circ + 234{\cdot}5}$$

where T° is the initial temperature and t° the rise above T°, both in degrees Centigrade, or

$$= \frac{(T^\circ + t^\circ) + 390}{T^\circ + 390}$$

where T° and t° are in degrees Fahr.

[1] The same figure is also adopted by the British Engineering Standards Committee (Standardisation Rules for Electrical Machinery No. 72 (1917), Art. 77). The sequence of the figures for Centigrade degrees renders them easy to remember.

Further, Messrs. Wolff and Dellinger[1] have shown that within practical limits the conductivity and temperature coefficient are proportional to a high degree of accuracy, so that, if the conductivity at 0° C. is 98 per cent. of the standard at 0° C., the temperature coefficient will be 98 per cent. of 0·00427 at 0° C., or its temperature of zero resistivity becomes

$$-\frac{1}{0{\cdot}98\times 0{\cdot}00427}=-\frac{234{\cdot}5}{0{\cdot}98}\ {}^\circ\mathrm{C}.$$

If R_{T1} = the measured resistance at a known temperature $T_1°$
R_{T2} = ,, ,, ,, ,, an unknown ,, $T_2°$

$$T_2° = \frac{R_{T2}}{R_{T1}}(T_1° + 234{\cdot}5) - 234{\cdot}5 \text{ in degrees Centigrade}$$

$$= \frac{R_{T2}}{R_{T1}}(T_1° + 390) - 390 \quad ,, \quad ,, \quad \text{Fahrenheit.}$$

From the two measured resistances therefore the mean temperature $T_2°$ of a winding which has resistance R_{T1} at temperature $T_1°$ can be at once determined, and also the *rise* in temperature as

$$t_r° = T_2° - T_1° = \frac{R_{T2} - R_{T1}}{R_{T1}}(234{\cdot}5 + T_1°) \text{ in degrees Centigrade} \qquad (207)$$

$$= 234{\cdot}5\,\frac{R_{T2} - R_{T1}}{R_{T1}} + \frac{R_{T2}}{R_{T1}}T_1° - T_1° \quad ,,$$

The temperature coefficient giving the increase in resistance per degree rise being $1/(234{\cdot}5 + T°\,\mathrm{C.})$ or $1/(390 + T°\,\mathrm{F.})$, the ratio of the increased resistance for a given rise of temperature to the original resistance evidently depends upon the starting point, *i.e.* upon the temperature at which the original resistance is measured, thus the percentage increase for each degree becomes greater as the starting point is lowered, and *vice versa*, as shown by the following table—

Temperature at which the given initial resistance holds.		*Increase in resistance per ohm of initial resistance.*	
°C.	°F.	per °C.	per °F.
0	32	0·00427	0·00237
5	41	0·00418	0·00232
10	50	0·00409	0·00227
15	59	0·00401	0·00223
20	68	0·00393	0·00218
25	77	0·00385	0·00214
30	86	0·00378	0·00210
35	95	0·00371	0·00206
40	104	0·00364	0·00202

[1] Bulletin of the U.S. Bureau of Standards, Vol. 7.

To take a numerical instance, the increase in the resistance of the copper wire on an armature which rises 72° F. (40° C.) above the temperature of the air of the dynamo room when this is 70° is

$$72 \times \frac{1}{390 + 70} = 72 \times 0{\cdot}00217 = 0{\cdot}156,$$

or 15½ per cent. of its initial resistance at 70° F. But if R is calculated during the process of design from a table of resistance at a standard temperature of 68° F. (20° C.), the percentage increase is 0·2183 for each degree Fahrenheit,[1] and R hot at 142° F. is $(1 + 0{\cdot}002183 \times 74) = 1{\cdot}1615$ times R cold at 68° F.; while from a standard initial temperature of 60° F. the temperature coefficient is 0·00222, so that the hot resistance would be 1·182 times its cold resistance, calculated from the wire table for 60° F.

In the case of coils with a large number of layers, as already explained in Chapter XVI, § 16, the mean temperature as deduced from measurement of the resistance is considerably higher than that of the surface as measured by a thermometer placed in contact with the outer insulating covering of the conductors. Thus our previous calculation (Chapter XVII, § 8), for a depth of winding of 2½″, was finally based on a mean rise of temperature for the well ventilated coil 1·59 times the surface rise, for with a surface rise of 45° F. the increase of its resistance will then be about $0{\cdot}218\,(45 \times 1{\cdot}59 + 2) = 16$ per cent. of its resistance cold at 68° F., the temperature of the surrounding air in the engine room when the machine is at work being assumed to be 70° F. Such considerable percentages show that the effects of heating must on no account be neglected in designing machines or in estimating their efficiency. Even in armatures with a single layer of conductors, if they are the rotating portion, there may be a divergence of some 30 per cent. between their actual temperature as deduced from measurements of their resistance immediately after stopping and the temperature measured by a thermometer laid on their exterior.[2] Measurements of the rise of resistance, therefore, if taken to a high degree of accuracy, give more information than the temperature of the exterior as measured by the thermometer, although in the case of armatures their very low resistance may necessitate the use of a Thomson double bridge or other suitable method.

§ 4. Deterioration of insulating materials.—But, thirdly, and of chief importance, if the temperature of any coil becomes very high, the cotton or other fibrous material commonly used for the insulating covering of the copper wires will be burnt or charred; the insulation

[1] The approximate figure 0·22 has been given in Chapter XVI, § 16, and employed in Chapter XVII, § 8.

[2] E. Wilson, "The Heating of Dynamos," *Electrician* (11th Oct., 1895).

between neighbouring turns is thus broken down, and the short-circuiting which ensues is only terminated by complete collapse. A "burnt-out" armature may be the result of an accidental short-circuiting of the machine, the heat from the excessive current almost instantaneously raising the temperature so much as literally to burn the insulation. Quite apart, however, from such accidental heating the result of continually working a machine at a high temperature is a gradual deterioration in the toughness and mechanical strength of all fibrous insulating coverings. Slowly but surely they become charred and rotten, the cotton or oiled linen crumbling away when touched, and the blackened paper and press-spahn becoming excessively brittle; so that, although the insulation resistances may still remain very high, the liability to a breakdown is enormously increased. The cotton covering of internal layers of wire which have been continuously subjected to high temperatures may be found to be a mere charred powder, which can be wiped away with the finger.[1]

Although fibrous materials in general have a very long life if an ultimate temperature of 90° C. is never exceeded, yet as soon as 100° C. is exceeded, their mechanical strength is so speedily and so seriously impaired that they fail to maintain the requisite mechanical distances between conductors and conducting structures. The longevity of even low-voltage machines may thus become reduced to a matter perhaps of weeks instead of years.[2]

§ 5. Maximum permissible temperature.—The ultimate temperature attained by a dynamo when at work is thus of the utmost importance, and practically it is the ultimate temperature of the *insulation* which is decisive when from considerations of durability we are led to fix a maximum temperature, which no part of the machine should exceed in continuous working.

By the use of insulating materials, such as mica and micanite, which may by contrast with cotton and paper be called fireproof, very high temperatures become permissible, such as 150° C. or 300° F. Field-magnet coils of thin and wide copper tape can be successfully insulated with thin mica or asbestos sheet, and drum toothed armatures with barrel winding of bars can be insulated with micanite troughs to take the bars within the slots, although such constructions are not suitable for high-voltage coils and small armature wires, owing to the room which they occupy. But even though the insulation may be quite satisfactory to withstand high temperatures, the useful field for a "fireproof" construction, at least in the case of continuous-current dynamos which are normally worked in reasonable temperatures, is very limited. The heat gradually spreads to the commutator and brushes, and impairs the

[1] *Cp.* Chap. XIII, § 17; and E. H. Rayner, *Journ. I.E.E.*, Vol. 34, p. 656.
[2] *Cp.* C. P. Steinmetz and B. G. Lamme, *Trans. A.I.E.E.*, Vol. 32, Part I, p. 81.

commutation, while the first consideration of § 2, namely, the great difference in the electrical resistances when hot and when cold, makes the regulation of the voltage more difficult. It is, therefore, mainly in connexion with large turbo-alternators that the possible use of materials of class B (described below) at very high temperatures has been discussed, and will again be alluded to in Chap. XXX.

Based on the recommendations of the International Electrotechnical Commission at its meeting of 1913, the British Standardisation Rules for Electrical Machinery issued by the British Engineering Standards Committee (No. 72—1917, Appendix I) and the Standardization Rules[1] of the American Institute of Electrical Engineers agree in fixing the maximum temperature to which various materials should be subjected as shown in the first column by their division into three classes—

	Maximum permissible temperature.	*Maximum permissible temperature rise.*
A. Cotton, silk, paper, and similar materials when impregnated or immersed in oil . . .	105° C. (221° F.)	65° C. (117° F.)
Cotton, silk, paper, and similar materials when *not* impregnated or immersed in oil . . .	90° C. (194° F.)	50° C. (90° F.)
B. Mica, asbestos, and other heat-resisting materials combined with A material as described below .	125° C. (257° F.)	85° C. (153° F.)
C. Materials capable of resisting higher temperatures than class B, such as pure mica	No limits yet specified.	

If a material or binder of class A is used in conjunction with class B material, but for structural purposes only, so that the former may be destroyed without impairing the insulating or mechanical qualities of the insulation, the combined material may be reckoned as belonging to class B.

When the insulation is composed of several different materials, the material with the lowest allowable temperature is to fix the temperature limit, but "material employed in small quantity in the construction and not relied upon continuously as a support for the insulating material" need not be regarded as part of the insulation under this rule. The latter rules, therefore, would appear to meet the case of micanite backed with thin paper or cloth, where the deterioration of the backing will not impair the insulation when once it is in place.[2]

When different insulating materials are used in different parts of

[1] *Trans. Amer. I.E.E.*, Vol. 40 (1921), p. 1571.

[2] Mr. J. S. Peck, in discussion on Mr. Everest's paper "Notes on International Standardization of Electrical Machinery," *Journ. I.E.E.*, Vol. 52, p. 244; and Messrs. Merrill, Powell, and Robbins, *Trans. Amer. I.E.E.*, Vol. 32, Part I, p. 94.

one winding (e.g. the slot and end portions of a coil), the temperature of each material must fall within its prescribed limits.

§ 6. Maximum permissible rise of temperature as limiting output.—*Having thus fixed upon a maximum permissible temperature, it is evident that the number of degrees by which a dynamo may be allowed to rise in temperature without exceeding the limit depends upon the starting point from which the rise takes place; in other words, upon the temperature of the surrounding air during the working of the dynamo. Thus, in the case of dynamos working in hot atmospheres, for instance, in the engine-room of a steamer in the tropics, where the normal temperature may be, and frequently is, as much as 46° C. (115° F.), the permissible rise is much smaller than in the case of a dynamo working in a well ventilated central station on land, where the temperature will seldom exceed 25° C. (77° F.). Since the maximum current of a dynamo is dependent upon the *rise* in temperature which is permitted, it follows that the output is indirectly limited by the normal temperature in which it is to work, and from which the rise is reckoned.

By the British and American Standardization Rules, 40° C. (104° F.) has been fixed as the standard maximum temperature which the cooling air is likely to have in temperate climes under working conditions, and hence by deducting 40° C. from the maxima to which the different materials may be subjected, the maximum permissible temperature rise as given in the end column of the table in the preceding section is obtained. The permissible *rise* having thus been once fixed, it has been decided that no correction need in practice be made in cases where the cooling air during test is at a different temperature from that which it will have when the dynamo is put into service. Further, the possible range of variation in the humidity of the cooling air under ordinary conditions is too small to require that it should be considered, the effect of humidity in increasing the cooling power being negligible unless with artificial ventilation the air is actually fog-laden.[1]

§ 7. Maximum observable temperature rise.—In any rules as to the ultimate allowable temperature it has to be borne in mind that in all probability there will be certain portions of the machine inaccessible to our measuring instruments, at which the temperature of the insulation will exceed that which can be directly observed. Owing to this fact the increase-of-resistance method of measuring the temperature of windings in order to determine durability is really only applicable to field coils and some stator windings where it is known fairly accurately by how much the temperature of the hottest part exceeds the average temperature.

[1] Blanchard and Anderson, *Trans. Amer. I.E.E.*, Vol. 32, Part I, p. 296; Skinner, Chubb, and Thomas, *loc. cit.*, p. 286; and Frank and Dwyer, *loc. cit.*, p. 249.

In order to take into account the possible presence of local "hot spots" in windings, the American Standardization Rules simply deduct from the maximum permissible temperatures and rises a margin of 15° C. when measurement is made by thermometer (except when applied directly to the surface of a bare winding, such as an edgewise strip conductor or cast copper winding, in which case a correction of 5° C. is permitted), and of 10° C. when measurement is made by the resistance method, so that a second analogous table results as follows:—

	Limiting observable temperature	*Limiting observable temperature rise.*
Class A material: by thermometer .	90° C. (194° F.)	50° C. (90° F.)
,, ,, ,, ,, resistance method	95° C. (203° F.)	55° C. (99° F.)
(If treated, impregnated, or immersed in oil, and 15° C. less if *not* treated, impregnated, or immersed in oil)		
Class B material: by thermometer .	110° C. (230° F.)	70° C. (126° F.)
,, ,, ,, ,, resistance method	115° C. (239° F.)	75° C. (135° F.)

The British Standardisation Rules require the temperature of the shunt or separately-excited field-windings of a continuous-current machine in general, and the temperature of the field-windings of an alternator always, to be ascertained by the resistance method, while the temperature of the armature, commutator, series field coils and commutating-pole coils of the continuous current machine may be measured by thermometer; the stator winding of an alternator for less than 5000 volts may also be measured by thermometer, but above this voltage, the measurement, whenever practicable, is to be by the resistance method. According then to the prescribed method of measurement, the same limits are laid down as above in the American Standardization Rules, if the rotating machine is wound for a pressure not exceeding 5000 volts, but if wound for a pressure exceeding this value, $1\frac{1}{2}$° C. is to be deducted from the limiting observable temperature for each 1000 volts or part thereof by which the rated pressure exceeds 5000 volts. For commutators the limiting observable temperature and limiting observable temperature rise are fixed at 90° C. and 50° C. respectively.

In a homogeneous body throughout the interior of which the source of heat is distributed uniformly, let

$$\frac{T^\circ_{max} - T_o^\circ}{T^\circ_{mean} - T_o^\circ} = a = \frac{1}{c}$$

where T_o° is the temperature of the outer surface,

T°_{max} is the temperature of the hottest spot,

T°_{mean} is the mean temperature of the whole.

Then $T^\circ_{max} = aT^\circ_{mean} + (1 - a)T_o^\circ = \dfrac{T^\circ_{mean} - (1 - c)T_o^\circ}{c}$

Prof. M. Vidmar[1] has thence proposed that the temperature T°_{max} of the hottest spot should be estimated from the measured mean temperature T°_{mean} and measured temperature T_o° of the outer surface upon the assumption that $a = 2$ or $c = \frac{1}{2}$,[2] so that,

$$T^\circ_{max} = 2T^\circ_{mean} - T_o^\circ$$

Deducting T_a°, the temperature of the ambient air, from both sides of the equation,

$$\begin{aligned} T^\circ_{max} - T_a^\circ = t_{r\,max} &= 2T^\circ_{mean} - T_o^\circ - T_a^\circ \\ &= 2T^\circ_{mean} - 2T_a^\circ - (T_o^\circ - T_a^\circ) \\ &= 2t_{r\,mean} - t_{ro} \end{aligned}$$

That is, the mean temperature-*rise*, $t_{r\,mean}$, is the mean of the rises of the hottest spot $t_{r\,max}$ and of the outer surface t_{ro}.

It has been found by calculation and actual experiment on coils of different forms that c does not, in fact, vary widely, and usually falls between 0·5 and 0·6. There remains, however, the difficulty that the surface temperature is assumed uniform, and further that the surface temperature to be inserted in the equation must for accuracy be that of the copper obtained by the use of a thermo-element or thin test-coil applied under the coil wrapping; ordinary measurement of surface temperature by thermometer will not yield the information required for a true estimate of the maximum temperature.[4]

§ 8. Embedded temperature-detectors.—By the employment of thermo-couples or resistance-thermometers, embedded in the slots as temperature detectors,[5] it may be expected that the highest temperature reached at any spot will be more nearly measured. Consequently the use of such embedded temperature-detectors is encouraged by allowing a closer approach to the real permissible limiting temperature. The American Standardization Rules, therefore, require a deduction of 5° C. only from the maxima appropriate to the class of material in the case of two-layer windings for all voltages with detectors between a coil-side and the core, and between the layers, but return to a deduction of 10° C. for single-layer windings for voltages not exceeding 5000, with detectors between coil-side and core and between coil-side and

[1] *E. u. M.*, Vol. 36, pp. 49 and 64.

[2] Either form is found in different papers.

[3] *Archiv für Elektrot.*, Vol. 7, p. 41 (W. Rogowski), and Vol. 8, p. 117 (M. Jakob).

[4] *Archiv für Elektrot.*, Vol. 8, pp. 126 and 362 (M. Jakob), and p. 329 (W. Rogowski and V. Vieweg); and *E.T.Z.*, Vol. 41, p. 646 (K. Lubowsky).

[5] For such special methods of measurement and the advantage of thermo-couples over exploring coils, see especially the papers of Messrs. L. W. Chubb, E. I. Chute, and O. W. A. Oetting *Trans. Amer. I.E.E.*, Vol. 32, Part I, p. 163; of Messrs. J. A. Capp and L. T. Robinson, *Trans. Amer. I.E.E.*, Vol. 32, Part I, p. 185; and discussion, pp. 362 ff.

wedge, while for more than 5000 volts, owing to the greater thicknesses of the insulation and high heat-gradient between copper and iron, an additional deduction of 1° C. must be made for each 1000 volts above 5000 volts rated pressure between terminals. Further, it is stated that embedded detectors are to be used in all stators with cores 50 cm. (20 in.) long or more, and this especially applies to the case of large turbo-alternators having long cores where the heat distribution may vary considerably from centre to either end; embedded temperature-detectors are also to be used in all machines for 5000 volts or more, if their output is over 500 kVA.

The thermo-couples for use as described above are usually made of iron and Eureka wire, the voltage being read on an accurate milli-voltmeter.[1]

§ 9. Rated output.—Thus the great importance of the safe *rise* of temperature lies in the limit which it places to the maximum current that may be passed through a given armature in continuous working, and in the majority of continuous-current dynamos it is the serious heating which would occur in everyday working if the armature current were increased that fixes their full normal output. By the British and American Standardization Rules a generator must give its rated output as stated on its nameplate in continuous service, with the addition in the former rules that it must be capable of carrying for 15 seconds a load in amperes 50 per cent. in excess of its standard rating without injurious sparking, and by the latter rules a momentary load of the same amount.

A stipulation which has been very largely employed is that the machine must run satisfactorily during a test of six hours' duration at full-load without undue heating of any part, and that at the end of the run the surface temperature of the armature or field-winding must not exceed the temperature of the surrounding air by more than 70° F. For short periods of, say, 1 to 2 hours, the normal full-load current may usually be exceeded by some 20 to 30 per cent. without raising the temperature of an armature in an excessive degree, and such a permissible overload enables the dynamo to deal with a large demand for current lasting a comparatively short time.

The rules adopted (1920) by the British Electric and Allied Manufacturers Association recommend as standard conditions for machines with unobstructed ventilation that they should be capable of withstanding an overload of 25 per cent. lasting half an hour when their continuous rating is below 25 kW, lasting 1 hour for ratings between 25 and 100 kW, and 2 hours for ratings of 100 kW or

[1] For further details, see Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 46; and F. D. Newbury and C. J. Fechheimer, "Practical Experience with Embedded Temperature Detectors," *Trans. Amer. I.E.E.*, Vol. 39, Part I, p. 971.

more. A standard air temperature of 25° C. (77° F.) is assumed as a starting point, and then for rating purposes a limiting rise of surface temperature is adopted of 40° C. (72° F.) for windings, or of 55° C. (99° F.) for commutators and slip-rings, both as measured by thermometer. A constant temperature is regarded as reached when the rate of its increase does not exceed 1° C. per hour. The same rules lay down that in general the temperature rise as measured by increase of resistance should not exceed 55° C. (99° F.) for alternator field coils, or 60° C. (108° F.) for shunt field coils of continuous-current machines.

Assuming a normal temperature of 25° C. (77° F.) for the surrounding air, it will be seen that the maximum temperature which the surface of the coils may attain in continuous work is 65° C. (149° F.), and this limit is found to give thoroughly satisfactory results in practice. It further results from such a rule that the temperature of an armature, as measured by rise of resistance, may differ considerably from its temperature as measured by thermometer, before a limit of 95° C. is exceeded. With stationary field-magnet coils having considerable depth of winding it might be thought advisable to fix an even lower limit of surface rise, such as 30° C. or 54° F., in order that the centre layers may not exceed the permissible maximum temperature, but the absence therein of the vibration and mechanical stresses to which the rotating armature is subjected permits of a closer approach to 95° C. without impairing the durability of their insulation.

In the case of machines for use at high altitudes above 3300ft., it is advisable to reduce the permissible temperature rise when tested near sea level by $2\frac{1}{2}$ to 3 per cent. for each 1000 ft. above 3300 ft.

§ 10. Testing dynamos for rise of temperature.—In order to determine the air temperature of the room several thermometers should be used, placed from 3 to 6 feet from the machine and on a level with its centre on opposite sides; they should be protected from draughts and abnormal heat radiation, and the mean of their readings taken at equal intervals of time during the last quarter of the duration of the test is to be adopted as the final air temperature. The time taken for the armature and field-coils of a dynamo to attain their ultimate temperatures is dependent upon their size. At the commencement of the run the rate at which heat is produced is almost as great as when the machine has attained its maximum temperature, but part of this heat is absorbed in raising the temperature, not only of the winding, but also of the core or magnet on which it is wound. The larger this mass which has to be heated, or the greater its specific capacity for heat, the longer will be the time taken in raising its temperature, until the final state is reached in which the heat has to be dissipated almost entirely by radiation

and convection currents in the surrounding air. A certain difference of temperature between the cooling surfaces and the surrounding air must then have been established sufficient to enable the heated masses to part with their heat as fast as it is generated. Strictly speaking, the temperature approaches its final value asymptotically, the rate of increase being a maximum at starting and thence gradually falling off; but if a machine be run with constant load for several hours, and the rise of temperature of armature or field-winding, as taken at intervals of, say, one hour, be plotted as ordinates to a horizontal axis of time, the curve so obtained will be found to bend gradually over and become more and more flat; finally, the readings will fall almost in a straight and horizontal line, showing that a steady temperature has then practically been attained (*cp.* Fig. 396). Such an experiment enables us to be certain that the final state has been reached, and the time which we find that a dynamo of given size takes to attain its maximum temperature will serve as a clue to the number of hours for which a machine of similar size should be run at full-load in order to test it thoroughly. While an armature, of which the core dimensions are 9″ diameter × 6″ length, will attain its final temperature after about four hours' run at full-load, an armature 15″ diameter × 8″ long will barely reach its maximum rise in six hours, and larger machines will require to be run for still longer periods. Even, however, in large machines, since they are multipolar and are usually barrel-wound or in other ways have their windings well exposed to the cooling effect of the air, there is but little rise of temperature after the first 8 or 10 hours.

The thermometer employed to measure the temperature of the surfaces should preferably be of a sensitive chemical type, the graduated glass stem having a very fine bore, and the small cylindrical bulb containing but little mercury; after being laid or held in close contact with the winding, it should be covered with some material (such as a piece of rag) which is a bad conductor of heat, and then allowed to remain undisturbed for several minutes until the mercury entirely ceases to rise.[1] When a rotating armature is stopped at the end of a run or at any time for the purpose of taking thermometer readings, the temperature of its exposed surface continues to rise for some minutes after the rotation has ceased. The generation of heat ceases with the rotation, but the simultaneous cessation of the air-currents set up by the revolving parts virtually amounts to a large reduction in the cooling power of the surfaces, and in consequence the fall of temperature between the inner or hottest parts and the external surface is reduced

[1] For other details in connexion with temperature measurement, *cp* Messrs. Reist and Eden, *Trans. Amer. I.E.E.*, Vol. 32, Part I, p. 177; and Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 41.

in other words, the outside rises by conduction to a higher temperature more nearly the same as that of the centre of heat. Both the cotton insulation of the wires and the shellac or other varnish with which they are coated are bad thermal conductors, and it therefore takes an appreciable time for the heat to penetrate through them to the outside of the armature. It may further be mentioned that, owing to the low thermal conductivity of insulating materials in general, if a heated armature be cursorily felt with the hand the bare metal of the binding wires, or even of the commutator, will appear hotter than the insulated wires, and these latter may seem comparatively cool to the touch; any such conclusion is, however, entirely illusory, and the continued application of the hand to the conductors will usually suffice to correct the error.

The speed of shutting-down has a decided effect on the maximum temperature recorded by a thermometer applied to armature or rotor after it has come to rest. It has therefore been proposed to keep the field excitation on where practicable in order to bring the machine to rest as quickly as possible.[1]

§ 11. The growth of the temperature rise.—Given a homogeneous body, in which heat is being generated at the rate of W watts, *i.e.* W joules per sec., let it be assumed that the rate of dissipation of heat from it by radiation, convection, and conduction is strictly proportional to the temperature rise; it could then be said *e.g.* to have a true "cooling coefficient" ξ, being the watts that can be dissipated per unit area of cooling surface per 1° rise, which when multiplied by the cooling surface S_c of the body will give its total rate of dissipation of heat by radiation, convection, and conduction. Although, as pointed out in the following section, not strictly true, the assumption is one that may safely be made for all practical purposes. The rise t_r of the temperature of the body above its initial temperature after any time t is then given by the equation

$$H\frac{dt_r}{dt} + \xi S_c t_r = W$$

where H is the heat capacity of the body, or the joules required to raise its whole volume 1°. The solution of the equation is

$$t_r = \frac{W}{\xi S_c}\left(1 - \varepsilon^{-\frac{t}{H/\xi S_c}}\right)$$

$$= t_{r\,max}\left(1 - \varepsilon^{-\frac{t}{T_c}}\right) \qquad (208)$$

where ε is the base of natural logarithms, and $T_c = H/\xi S_c$ is the "time-constant" of the body which is being heated. It is evident that the curve of the rise of temperature in relation to time is an

[1] Messrs. Chubb, Chute, and Oetting, *Trans. A.I.E.E.*, Vol. 32, Part I, p. 175.

exponential curve exactly analogous to that of the rise of current in an inductive circuit. It will therefore have to some scale the shape of the curve of Fig. 396. The final rise of temperature which the body will ultimately attain and to which the curve approaches asymptotically is of course simply

$$\frac{\text{rate of generation of heat in watts}}{\text{rate of dissipation by radiation, convection, and conduction per } 1^\circ \text{ rise in temperature}}$$

$$= \frac{W}{\xi S_c} = t_{r\,max}$$

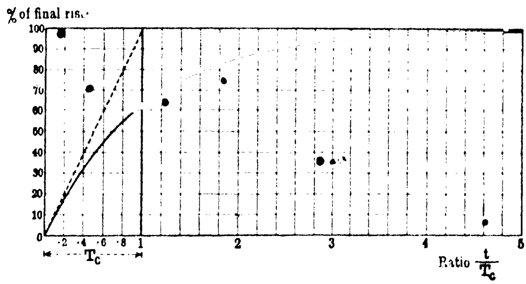


FIG. 396. Curve of temperature rise in relation to time.

From the nature of equation (208) the "time-constant," $T_c = H_t \xi S_c$ is the time in which the rise from the initial temperature of the surrounding air actually reaches $\frac{\varepsilon - 1}{\varepsilon} = \frac{1{\cdot}718}{2{\cdot}718}$, or 63·2 per cent. of the final excess temperature, as is evident when t is made equal to T_c. Or in general, for convenience in plotting the curve, with t expressed in multiples of T_c, the rise when $t = T_c/2$ will be 39·3 per cent. of the maximum, at the end of time T_c will be 63·2 per cent., at $t = 2T_c$, 86·5 per cent., at $t = 3T_c$, 95 per cent., and at $t = 4T_c$ will be 98·2 per cent., so that it then only differs by 1·8 per cent. from the final value $t_{r\,max}$.

The cooling equation is conversely

$$t_r = t_{r\,max}\,\varepsilon^{-\frac{t}{T_c}}$$

The time-constant, being the ratio

$$\frac{\text{joules required to raise the whole volume } 1^\circ}{\text{joules dissipated by radiation, convection, and conduction in unit time per } 1^\circ \text{ rise}}$$

is given in seconds, minutes, or hours according as its denominator

is ξS_c, $\xi S_c \times 60$ or $\xi S_c \times 3600$, and t must also of course be reckoned in the same unit of time. The time-constant is also the time in which the body that is being heated would reach its final temperature if the initial rate of increase was steadily maintained and there was no cooling action from any of the three causes, radiation, convection, or conduction, due to the rising temperature (*cp.* the dotted line in Fig. 396). The equivalence is not immediately apparent, since at the starting point there is no dissipation of heat by radiation, convection, or conduction from the body owing to its being at the same temperature as the surrounding air. But if $\dfrac{H}{\xi S_c}$ is multiplied by $\dfrac{W}{\dfrac{\text{joules expended in unit time}}{\text{unit time expressed in seconds}}}$, the numerator and denominator of the multiplier being equivalent expressions, the time-constant becomes

$$\frac{\dfrac{W}{\xi S_c}}{\dfrac{\text{rate of expenditure of joules in unit time}}{\text{joules required to raise whole volume } 1^\circ}}$$

that is,

$$T_c = \frac{H}{\xi S_c} = \frac{\text{final excess temperature}}{\text{initial rate of temperature increase}}$$

the initial rate being given in degrees per second, minute, or hour according to the unit of time selected for the calculation of T_c and t.

With a knowledge of the total rate of loss in watts under any given conditions and also of T_c, any question as to the heating or cooling of a dynamo as a whole or of any part could be solved, provided that the heat is generated fairly uniformly throughout the part considered. Actually the latter condition is not very closely fulfilled, and at starting there is a considerable amount of conduction of heat from the parts of small thermal capacity to parts of greater thermal capacity. It is only, therefore, in the later stages of a heat-run that the exponential curve for the rise of temperature is reproduced to any close degree of accuracy.

For a given material the value of H is dependent on its specific heat and is therefore known as a fact of physics. Taking the specific heat of copper per unit mass as 0·095 (calories required to raise 1 gramme of mass 1° C.), and the weight of a cubic inch as 0·32 lb., then since 1 calorie = 4·19 joules, the joules required to raise one cubic inch 1° C. are $0{\cdot}095 \times 4{\cdot}19 \times 0{\cdot}32 \times 453{\cdot}6 = 57{\cdot}6$, or 3·5 joules per cubic centimetre. Taking the specific heat of iron or steel per unit mass as 0·113, and the weight of a cubic inch as

0·282 lb., the joules required to raise one cubic inch of iron 1° C. are 0·113 × 4·19 × 0·282 × 453·6 = 60·4, or 3·68 joules per cubic centimetre. There is therefore but little difference between iron and copper, and an average figure of 59 joules per cubic inch, or 3·6 per cubic centimetre, may be taken for a composite mass such as an armature. The total watts of the armature are then to be averaged over its entire volume, and the truth of the above approximation for the joint specific heat is of course dependent upon how far the heat is actually developed in the core and winding in proportion to their respective volumes.

In the case of field-magnet coils, with numerous turns of cotton-covered wire, allowance must be made for the higher specific heat of the cotton. The specific heat of cotton is about four times that of copper, say, 0·38, and the joint specific heat of the copper and cotton will depend upon the ratio of the copper to the total. The heat is only developed in the copper, and for calculating either H or the initial rate of rise, if the watts are averaged over the entire mass of copper and cotton, their joint specific heat per unit mass is higher, but if the watts are averaged over the entire volume, it is lower than that of solid copper. With closely bedded rectangular wire the ratio of the copper volume to the total volume is the "space-factor" σ, and the weight of a cubic inch of cotton is 0·0323 lb., so that the ratio of the weights of unit volume of copper and cotton is 9·9; the specific heat per gramme of the combined copper and cotton may then be calculated from the formula $0{\cdot}095\left(\dfrac{4 + 5{\cdot}9\sigma}{1 + 8{\cdot}9\sigma}\right)$. Or analogously to the above expressions, the joules required to raise a cubic inch of cotton 1° C. are 0·38 × 4·19 × 0·0323 × 453·6 = 23·3, and the joules for any ratio σ of copper volume to total volume are

$$23{\cdot}3 + 34{\cdot}3\sigma$$

as plotted in Fig. 396a.[1] The result is that the joint specific heat of field-magnet coils per unit volume is appreciably less than that of solid copper, and if the wire is round, the air-interstices will tend to decrease it still further. On the other hand, the thermal capacity of the adjacent metal of the magnet which is heated by conduction has above been neglected, and this always tends to increase the apparent specific heat.

Since T_c is usually required in minutes, we thus have

$$T_c = \frac{j \times \text{cubic inches of total volume}}{60 \times \xi S_c} \text{ minutes}$$

where j = the joules to raise 1 cubic inch 1° C. = 59 for armatures

[1] C. W. Hill, "Crane Motors and Controllers," *Journ. I.E.E.*, Vol. 36, pp. 294 and 307.

and is taken from Fig. 396a for field coils in relation to the space factor. Since the initial rate of temperature increase in degrees Centigrade per minute

$$= \frac{\text{watts per cubic inch of total volume} \times 60}{j}$$

an alternative expression for T_c is

$$T_c = \frac{j \times t_{r\ max}}{\text{watts per cubic inch of total volume} \times 60} \text{ minutes.}$$

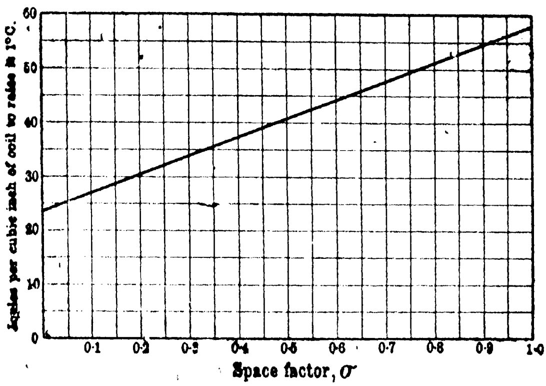


FIG. 396a. Specific heat of cotton-insulated coil in relation to space factor.

If, therefore, ξS_c is unknown, it still remains necessary that the final excess temperature should be known, and it is, in fact, mainly to check this that the dynamo is tested by a run of several hours' duration. Experiment cannot therefore be dispensed with unless from previous cases the final excess temperature and the total watts dissipated as heat can be regarded as known, and this is more usually the case with field coils than with armatures. An approximate calculation can, however, thence be made as to the time for which a given machine must be run in order to reach practically its final temperature or any particular percentage of it. The longer period for which large machines must be run in order that they may attain their final temperature is obviously due to the ratio of their volume to their surface, which increases with an increase in size, although this tendency is partially counteracted by the fact that there is usually at the same time an increase in the number of poles.

Further, if the temperature rises at different times during a run of a few hours are plotted, it usually becomes possible to predict

the final temperature from inspection of the shape of the curve with fair accuracy. Or by taking the Napierian logarithms of the slopes of the tangents to two points on the rise-of-temperature curve corresponding to times t_2 and t_1, the value of the ratio $H/\xi S_c$ can be found without a knowledge of either of its components; for

$$\frac{H}{\xi S_c} = \frac{t_2 - t_1}{\log_\varepsilon\left(\frac{dt_r}{dt}\right)_1 - \log_\varepsilon\left(\frac{dt_r}{dt}\right)_2}.$$

We can thence find $\varepsilon^{-\frac{t}{H/\xi S_c}}$ and state by how much after the end of the run the temperature falls short of its final value.[1]

Lastly, while the exponential curve reproduces the facts sufficiently closely for most practical purposes,[2] it must be borne in mind that it cannot be strictly true, owing to the fact that the total rate of dissipation in watts in the machine as a whole or in any part of it is not usually constant, but is varying as it gradually warms up.

§ 12. The heating of stationary field-magnet coils.—The final rise of temperature of a field coil is primarily determined by the ratio of the cooling surface S_c to the total watts to be dissipated, or by its reciprocal, the ratio of the total watts to the cooling surface, and under given conditions it is approximately inversely proportional to the former and directly proportional to the latter. It may therefore be expressed either through the "cooling coefficient," ξ, as $t^\circ = \frac{1}{\xi} \cdot \frac{W}{S_c}$, or through its reciprocal, the "heating coefficient," as $t^\circ = k \cdot \frac{W}{S_c}$, where $k = 1/\xi$ is the rise of temperature for a ratio of one watt dissipated per unit area, *i.e.* per watt per unit area. In either form the coefficient is not really a constant, but itself depends upon the final temperature which is reached. This is due to the increased effect from radiation, as the difference of temperature of the surface from that of the surrounding air rises[3]; if the whole cooling effect resulted from radiation, the reduction in the heating coefficient when a high final temperature was attained would be considerable, but in the dynamo at work it is of less account owing to the preponderating effect from conduction and convection.

[1] See Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 45.

[2] *Cp.* R. Goldschmidt, "Temperature Curves and the Rating of Electrical Machinery," *Journ. I.E.E.*, Vol. 34, p. 672 ff., where curves of actual machines are given, and the amount of their divergence from the exponential shape is noted.

[3] G. A. Lister, "The Heating Coefficient of Magnet Coils," *Journ. I.E.E.*, Vol. 38, p. 402; and Prof. Magnus Maclean, *Journ. I.E.E.*, Vol. 53, p. 526.

But apart from this variation of the heating coefficient, and when it is assumed that the final rise is strictly proportional to the watts per square inch of cooling surface, the value of k is dependent upon a number of conditions of which the chief are as follows—

(1) The construction of the field coil, whether a solid mass or divided by ventilating spaces into sections as described in Chapter XVII, § 8, and whether wound on spools or otherwise. Sheet-metal spools or bobbins assist in conducting heat to the iron core if closely fitting. When furnished with wooden end-flanges, the latter act as partial heat insulators and raise the temperature. On the other hand, brass or iron end-flanges assist considerably in conveying away the heat. Thick insulating bobbins, such as those moulded from vulcanabest, although convenient for the winder, have the disadvantage of increasing the heating coefficient.

(2) The nature and thickness of the insulating wrappings, especially in such coils as are self-supporting[1] and after winding are slipped directly over the pole. Thus with a coil of double-cotton-covered wire which is merely varnished on the outside, the mean rise may be only about $1\frac{1}{4}$ to $1\frac{1}{3}$ times the surface rise; if wrapped with thin tape, the mean rise will be increased to $1\frac{1}{2}$ or $1\frac{2}{3}$ times, while if the coil be again overlapped with canvas and string to a considerable thickness the mean rise will be further increased by some 25 to 50 per cent.[1] Thus any insulating wrappings, owing to their low thermal conductivity, have a prejudicial effect upon the mean rise and also upon the maximum internal rise, even though they may tend to lower the rise of the outside. The bare edges of copper tape wound on edge in a single layer, as in the field-magnet coils of multipolar engine-driven alternators, add considerably to their power of conducting the heat immediately to the outside and there radiating it away.

(3) The peripheral speed of a rotating armature, which causes a more or less efficient circulation of the air round the field coils. This fanning effect is especially marked in multipolar machines with armatures of large diameter.

(4) The load of the armature; as this is increased, the temperature of the armature rises, and the air which is thrown off from its surface and circulates round the stationary coils becomes hotter, so that the dissipation of heat from the field is checked.

(5) The type of machine—whether bipolar or multipolar, semi-enclosed or open. In a small multipolar machine, if the number of poles is large, the opposing faces of the coils are brought close together, and this is especially true with commutating poles which nearly fill up all the space between the main poles.

[1] *Cp.* throughout E. H. Rayner, "Report on Temperature Experiments carried out at the National Physical Laboratory," *Journ. I.E.E.*, Vol. 34, p. 628 ff.

(6) And lastly, the position of the coil, whether at the bottom of the machine or at the top to which the air heated by the armature ascends.

The basis of calculation for the cooling surface of the magnet coils varies in the experiments of different observers, so that in comparing them it must be noted whether only the external cylinder surface is employed or whether the exposed surface of one or both end-flanges is added thereto, or lastly, whether the whole of the surface, external and internal, is adopted.

Probably the best basis is to be found in the outside surface plus both the end-flanges, as in Chapter XV, § 5. With the comparatively short coils of modern multipolar machines having a considerable depth of winding, the cooling effect from the ends plays an important part, and the combination of the external cylindrical surface with that of both end-flanges is here adopted.[1] In the case of coils divided into sections, the same basis may be taken, the total axial length of the coil being reckoned without exact calculation of the air-ducts, but in combination with a low value for the heating coefficient.

The rate at which heat is developed in the magnet coils admits of easy measurement and calculation as the product of the square of the current into the resistance. The ratio of the watts to the area of cooling surface is therefore much more accurately known than in armatures; and further, as the field-magnet coils are stationary in continuous-current dynamos, and therefore less affected by currents in the surrounding air than is the rotating armature, their temperature rise can be predicted with greater certainty.

In the values to be taken for the "heating coefficient" the distinction must be carefully noted between the rise of the exterior surface of the coil and the mean rise of the whole of the copper wire, and again between these and the maximum temperature rise at any spot within the coil; for each case the value will be different.

Taking any one layer, as we proceed from the centre along the axial length of the coil towards either end, the temperature always declines towards either end-flange, which shows the influence of the end-flanges in assisting the dissipation of the heat. On the other hand, as we pass from the outside radially through the layers the

[1] Mr. Lister (*Journ. I.E.E.*, Vol. 38, p. 401) advocated the adoption of the entire surface, external and internal, and the iron of the pole undoubtedly assists in conducting away the heat. Yet since when the machine is at work under full load the pole gradually becomes heated by the hot air and radiation from the armature and also by any eddy-currents in the pole face, this action is to some extent checked, so that an air-space between coil and pole may even become of value. In ordinary cases the external surface is at least as good, if not a more reliable, guide to which the cooling effect will be proportional.

temperature rises, reaches a maximum, and again falls, but to a lesser degree, as we approach the inner layer next to the iron of the poles. The innermost layers thus part with some of their heat by conduction to the iron of the magnet, so that their temperature falls about midway between that of the outer and that of the hottest layer. Fig. 397 shows typical readings for the rise of temperature along a radial line passing through the centre of the axial length of a field-magnet coil, and along an axial line passing through the hottest spot. The hottest spot in the coil is approximately in the centre of the axial length, and about three-quarters of the total depth away from the outside, and the maximum temperature rise at this spot exceeds the surface rise by an amount which is from 50 to 60 per cent. higher than the difference between the mean rise and the surface rise; thus if the temperature rise, measured by change of resistance, is 20° higher than the surface rise as measured by a thermometer, the maximum temperature rise will be about $1\frac{1}{2} \times 20 = 30°$ higher than the surface rise.[1] On the average the maximum temperature rise is about 18 to 20 per cent. higher than the mean rise, when the machine is fully loaded. While the highest temperature attained has a practical bearing upon the durability of the insulating materials, the mean rise throughout the coil has the greatest influence upon the regulation of the dynamo and its general working, so that heating coefficients which give the mean rise have perhaps the greatest scientific value and are frequently adopted. On the other hand, so long as in specifications it is the rise of the exterior surface as measured by a thermometer for which a limit is laid down, this must continue to have interest for the designer; also, it remains a rough and ready test upon which immediate agreement can be come to between purchaser and manufacturer. Measurement of the mean rise of temperature by the observed increase of resistance (equation 207) requires great accuracy in the readings of the voltmeter and ammeter, since it is only a difference of which use is finally made.

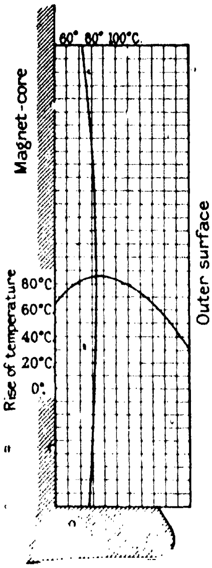


FIG. 397. —Temperature-rise along a transverse and a longitudinal section through the hottest spot of a field-magnet coil.

[1] R. Goldschmidt, *Journ. I.E.E.*, Vol. 34, p. 720.

The difference between the hot and cold resistances being usually about 20 per cent. of the cold resistance, any error in the measurement may lead to an error five times as large in the inferred temperature rise. Moreover, it is not infrequently the case that the actual cold temperature of the winding is different from that of the air, and is not known with certainty.

On an average, perhaps, it may be said that the mean rise is as much as $1\frac{1}{2}$ to $1\frac{3}{4}$ times the surface rise, so that, to calculate the latter, the heating coefficient for mean rise must be reduced in this proportion.

§ 13. The thermal conductivities of different materials.—For the approximate calculation of the temperature of the hottest spot in a field-magnet coil and for its location, a knowledge of the heat conductivity of copper and of various insulating materials is necessary. For electrical purposes this is expressed as the rate in watts at which heat will pass through a unit cube of the material under a difference of temperature of 1° C. steadily maintained between opposing faces, the other sides of the cube being perfectly heat-insulated.

In inch-units, *i.e.* per sq. inch of area, per one inch length of path and for 1° C., the thermal conductivity in watts is for copper $k_c = 9$ to 9·6.

Valuable experimental data on the transverse thermal conductivity k_t for various insulating materials have been recorded by Messrs. Symons and Walker[1] and by others.[2] They will not here be repeated, and for their use the reader is referred to Prof. Miles Walker's book on *The Specification and Design of Dynamo-electric Machinery*, pp. 219–241 (partly repeated in the same author's *The Diagnosing of Troubles in Electrical Machines*, pp. 62–67).

Solid impregnated coils, owing to higher thermal conductivity of the impregnating material, may have an internal conductivity three times that of a coil with plain cotton-covered round wire or $1\frac{1}{2}$ times that of a coil with shellacked cotton covering.

§ 14. Predetermination of temperature-rise of stationary magnet coils.—It will be evident that no exact figures can be given which can be applied without consideration of the particular conditions of the design and type of machine, but that the influence of peripheral speed, etc., can to some extent be taken into account by the use of such curves as those of Fig. 398, which indicate

[1] *Journ. I.E.E.*, Vol. 48, p. 674 ff., and tabulated in Tables XII and XIII of Miles Walker, *The Specification and Design of Dynamo-electric Machinery* (pp. 220 and 221), where also is given in Table XIV (p. 239) the values specially applicable to the case of wire-wound coils.

[2] See especially R. B. Williamson, *Trans. Amer. I.E.E.*, Vol. 32, Part I, pp. 303 and 405; T. S. Taylor, *Elec. Journal*, Vol. 16, pp. 526–532 (Dec., 1919); and *Elec. World*, Vol. 75, pp. 369–371 (14th Feb., 1920).

approximately the limits which usually occur in practice. The lower dotted curve for the mean rise of well ventilated coils may approximately be expressed in terms of square inches and degrees Centigrade as $k = \frac{144}{1 + 0{\cdot}35 . (v/1000)}$ where v is the peripheral speed of the armature in feet per minute. The mean temperature rise is also affected by the amount of cotton in coils which are of the same size and otherwise alike, and is less with a larger size of wire and smaller amount of cotton. Shunt coils are appreciably benefited by subdivision into sections by ventilating spaces; by their use the mean rise is made to approach more closely to the surface rise, so that it may nearly coincide with the highest of the curves given in Fig. 398 for the surface rise. Such subdivided coils are slightly detrimental to the efficiency of the machine, since, if the intervening gaps were filled with copper, the watts would be reduced, but in a large machine this has only a very small effect on the total efficiency.

The mean rise should preferably not exceed 50° C. or 90° F., so that the maximum temperature at any spot may not exceed about 90° C. or 194° F.

If the permissible rise of surface temperature be taken as about 30° C., it will be seen that from 2 to $2\frac{3}{4}$ square inches of cooling surface must be allowed per watt with solid coils, and $1\frac{1}{2}$ to 2 with coils in sections; or the watts per sq. inch must not exceed from 0·5 to 0·36 in the former, or 0·66 to 0·5 in the latter case. A greater rise, although perhaps admissible in respect of the final temperature reached, is seldom advisable owing to the difference which it causes in the excitation according to the time during which the machine has been at work. If a number of separate bobbins are revolved at a high peripheral speed, as in alternators of the type shown in Fig 71, a much higher ratio of watts to square inches is permissible, owing to the great cooling effect from their rapid movement through the air, but this case will again be referred to in Chapter XXVII.

§ 15. Sources of heat in the armature, and their relative importance.—The final rise of temperature in an armature is a much more complex problem, and this for two reasons.

In the first place, it is difficult to calculate with accuracy the rate in watts at which the armature is actually being heated. The sources of the heat are threefold, namely: (1) the loss of electrical energy due to the passage of the total current through the armature resistance; (2) eddy currents set up in the copper winding, and in the armature core and teeth; and (3) the magnetic loss due to hysteresis. The first of these can be calculated to a close degree of approximation, and the third fairly closely, but the second loss is very largely indeterminate, inasmuch as it is dependent upon a number

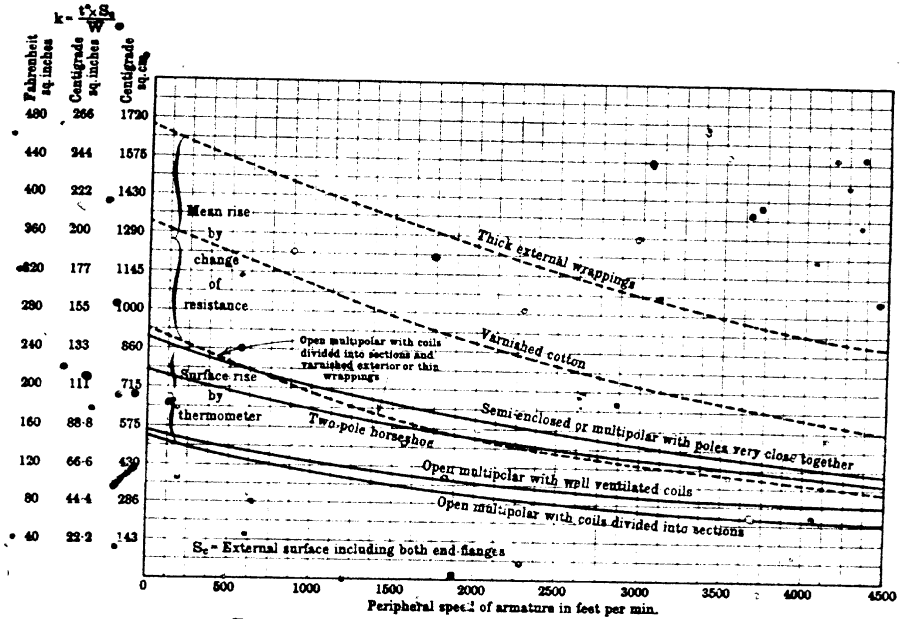


FIG. 398.—Heating coefficient of field-magnet coils.

of complex conditions, such as the width of the armature conductors, the proportions of teeth and slots, and the strength of field.[1]

To show how variable is the relative proportion of the three losses, Table XIV gives a few results of actual tests made on machines of different construction and size. The figures show also that the eddy-current loss is usually of considerable importance, and equal to or even greater than the $I_a{}^2R_a$ loss. Were it not for this, the current-density in the armature conductors would be a more fundamental guide to the necessary dimensions of a machine in place of ac used in Chapter XXII. The precautions necessary to minimize eddies have been already emphasized in Chapter XIII, §§ 4 and 19, but their complete elimination cannot be attained, and the probable loss due to these must be closely known from previous experience if the rise of temperature of the armature is to be predicted with any certainty.

§ 16. Effect of peripheral speed.—In the second place, even when the rate at which heat is developed in the armature is accurately known, the rate at which it can be dissipated is most materially affected by the speed of rotation, and also by the shape and number of the pole-pieces which surround the armature, in so far as they allow of or actively promote a more or less free circulation of the air over the surface of the winding. The exact effect of either of these causes does not admit of very accurate generalization, since it necessarily varies under different conditions. How important a part is played by the peripheral speed of a rotating armature in increasing its cooling power is at once evident from a comparison of the watts which it can dissipate with the watts which a stationary field coil can dissipate: their exterior surfaces being reckoned in each case as effective in cooling, if the rotating armature have a peripheral speed of 2000 feet per minute, it will be found that, roughly speaking, it can dissipate at least twice as many watts per square inch of cooling surface as the stationary field coil for the same rise of temperature. If the surface of the armature winding be broken up, so that the air can play upon more than the outer face of the conductors, and can reach freely to the core, the cooling action is much assisted. Again, the influence of eddy-currents in the pole-pieces, when these latter are not laminated, reacts upon the heating of the armature. It is therefore impossible to lay down any general formula, expressing the cooling power of a given surface at different speeds, which will meet the case of entirely different types of machines, and all curves connecting rise of temperature with certain values for the watts per square inch must necessarily be only approximate.

[1] See especially B. G. Lamme, "Iron Losses in Direct-current Machines," *Trans. Amer. I.E.E.*, Vol. 35, Part I, p. 261.

TABLE XIV

	Dimensions of Armature Core in Inches.		kW.	Volts.	Amps.	Revs. per Min.	Peripheral Speed in Ft. per Min.	Watts.			B_g	B_c	**Uncorrected B_{tz}**
	Diam.	Length.						$I_a^2 R_a$	Hyst.	Eddies.			
4-pole drum, slotted . .	10½	6	9½	140	66	780	2140	350	100	340	7000	10,300	**22,620**
" " . .	12	7	13½	105	130	780	2450	440	107	574	7200	14,400	**21,800**
" " . .	14	8	27	220	123	780	2860	640	210	425	7180	12,300	**23,000**
" " . .	21	11	50	550	91	400	2200	1360	320	600	7500	14,650	**24,200**
6-pole drum, slotted . .	19·3	8·85	45	120	375	550	2780	687	660	1240	7160	9000	**20,700**

§ 17. The loss over the ohmic resistance of the armature.—Considering in detail the three sources of heating in an armature, the first, or the rate of electrical loss over the copper resistance of the winding, is entirely independent of the speed, being simply the product of the square of the total armature current and the resistance of the armature from positive to negative brushes, *i.e.* $I_a^2R_a$ watts. In the process of designing a dynamo the resistance of the armature can easily be calculated, and from it, after making allowance for its increased value when hot, the loss in watts due to the passage of the normal armature current. It is only necessary to remark that if the resistance of an armature from brush to brush be actually measured, it will usually be found to be slightly higher than the resistance as calculated from the length and area of copper used. This discrepancy is due to the inferior conductivity of the soldered joints, even when carefully made. In this respect armatures wound with former-shaped coils have the advantage that in them the soldered joints are reduced to a minimum, and need only occur at the unions with the commutator sectors.

Owing to the low resistance of a large armature with few bars, the most convenient method of measuring its actual value is by soldering leading-in strips on to two commutator sectors at such a distance apart that they divide the winding into two parallel paths, passing through them a known current of considerable strength, and measuring the difference of potential between the points where the current leaves and enters. From the quotient obtained by dividing the measured difference of potential by the known current, the resistance of the armature can be calculated according to the nature of the winding, and the $I_a^2R_a$ loss when the armature is hot can be accurately determined.

§ 18. The eddy-current loss as depending on speed.—It is in the second or eddy-current loss that most difficulty lies. With a given core, a given winding, and a given field (magnetic screening in iron assumed to be negligible), this loss will be directly proportional to the square of the speed; since, if the speed be increased x times, the E.M.F. of an eddy will be equally increased, and this will increase the current of the eddy x times, so that the product of eddy E.M.F. × eddy-current will be increased x^2 times. The rate of loss in watts may therefore be expressed by a coefficient F multiplied by the square of the speed, *i.e.* by FN^2, where N = revolutions per minute, the value of F being different in different armatures and being also dependent upon the particular excitation which is normal for a given armature. During the process of designing, the value of F must be estimated from the coefficients of machines previously tested. A brief description of the experimental methods by which the eddy-current loss in a given dynamo when run at different speeds can be measured with considerable accuracy will be found

in Chapter XXII, §§ 13–15. Two points alone require to be further mentioned. In the first place, the value of the eddy-current loss, as thus determined, includes any eddies set up in the pole-pieces of the magnets by the rotation of the armature. Such loss of energy in the pole-pieces will be considered in §§ 28 29 of the present chapter, and does not here concern us, so that with toothed armatures and solid pole-pieces some deduction may require to be made from the total loss in watts as measured by experiments on the machine as a whole. In the second place, during the experiments the distribution of the field is approximately the same as that at no-load, and it has already been shown, in Chapter XIX, that this distribution is considerably modified at full-load. Hence, on this score, the eddy-current loss at full-load may be greater than the experiment would show, and when the results of such tests have been checked by other methods it has been found in practice that this difference in the distribution of the field does sensibly increase the amount of the eddy currents.[1]

It is, however, justifiable to assume that the methods above alluded to furnish evidence for determining, at least approximately, the eddy-current loss at full-load. In applying such results to new machines, not only must any alteration in the length and volume of the core be taken into account, but, as previously stated, it is equally important to estimate the probable effects of alteration in the value of the flux-density in the air-gap or armature, the degree of lamination of the core, the width of the bars, and the flux-density in the teeth of slotted armatures. An alteration which at first sight might seem unimportant in any one of a number of conditions may radically affect the amount of the eddy-currents.

§ 19. Eddy-currents in the armature core.—The eddy-current loss may be divided into (*a*) that within the armature core, (*b*) that in the copper winding, and (*c*) that in the binding wire on the circumference of the armature. To allocate to these sources their respective shares in the total loss is an extremely difficult matter, as any change in one of the conditions such as can be tried in a practical machine produces but a small change in the total which can scarcely be isolated; yet the combined effect of all the losses adds up to an amount which seriously reduces the possible output of an armature from the heating point of view.

Taking them seriatim, we have first the armature core with its supporting framework and surroundings. The eddy-current loss produced by an alternating field in thin iron plates such as are used in transformers and laminated armature cores is proportional to

[1] In a test on a Siemens 1500-kilowatt dynamo the losses deduced from a Hopkinson test showed an increase of 15 per cent. at full load as compared with the sum of the no-load and I^2R losses (*Electr. Eng.*, Vol 62, p. 15). *Cp.* Chap. XXII, § 17.

the square of the maximum flux-density and also to the square of the thickness of the plates, and the same law has been experimentally established by Mr. Holden[1] when the field rotates about a stationary armature, or *vice versa*. The energy consumed in thin plates or discs in an alternating field per cycle and per cubic centimetre of iron is $\dfrac{\pi^2 t^2 . f . B_c^2}{6\rho} \times 10^{-16}$ joules, where t is the thickness of the plates in centimetres, f is the periodicity or frequency, and ρ is the resistivity of the iron. The latter quantity may be taken as from 1×10^{-5} to $1{\cdot}5 \times 10^{-5}$ ohms with wrought iron or thin sheet steel,[2] whence the loss of energy per cycle and per cubic centimetre is approximately $1{\cdot}645\, t^2 . f . B_c^2 . 10^{-11}$ joules. The rate of loss at any frequency $f = pN/60$ in a total volume of V_c cubic centimetres of iron is therefore in an alternating field

$$1{\cdot}645 t^2 . f^2 . B_c^2 . V_c \times 10^{-11} \text{ watts.}$$

The same form of expression may also be applied to the case of a rotating field, but owing to the E.M.F.'s being on the whole greater in the rotating field, and the average length of path and its resistance smaller, the first numerical constant requires to be increased. For circular plates of large diameter as compared with their thickness the best approximation gives for a rotating field[3]

$$2{\cdot}78 t^2 f^2 B_c^2 . V_c \times 10^{-11} \text{ watts} \quad . \quad . \quad . \quad (209)$$

But no rigid mathematical solution can be given, since the numerical constant is dependent to a certain extent upon the distribution of the field in the air-gap, and further, as has been pointed out by Professor W. M. Thornton, such curves as those of Fig. 220 show that between B and A the flux at one depth may be increasing while at another depth it is decreasing, so that the rate of cutting lines is not simply proportional to the distance from the centre, as it would be if the induction was uniform over any radial section. For the volume of the teeth the coefficient would again be higher.

When the above formula is worked out, the watts, even in the case of a high-speed multipolar dynamo, are found to be small in comparison with other more serious losses, and in practice the actual loss from eddy-currents in the discs would be even smaller, owing to any magnetic screening therein. The reduction in the eddy loss through the increase of resistance when the iron becomes heated is about 5 per cent. for each 10° C. rise of temperature.

[1] *Electrician*, Vol. 35, p. 327.

[2] For high-resistance alloys, see Chap. XIV, § 9.

[3] See Prof. E. Wilson, *Proc. R.S.*, Vol. 70, p. 359, and *Electr. Eng.*, Vol. 30, p. 226; and Prof. F. G. Baily, *Phil. Trans.* (1896), Vol. 187 A., pp. 715-746. *Cp.* also M. B. Field, "Eddy Currents in Solid and Laminated Masses," *Journ. I.E.E.*, Vol. 33, p. 1125 (Case II, p. 1141).

It must not, however, be assumed that the loss in the armature core as a whole is by any means negligible. The end-fringe from the flanks of the pole-pieces is quite appreciable, and unless the end-plates which clamp together the discs are kept well outside the edges of the poles the lines which curve round into the ends of the armature will set up considerable eddy-currents.[1] There is too a loss from each ventilating air-duct, wherein the lines bend into the flat surface of the discs at the sides of the duct. Again, in barrel-wound armatures the cylindrical structure which supports the end-connexions of the bars should not be a solid sheet of metal, but as far as possible should be cut away near to the armature core so as to remove the iron into which the lines of the fringe might stray (*cp.* Figs. 131, 406, and 414).

Next, the operation of turning the surface of a smooth armature-core increases the eddy loss by burring over the edges of the discs, and requires to be carried out with considerable judgment. Analogous to this in a toothed armature core is the operation of drifting or filing out the slots so as to remove any roughness or inequality of their sides. Any such mechanical treatment requires to be both lightly and cleanly executed with the minimum amount of injury to the laminations; indeed, unless absolutely necessary, it should be entirely avoided. On this account the longitudinal grooves on the assembled core are not milled out, and even when the notches are stamped in the discs before assembling, they must be carefully inspected to see that they are free from burrs, by reason of which the rough edges of neighbouring discs would be driven into good contact with one another.[2] The effect of the pressure with which long armature cores as in large turbo-generators are squeezed together between the end-plates is to drive the burred edges of punched discs into contact with one another, and this may give rise to high local temperatures at places where the edges of the laminations are short-circuited, especially at each end of the core[3] where the applied pressure is greatest. There may be an E.M.F. of as much as 1 volt acting on the short-circuit, and if there is only one or a few points of contact between adjoining plates, they may be raised to incandescence. The insulating paper is then burnt out, and the area of the damage extends with increased iron loss.[4]

§ 20. Eddy-currents in bolts through armature core. If the body of an armature core is traversed by a number of bolts, and these, although insulated from the discs through which they pass, are in metallic connexion with the

[1] *Cp.* Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 91.

[2] For the eddy-loss specially in the laminated teeth of a toothed core, *cp.* F. E. Meurer, *Electr. World*, Vol. 49, p. 792.

[3] B. G. Lamme, *Trans. Amer. I.E.E.*, Vol. 35, Part I, p. 266.

[4] M. Vidmar, *E.u.M.*, Vol. 37, pp. 1 and 17.

end-plates at either end of the core, not only is the effective area of the core *periodically reduced* by the presence of the bolts, but there results a conducting system analogous to a squirrel-cage rotor in an induction motor. Each bolt becomes the seat of an alternating impressed E.M.F. proportional to the flux which it cuts, the latter depending upon the radial depth within the core at which it is situated; this E.M.F. of periodicity $f = pN/60$ will pass through zero midway in the interpolar gaps, and will have a double-peaked maximum under each pole. The resistance of the portions of the end-plates corresponding to each bolt, and forming part of the circuit, will be low owing to their large area, and may be neglected. When the resistance of their contact-area with the head of the bolt is also neglected, and the specific resistivity of wrought-iron bolts is taken to be 1×10^{-5} ohm, the resistance of each bolt of diameter d centimetres and length l centimetres is $\dfrac{l \times 10^{-5}}{\pi d^2/4}$ ohms.

If the bolts pass nearly through the centre of the radial depth of the core, and so are completely surrounded by iron, the inductance of each will be so high as compared with the above resistance that its self-induced E.M.F. will be nearly equal to its impressed E.M.F.; the resultant E.M.F. will be low, and the current will lag behind the impressed E.M.F. by some angle approaching 90°. Hence even if the current be large, the watts $E_i I \cos \phi$ will be so small as to be negligible by comparison with other more important losses.

But this condition also implies that the self-induced flux due to the current through the bolt is almost equal to the original field which it cuts; the current, lagging nearly 90 electrical degrees, reaches its maximum magnetizing effect in the centre of each interpolar gap in the position best calculated to oppose the passage of all lines below the bolt circle and to concentrate them above the bolt circle. Almost all the flux is driven into the outer layers of the core, and only a small percentage is left within the bolt circle sufficient to supply the loss of volts over the ohmic resistance of the bolt. The area of the core is thus in effect reduced, and the current reaches such a magnitude that its ampere-turns in combination with those of the field winding are sufficient to drive the flux through the armature path of which the permeability is reduced by reason of the density being nearly doubled. The eddy-currents in the bolts will supply half the total number which are required, but in order to retain the same total flux the field ampere-turns must themselves be raised as compared with the number which would be required over the armature path if the bolts were absent. Thus the correlative of the negligible expenditure of watts in the bolts is an appreciable increase in the loss from eddy-currents, and possibly also from hysteresis, in the discs.

On this account either the bolts passing through the body of the core must be insulated from the end-plates, as shown in Fig. 132, or the effective area of the core must be reckoned practically as that between the bottom of the slots and the bolt circle.

As the bolt circle is brought nearer to the inner edge of the discs their inductance and magnetizing effect decrease, but on the other hand the expenditure of watts in the bolts themselves increases. If the bolts are immediately under the inner edge of the discs we approach the condition of a conductor lying on the surface of an iron core, and the inductance of each bolt is so low as compared with its resistance as to be negligible. If for a rough estimate the distribution of the lines which leak out of the core is assumed to be sinusoidal, with an average induction B, the virtual E.M.F. of each bolt would be $2{\cdot}22\ (B \times \text{pole-pitch})\ f \cdot l \times 10^{-8} = E$, the pole-pitch being of course measured on the inner circumference of the discs. The lag of the current being negligible, the expenditure of power in each bolt is then E^2/R, or by substitution of the above expressions is $3{\cdot}9\ (B \times \text{pole-pitch})^2 f^2 \cdot d^2 l \times 10^{-11}$ watts. If in practice such a figure as $B = 100$, and a pole-pitch of 20 centimetres in a multipolar machine of fairly large size are assumed, the loss in each bolt of diameter 2·54 centimetres for a frequency of 30 works out to 0·9 watts per centimetre of its length. The interest of such a case lies in the fact that similar or larger losses may be debited to an armature core supported on a central hub, since in effect each arm of the hub corresponds to a bolt lying immediately under the discs.

§ 21. Eddy-currents in the copper winding of armatures.—(*b*) The origin and nature of the eddy-currents which occur in solid copper conductors embedded in slots during their passage through the field of a dynamo were first investigated by S. Ottenstein,[1] and from his experiments the following conclusions are mainly derived. Since the eddies are due to the change in the number and direction of the lines within the slot as its position relatively to a pole is altered by rotation of the armature, the total loss can be divided into two components: (1) due to the rate of alteration of a supposed radial field passing straight down the length of a slot, and (2) due to the rate of alteration of a supposed transverse field passing straight across the slot. To correspond with this and to isolate each effect separately, the actual field within the slot, whatever its nature, must be resolved into two components at right angles, their combination at any moment and at any particular spot within the slot reproducing the actual slanting direction of the lines together with their number. The E.M.F.'s are at right angles to the direction of each component field, and the eddies are reduced by subdivision of the copper into a number of thin strips side by side in the case of the radial field and into a number of layers in the case of the transverse field.

The maximum value of either the radial or the transverse flux density will be different for each bar within the slot according to its position; thus the former will be small near the middle of the slot, greater at the bottom due to lines leaking out again from the teeth when highly saturated, and greatest immediately at the top where the lines enter from the air gap. With normal proportions of slot the maximum value of the radial flux-density at the top of the slot is twice or thrice that of the cross density, but it very greatly diminishes both from the top and bottom of the slot towards its middle, so that its influence on the whole is not so preponderant. The cross field does not show nearly so much difference in its values for the upper and lower halves of the slot. Also, the surface exposed to the cross density is about three times the width of the bar in an armature with narrow deep slots and two layers of winding. Hence in such armatures a further transverse division into more than two layers might give a lower total loss than an equal number of radial subdivisions; in other words, four bars one above the other would be preferable to two layers of two thinner bars abreast. On the other hand, with wide shallow slots, radial subdivision into several sections abreast would be the more advantageous, as the radial flux-density preponderates. In either case, soldering of the bars together at their ends largely nullifies all the advantage of the division, so that it is of little use to divide a thick bar into several laminæ in parallel which are immediately united outside the limits of the core length. Lastly, the top of the bars should be kept well below the level of the top of the slot, where there is a strong field of curving lines; the use of hard wood wedges for securing the bars in a slotted armature finds in this an additional recommendation. Shading off the field at the pole-tips, as by cutting away half the pole-tip laminations, is found to produce little or no effect on the total loss.

As an empirical approximation, the loss per slot filled with a single solid bar with normal insulation from the walls and distance from the top may be

[1] "Das Nutenfeld in Zahnarmaturen und die Wirbelstromverluste in massiven Armatur-Kupferleitern," *Sammlung Elektrotechnischer Vorträge*, Vol. 5 (Ferdinand Enke, Stuttgart).

said to be proportional to the depth and width of the slot, and when the conductor is subdivided, the loss, roughly speaking, varies inversely as the number of layers and as the number of conductors abreast, *i.e.* inversely to their product $z = Z/S$. The two fields co-exist, and as a practical clue to the amount of either field within the slot may be taken the difference between the uncorrected flux-density at the roots of the teeth and a fixed flux-density of, say, 17,000, at which but few lines spread into the slot except at its extreme top. The total loss in the S slots is so far

$$\propto S^2 \frac{w_s \cdot h_s}{Z} L \text{ (uncorr. } B_{t2} - 17{,}000)^2.$$

The radial field declines from a maximum under the centre of the pole to zero on the neutral line. The transverse field, on the other hand, at least so far as the centre of the slot is concerned, increases from zero at the centre of the pole to a maximum at the pole-edges. The average value of the transverse slot field along the pole-pitch is indicated in Fig. 398*a*. It will be

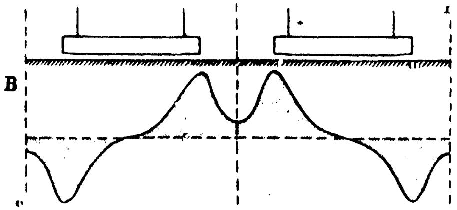


FIG. 398*a*.—Curve of average value of slot transverse field at no-load.

seen that when the teeth of a continuous-current armature under the pole-face are worked with a high density at the root, say, B_{t2} over 20,000, there may be a considerable cross flux within the slots close to the pole-edge causing eddy-currents in solid conductors. The ampere-turns over a tooth under the pole may be 2000, while an adjacent tooth just beyond the pole-tip, with only a comparatively slight falling off in the density, may only require perhaps 400 ampere-turns; there will then be a difference of magnetic potential across the slot of 1600 ampere-turns, and just as in the case of the rotor of a turbo-alternator, flux will be shunted transversely from the highly saturated root across the slot into the less saturated tooth. Hence near each pole-edge the cross flux suffers a rapid rise and fall, and as the difference in the ampere-turns of two adjacent teeth may increase faster than the square of the total main flux, the loss from the above cause may increase faster than the fourth power of the flux per pole.

The transverse slot field at no-load and the eddy-current loss arising therefrom in solid conductors has since been calculated in detail by L. Dreyfus,[1] who has shown that the eddy-currents react on the flux-density in the teeth and drive the lines within the slot back into the teeth. A check is thus automatically applied to their increasing magnitude, and the total copper losses do not continually increase with increasing slot depth. Under load, when the conductors themselves carry a normal current, the conditions become even more complex.

An approximate formula for the amount of the loss in watts per cubic cm. of copper in the slot has been proposed by Prof. Miles Walker, and an actual case has been analysed, in *The Diagnosing of Troubles in Electrical Machines*, pp. 92-95. As a purely empirical formula Mr. B. G. Lamme (*Trans. Amer. I.E.E.*, Vol. 35, Part I, p. 272 ff., *q.v.*) has given for the total effect in a two-layer winding

$$\text{watts} = 6pN\,(1000 + AT_t)^2 V \times 10^{-8},$$

[1] *Archiv. für Elektrot.*, Vol. 6, pp. 165 and 327.

where p = no. of pairs of poles, N = revs. per min., AT_t = maximum ampere-turns for one tooth, and V = total volume of copper in cubic inches in one slot.

Thus in the case of either the radial or the transverse slot fluxes the rate of change or the rise and fall of the field is largely localized in close proximity to the pole-tips. An examination of the gradient of the field as dependent upon the length of air-gap and of the interpolar zone shows that with a usual ratio of pole-arc to pole-pitch the loss does not increase so fast as the diameter of the armature, and only increases in proportion to the ¼th power of the air-gap; further, that it becomes proportional to the 1·25th power of the number of poles. The conclusion that the length of the air-gap has but little effect, although an increase in it tends to increase the loss, is borne out by experiment.

In default, therefore, of more complete knowledge, the eddy-current coefficient, which must be multiplied by the square of the number of revolutions per minute to determine the loss in the bars of a slotted armature may be given as

$$k_3 \cdot \left(\frac{\text{uncorrected } B_{t2} - 17{,}000}{1000}\right)^2 \times S^2 \cdot \frac{w_s \cdot h_s}{Z} \cdot L \cdot D^{\frac{3}{4}} \cdot l_g^{\frac{1}{4}} \cdot p^{\frac{5}{4}}$$

Under load when the armature is carrying current, the distortion of the field by armature reaction increases the loss. The maximum value of either the radial or the transverse field at the trailing pole-corner is increased and at the leading pole-corner is decreased, so that these two changes largely counterbalance one another and the loss in the interpolar bars is but little increased. But the fields now vary almost continuously under the poles, and the loss in the slots under the poles may be regarded practically as an entirely additional quantity. Owing to the complexity of the conditions, neither theory nor experiment has up to the present provided a simple means for estimating the loss under load by a rational formula. Tests under short-circuit do not help in this matter, since there is then practically only a cross field from the armature conductors present, and the teeth are not highly saturated.

Finally, the copper bars of a section during the period of its commutation are traversed by an alternating flux self and mutually induced by the armature current and crossing the slot from side to side. The current is then no longer uniformly distributed over the cross section of a large solid conductor, and since this effect produces a loss proportional to the square of the load-current, it may also be derived by assuming a virtual increase in the resistance of the conductor. The energy lost thereby, although assisting to heat the armature, is, however, rather to be considered under the question of commutation, being closely connected with the apparent inductance of the section during short-circuit. Yet under commutating poles, the bars may become seriously heated at the periods of short-circuit.

It is not only along the active length of the bars that eddy-currents are set up. Where the solid bars project beyond the core of the toothed armature, or where the stranded bars are soldered to their

end-connectors in the bar-wound drum with evolute connectors, the fringes from the flanks of the poles contribute small quota to the total loss.

(*c*) The bands of binding wire, especially where soldered together, are a further source of loss. On smooth-core armatures they are of minor importance, but when employed in connexion with toothed armatures they may give rise to considerable losses. By the use of a material of high resistivity such losses can, however, be reduced to an almost negligible quantity even in toothed armatures.*

§ 22. Approximate formulæ for eddy-current loss in armatures.—Enough has been said to show the great difficulty of expressing in any simple formula the combined effect of the numerous variables which enter into the question of the amount of the eddy-current loss. The law which governs any one constituent may be unknown, or, if known, the magnitude of its effect may be quite uncertain owing to its being dependent upon varying conditions in practice, such as the stamping of the discs or filing of the slots. Either of two methods of treating the subject may be adopted. By the first the hysteresis and eddy-current losses in the core and teeth of the armature are grouped together, and an approximate expression for these "iron losses," as they are called, is constructed, which should bear a reasonable relation to the losses experimentally measured on no-load when the unwound machine is driven in the excited field by an electric motor. To these losses are subsequently added an allowance for eddy-currents in the copper winding on no-load and on full-load. By the second method, all the eddy-current losses proportional to the square of the speed are grouped together as a whole, and an approximate expression is constructed for the coefficient F, which should, after deduction of the pole-face loss, agree with that deduced from an experimental test on the Kapp-Hopkinson method to be subsequently described.

In either case some formula which when applied under changed conditions shall give fairly reliable results is so necessary to the designer that the required forms of expression must at least be tentatively suggested as a starting point for purposes of comparison, even if the constants require to be afterwards corrected as experience accumulates.

The first method is perhaps more suitable for alternators, and its discussion is therefore reserved until Chapter XXIX, § 13. The second method will here be followed, although it must be understood that the results of Chapter XXIX are also applicable *mutatis mutandis* to the continuous-current dynamo, as there explained. The problem, therefore, is to construct a formula for F, or the watts lost in eddy-currents at one revolution per min., which will in some

degree differentiate between the different localities of eddy-currents and the different causes of the loss of energy by them. The losses other than those in the active length of the copper winding may be divided into a portion due to the active length of the iron under the poles, and a portion due to the end-fringe from the flanks of the poles. The first for a given linear speed is probably best regarded as proportional to the cylindrical surface of the core, *i.e.* to DL, since it is largely due to the extent of the surface contacts between the core laminations caused by mechanical processes of manufacture. Indeed, in toothed armatures a good deal of evidence might be brought to show that this portion of the loss is proportional to the number of slots, each as causing continuity of surface, or to their own area of surface. Next, this loss will vary as the square of the density on the surface at different parts of the teeth, yet since this diverges from proportionality to the air-gap density only when the apparent density in the teeth has been pressed very high, the term B_g^2 may be retained approximately. Lastly, it will vary as the square of the linear speed, and therefore for a given number of revolutions per minute as D^2, so that it finally becomes

$$= k_1 \,.\, N^2 \,.\, D^3 L \left(\frac{B_{g\,max}}{1000}\right)^2$$

In the second portion due to the end-fringe may be grouped the losses in the end-connexions and solid ends of the bars where they project past the core, and the whole is roughly proportional to the circumference, or to D. All these losses will be proportional not simply to B_g^2, but to $(AT_g + AT_t)^2$ as causing the stray lines between the pole and the body of the armature core or supports of the end-winding. They will also vary as the square of the linear speed, and so become $= k_2 \,.\, N^2 \,.\, D^3 \left(\frac{AT_g + AT_t}{1000}\right)^2$

For the toothed armature k_1 varies between 4×10^{-10} and 6×10^{-10}, k_2 is about 12×10^{-10}, and k_3 the factor in the last expression of the previous section for the copper winding has such a value as 80×10^{-10}, all dimensions being reckoned in inches, and flux-densities being measured in lines per square centimetre. An equation of the form

$$F = \left[D^3 \left\{5\left(\frac{B_{g\,max}}{1000}\right)^2 L + 12\left(\frac{AT_g + AT_t}{1000}\right)^2\right\} + 80\left(\frac{\text{uncorr. } B_{t2} - 17{,}000}{1000}\right)^2 S^2 \,.\, \frac{w_s h_s}{Z} L D^{\frac{3}{4}} l_g^{\frac{1}{4}} p^{\frac{5}{4}}\right] 10^{-10} \quad (210)$$

may thus be suggested for the toothed drum. In practical cases the first term is by far the greatest, the second about 5 per cent. of the first, while the third term never reaches any appreciable amount

until the uncorrected B_{t2} exceeds 22,000 with fairly narrow slots and deep bars in two layers, or 24,000 with four layers of shallow bars. A wide slot should only be used in combination with thin bars and with low values of B_{t2}. Analysis of numerous experiments on toothed armatures shows that the total loss is not far from proportional to $B_{g\ max}^2$, and when the total measured loss in both armature and pole-pieces does not rise so rapidly the reason may be traced to magnetic screening reducing the loss in the poles (*cp.* § 29).

In conclusion, it may again be repeated that further direct experiment in the laboratory as to the various sources of eddy-currents and their suppression is much needed, and the lack of such a firm experimental basis must be the excuse for the approximate nature of the formulæ which have been above hazarded. In commercial work machines can seldom be designed so as to test separately the influence of the several varying factors, yet it is only to the more complete elimination of eddy-currents that we must look if the efficiency and output of dynamos for a given mass of iron and copper are to be appreciably increased in the future.

§ 23. Hysteresis loss in armatures.—Returning to the third source of heat, or the "magnetic" loss by hysteresis, the amount of power spent in changing the direction of magnetization of the core must be calculated, as explained in Chapter XIV, § 14. In the core below the teeth the density and hysteresis loss is greatest in the outer layers. The nominal maximum density when the flux is averaged over the whole of a cross section midway between the poles is only about 82 per cent. of the true maximum near the bottom of the teeth. But since the curve of Fig. 218 for rotating magnetization is based on the nominal maximum B_c averaged over an interpolar cross section, the values of h for other armatures with the same nominal maxima B_c may thence be obtained. The loss in the core is then for any given maximum density simply proportional to the number of complete cycles per second, and to the volume of the core; it may therefore be expressed by a coefficient H multiplied by the number of revolutions per minute, or in terms of the symbols of equation (97), $H = h\,\frac{p}{60}\,V_c$ and $HN = H_w$. The cross-magnetization of the armature under load here also increases the loss as compared with that at no-load. The total volume of the teeth must be calculated separately from the rest of the armature core, and in default of experimental figures strictly applicable to their case h must again be taken from Fig. 218, as explained in Chapter XIV, § 14, *qu.v.*

The punching of the slots in the discs hardens the edges of the notches where the iron is sheared off, and the hysteresis loss is thereby as well as by any bending or mechanical stress increased

as compared with laboratory results on plain strips of the same quality of iron. But all such sources of increased loss are practically indeterminate.

§ 24. Predetermination of rise of temperature in armatures.—Combining the three losses together, the total watts expended in heating the armature are

$$W = I_a^2R_a + FN^2 + HN \quad . \quad . \quad . \quad . \quad . \quad (211)$$

and it is the ratio of this to the cooling surface of the rotating armature that determines its rise of temperature.

In estimating the relative cooling value of any surface the general principle must be to allow approximately for differences in their degree of exposure to the outer air, and also for differences in their peripheral speed. Thus the end-windings of multipolar barrel-wound armatures are of more importance than the centre portion of the core which is directly under the poles. Again, in large multipolar armatures in which the discs are supported on the arms of a hub, not only is a clear air-space secured between the internal circumference of the core and the cylindrical hub, but the percentage difference between the peripheral speeds of the inside and outside grows less as the diameter of the armature is increased. Finally, the number and width of the radial air-ducts through the armature core will exercise a considerable effect, and may require to be taken into account in the calculation. It is, however, useless to adopt any great degree of refinement in the calculation of the cooling surface owing to the number of secondary conditions which affect the result. Different types of machines will require different methods of reckoning the cooling surface, but in general the curve connecting the heating coefficient with the peripheral speed will have the shape of those in Fig. 399 when the cooling surfaces are reckoned on some consistent basis. Practical experience will determine a method of calculating a figure to which the cooling effect may be regarded as proportional, even though all the factors which enter into it may not be strictly taken into account. The general effect of taking close account of the surfaces of the inside of the core and of the air-ducts will merely be to raise the value which must be assigned to the coefficient when the armature is stationary.

The two cases of smooth-core drum armatures and toothed barrel-wound multipolars in which the armature core is well ventilated by air-ducts at intervals of 4 or 5 inches may, however, be distinguished. In some designs the ventilating ducts have been made so numerous and so wide that the effective length of the iron is only two-thirds of the gross length of the core. In small machines less than 15 inches in diameter, owing to the confined nature of the internal apertures into the core, the effect of such ducts is not so marked as in large machines, although they still remain of value

when made of considerable width and accompanied by a high peripheral speed. On an average the curves given in Fig. 399 have been found by the writer to give good results in the case of continuous-current multipolar armatures of usual construction up to peripheral speeds of 4000 feet per minute. The value of the heating coefficient $k = t^\circ \times S_c/W$, or the temperature rise in degrees Centigrade per square inch per watt is for barrel-wound toothed armatures $\frac{65}{1 + 0 \cdot 8\,(v/1000)^{1 \cdot 3}}$, where v is the peripheral speed of the external surface of the armature in feet per minute, and S_c is the cooling surface in square inches. The final maximum rise of temperature of the outside of the armature as measured by thermometer is thus

$$t^\circ C = \frac{65W}{S_c\left\{1 + 0 \cdot 8\left(\frac{v}{1000}\right)^{1 \cdot 3}\right\}} \quad . \quad . \quad . \quad (212)$$

The effect of the peripheral speed has in many formulae been given by a factor of the form $1 + 0 \cdot 5\ (v/1000)$, but the use of the first power of the peripheral speed appears to underestimate its great influence.[1] The constant value when $v = 0$, or the armature is at rest, namely, 65, is lower than the average value for stationary field-magnet bobbins owing to the lesser thickness of the layers of copper in the armature; for any given type of machine it may be determined by measuring the rise of temperature for a given number of watts when the armature is placed in its appropriate field-magnet and is stationary.

The cooling surface to be used in connexion with the curves of Fig. 399 is in the case of toothed armatures with barrel winding the external cylindrical surface, *plus* the internal cylindrical surface of the winding at both ends so far as it projects beyond the core; the latter must be again reduced in proportion to its lower speed when there is considerable difference between the external and internal diameters. The former surface is $\pi D_1 L_1$ where L_1 is taken from the outer edge of the commutator lug to the extreme opposite end of the armature, *i.e.* $= L + 2l_e$ nearly. The latter is approximately $\pi D_2(L_1 - L) = \pi D_2 \cdot 2l_e$, or when reckoned as of less value in proportion to its lower peripheral speed, $\pi \cdot \frac{D_2^2}{D_1} \cdot 2l_e$.

In all cases the question of the heating of the commutator must be also considered in relation to that of the armature. The above figures assume that the temperature of the commutator is lower than that of the armature. With carbon brushes, however, the final temperature of the commutator may exceed that of the armature.

[1] Constants have also been used to suit a factor of the form $(1 + \beta\sqrt{v})$, *cp.* J. Fischer-Hinnen, *E.u.M.*, Vol. 36 (1918), pp. 205 and 213.

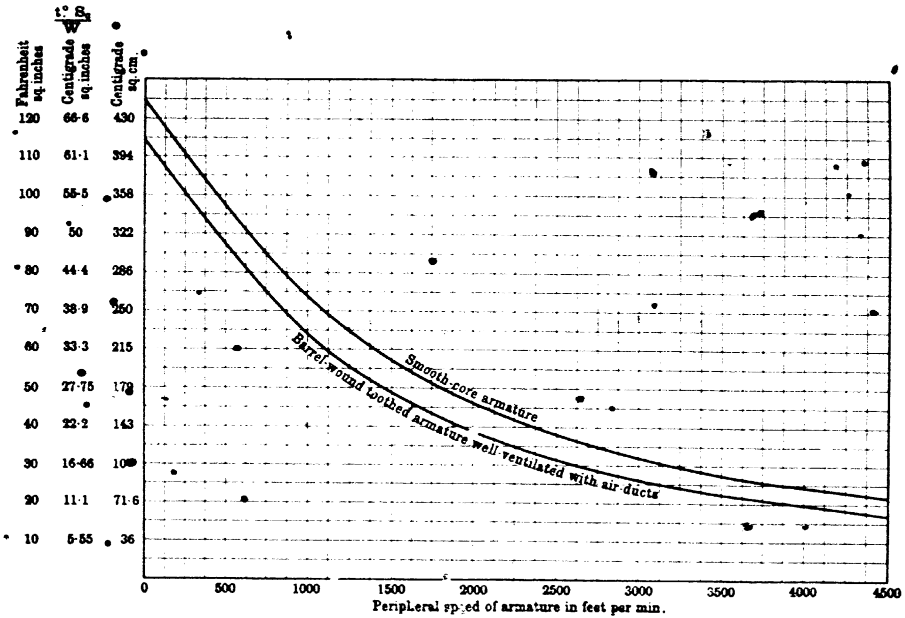


FIG. 399.—Heating coefficients of armatures.

Hence the commutator, so far from helping to dissipate the heat of the armature, positively assists in raising the temperature of the near end by conduction, unless there are long radial commutator connexions which exert a powerful fanning action.

For a peripheral speed of 2000 feet per minute, a rise of 36° C. is approximately obtained if the watts per square inch, W/S_c, = 1·41 to 1·5, or *vice versa* if S_c = 0·71 W to 0·66 W; in other words, about three-quarters or two-thirds of a square inch of cooling surface, as above reckoned, must be allowed per watt expended over the armature.

It may here be remarked that a cool machine is by no means necessarily efficient; although in most cases these two desirable qualities are attained by the same means, still it should be remembered that, while the efficiency is dependent on the ratio of the lost watts to the useful output, the rise of temperature is determined by the ratio which the lost watts bear to the cooling power of the surfaces.

Finally, with a given armature, since the eddy-current loss is dependent upon the square of the revolutions, the amount of current that can be taken out of it at different speeds for a fixed rise of temperature depends largely upon the proportion of the copper loss to the eddy loss, and upon the way in which the effectiveness of the cooling surface is modified by alterations of the peripheral speed.

§ 25. Heating of the commutator.—The heating of the commutator with carbon brushes has an importance second only to that of the armature. The sources of heat within the commutator itself are fourfold, namely—

(1) The loss of energy due to the passage of the armature current over the contact resistance of the brushes, this current being assumed to be divided between the several sets of brushes of the same sign and between the portions of any one brush in strict proportion to their areas; *i.e.* on the supposition of a uniform current-density under the brushes.

(2) The additional loss due to the unequal division of the current over the surface of the brush contact, and to sparking if commutation is not properly performed. Apart from "additional" currents, unequal distribution of the current between the brushes on one arm quickly arises, due to slight differences in their pressures or in the state of their surfaces; for the drop over the contact-resistance, as has been shown in Chapter XX, does not vary in proportion to the current-density, even with the same pressure.

(3) The loss from the mechanical friction of the brushes.

(4) And the loss from eddy-currents in the sectors which are at any moment carrying the armature current and the adjacent sectors. As the current flows along the sectors which are undergoing

commutation, and is gradually tapped off into the brushes, these sectors are practically situated in a stationary magnetic field, while they themselves are moving forwards. Eddy-currents are thereby set up in the mass of the copper plates forming the commutator.

With carbon brushes the great increase which is possible in the first and second items renders it imperative for the designer to consider carefully the heating due to the combined effect of the four causes, and this is especially the case in machines of low voltage and large current.

§ 26. Calculation of commutator losses.—The commutator losses are calculated as follows: (1) Let p_1 = the number of rows of brushes of one polarity; then, assuming that the total armature current I_a is equally divided between them, the average current-density is $s_u = \frac{I_a}{p_1 \cdot F_u}$, and the current to be collected at one row is $\frac{I_a}{p_1}$.

In the absence of complete data as to the shape of the short-circuit current curve, the virtual current-density and the form factor, R_k must be taken from the curves of Chapter XX, § 10, on the assumption of a uniform current-density s_u corresponding to $\frac{I_a}{p_1 \cdot b_1 \cdot l_b}$, where l_b is the length of the brush surface in one row measured parallel to the axis of rotation, and b_1 is the width of contact in the direction of rotation. The total loss of watts over the two sets of brushes is then

$$2 I_a^2 \cdot \frac{R_k}{p_1 b_1 l_b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (213)$$

where $\frac{R_k}{b_1 \cdot l_b}$ is the resistance of one row, and $\frac{R_k}{p_1 b_1 l_b}$ is the resistance of the p_1 rows of one polarity. In the bipolar machine or the wave-wound machine with two sets of brushes, $p_1 = 1$; in the lap-wound multipolar there must be as many sets of brushes as there are poles, and $p_1 = p$, but in the wave-wound multipolar, even though there are several rows of brushes of one polarity, some rows may be omitted, and p_1 need not be equal to p.

(2) The additional loss due to want of uniformity of current-density cannot reach any great amount with copper brushes owing to the sparking that would result, but with carbon brushes it may form a very considerable item without evident overheating of the brush edges. Since the increase depends upon the square of the form factor, it may easily amount to 30 per cent. of the normal loss, or in extreme cases may double it. The advantage of carbon in reducing the sparking is, in fact, secured at the expense of the efficiency of the machine.

(3) If p be the pressure on the brushes in lb. per sq. inch, and a be the total area of surface of all the brushes on the machine, the total pressure is $P = pa$, and the loss in watts from the mechanical friction is

$$\frac{\mu P \times v_c' \times 746}{33{,}000} = \mu P \times v_c' \times 0{\cdot}0226 \text{ watts} \quad . \quad (214)$$

where v_c' is the peripheral speed in ft. per min., and μ is the coefficient of friction.

With low voltages (when the brush surface is large as in electroplating machines) and with high speeds, the brush friction loss may be a very appreciable percentage of the useful output of the dynamo, and may exceed the loss under (1).

The exhaustive experiments of J. Liska[1] on dry and oiled bronze rings and copper commutators with brushes of many kinds show great divergences between brushes of different materials and entirely dissimilar behaviour under varying conditions of pressure, speed, temperature, current density, and current direction. It can only be said that the coefficient of friction increases with increasing pressure and on a commutator diminishes with increasing speed owing to the mechanical vibration due to the passage of the mica and copper strips.[2] Assuming $p = 1\frac{1}{2}$ to 2 lb., average values for μ for rough calculations will be—

Hard carbon . .	0·4 (at 500 ft. per min.)	to 0·2 (at 3000 ft. per min.)
Electrographitic .	0·35 (,, 1000 ,,)	,, 0·2 (,, 5000 ,,)
Soft graphitic .	0·2 (,, 2000 ,,)	,, 0·15 (,, 6000 ,,)
Copper & graphite mixed . . .	0·28 (,, 500 ,,)	,, 0·15 (,, 4000 ,,)
Copper gauze .	0·3 to 0.2	

The coefficient of friction on a perfectly smooth bronze ring is higher, and both on the ring and on the commutator is, as a rule, much reduced by a little lubricating oil.[3]

(4) Definite experiments as to the magnitude of the fourth loss from eddy-currents are wanting, but probably they are but small, although increasing with increased thickness of the commutator plates.

[1] *Arbeiten aus dem Elektrotechnischen Institut zu Karlsruhe*, Vol. 1, p. 48. *Cp.* also Prof. F. G. Baily and Mr. W. S. H. Cleghorne, *Journ. I.E.E.*, Vol. 38, p. 157.

[2] Confirmed also by Mr. P. Hunter Brown, *Journ. I.E.E.*, Vol. 57, p. 195. At high speeds the lower friction is probably due to the contact being in fact intermittent to some degree; intermittently therefore a thin air-film intervenes between brush and commutator.

[3] The paper of Prof. Baily and Mr. Cleghorne quoted in a previous note also shows that the application of a small amount of paraffin wax as a lubricant reduces the coefficient of friction of a dense electrographitic brush (Le Carbone X) by 80 per cent., *i.e.* to 0·08 without increasing the electrical losses. But the lubricant must be very sparingly used (*cp.* Chap. XXIII, § 9).

§ 27. Temperature rise of commutator.—The rise of temperature of the commutator surface should preferably not exceed 70° F., or as a maximum may be allowed by the British Standardisation Rules to reach 90° F. (50° C.) in continuous running at full-load. It may be calculated by a formula similar in its construction to that for the rotating armature. The constant of the numerator which determines the rise per watt per sq. inch when the commutator is at rest is lower than in the armature, owing to the better exposure of the former to the air. On the other hand, owing to its smooth surface, the influence of the peripheral speed is not so marked as in the case of armatures, although still considerable if there are separate connectors to the armature winding. If W the total watts expended over the commutator, and the cooling surface be reckoned in sq. inches as the external cylindrical surface *plus* the area of one side of the radial connectors up to a limiting length of, say, 3″ from the commutator surface

$$t°C = \frac{55W}{S_c\left\{1 + 0{\cdot}3\left(\frac{v_c'}{1900}\right)^{1\cdot3}\right\}}$$

where v_c' is the peripheral speed of the commutator in ft. per minute.

Any such formula is, however, liable to many disturbing conditions, among which especial importance must be given to the number and shape of the connexions which lead from the armature winding to the sectors. If these are numerous, and are thin but wide blades of copper, they have a powerful fanning action, which very greatly assists in dissipating the heat of both the commutator and the armature.

A cool commutator is of great assistance in any case of difficult commutation, and on this account the commutation of turbo-generators is much improved by special means for ventilation both inside and outside the commutator; an instance is found in Siemens' commutator for turbo-generators, which is divided into two halves united by copper radial blades that serve also as fans (Brit. Patent No. 19891 (1908)).

§ 28. Eddy-currents in pole-pieces.—There remains the question of eddy-currents as set up in the pole-pieces when a rotating toothed armature causes the density of the field over their bored face to vary rhythmically. Such currents do not spread to any great depth within the metal mass, but whirl round near the surface facing the armature. One complete cycle of varying flux-density at any spot corresponds to the passage of one tooth and one slot past a fixed point on the pole-face; or, in other words, a period corresponds to the time taken by the armature in moving through

the pitch of the teeth ($t_1 = w_{t1} + w_3$) (*cp.* Figs. 481, 482). If S = the total number of slots, the periodicity is $f = \frac{N}{60} \cdot S$. If the local paths had no inductance, the currents would run along the pole-face opposite to each projecting tooth, and then dividing would curve round to complete their circuit opposite the slots. Owing, however, to the action of the inductance which causes the current to lag behind the impressed E.M.F., the positions of the eddies are displaced relatively to the teeth; if the reactance ($2\pi f \mathcal{L}$) were very high, or the resistance very low, the angle of lag would approach a quarter of a period, so that the currents would embrace the teeth and openings of the slots. The M.M.F. of the eddy-current would then act to reduce the density opposite the teeth, and to raise it opposite to the slots. Thus by the effect of magnetic screening the distribution is rendered more uniform, and the eddy-currents are prevented by their own inductance from reaching any great amount.

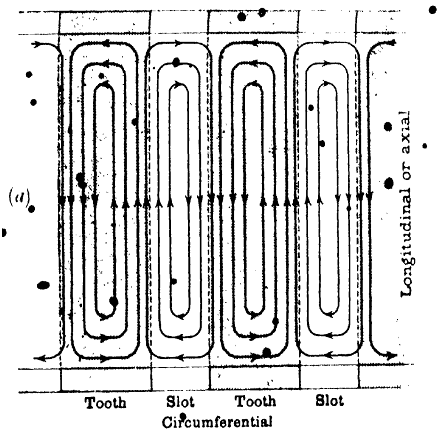
There is, however, a difference between the two cases of a solid and of a laminated pole-piece. In the former case the greater part of the path of the eddy currents is longitudinal or along the axial length of the armature core (Fig. 400 (*a*)), and currents in such paths lessen the penetration of the flux-density variation into the pole-face. In the latter case, the greater part of the eddy current path is circumferential (Fig. 400 (*b*)), and such circumferential currents lessen the penetration of the flux variation at the sides of the laminations as compared with their central portions.[1] The flux variation, therefore, penetrates more deeply into the laminated pole, although in either case it extends over a very small depth of iron.

In a solid pole, the eddy-currents flow longitudinally across the pole through a strip of width equal to a quarter of the tooth-pitch or $t_1/4$ centimetres, and complete their circuit by returning through another adjacent strip of the same width (Fig. 400*a*). The maximum range of induction on the pole face being from B_{max} to B_{min}, the maximum E.M.F. acting round the edges of the circuit corresponding to half the tooth-pitch is $\propto B_{max} - B_{min}$, or along the one edge is $\propto \frac{1}{2}(B_{max} - B_{min})$; it is also proportional to the speed of the moving teeth, *i.e.* to the peripheral speed v of the armature $= \pi DN/60$ cm. per sec., and to the length L_f of the pole-piece axially in cm. The maximum E.M.F. down one strip is therefore

$$v \cdot \frac{B_{max} - B_{min}}{2} \cdot L_f \times 10^{-8} \text{ volts}$$

Now the amount of the eddy-current loss depends essentially on the nature of the curve of the non-uniform flux-density, and as a first approximation let the variation be assumed to be sinusoidal, so that it gradually diminishes after a sine law to zero midway between the two outer edges of our circuit. The loss of power in any elementary strip is proportional to the square of the E.M.F. divided by its resistance, and the average value of the squares of the ordinates of a sine curve is half the square of its maximum value. The

[1] Messrs. Adams, Lanier, Pope, and Schooley, "Pole-face Losses," *Trans. Amer. I.E.E.*, Vol. 28, Part II, p. 1133, where the different effect of the longitudinal (axial) and the circumferential (tangential) currents is emphasized by calling the former "damping" and the latter "screening" currents.



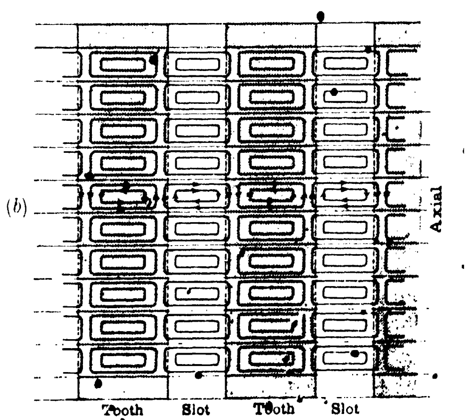


FIG. 400.—Eddy-currents (*a*) in solid pole piece, with large angle of lag; (*b*) in laminated pole-piece, with large angle of lag.

resistance of the single strip to a distance h cm. within the pole-piece (Fig. 401) is $\frac{L_f\rho}{\frac{1}{2}t_1 \cdot h}$, where ρ is the resistivity in ohms of a cm. cube of the material of which the pole-piece is composed; the resistance of the end or circumferential portions of the circuit may be entirely neglected as compared with that of the axial lengths. With sinusoidal variation the loss of power in the one strip is therefore in watts

$$\frac{1}{2} v^2 \left(\frac{B_{max} - B_{min}}{2}\right)^2 L_f^2 \times \frac{\frac{1}{2}t_1 h}{L_f \rho} \times 10^{-16}$$

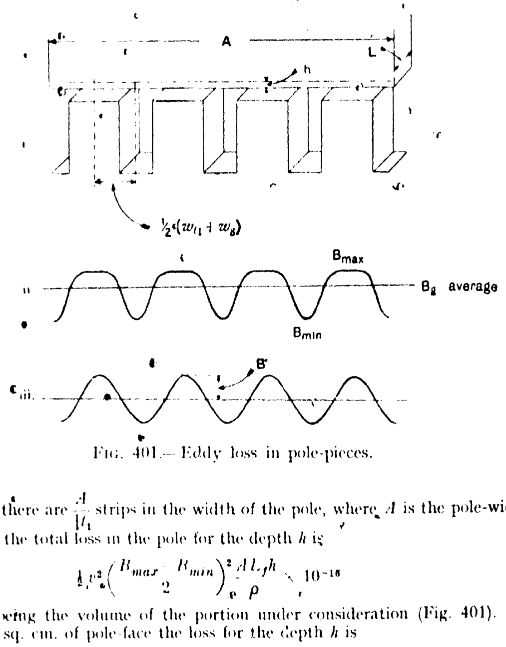


FIG. 401.—Eddy loss in pole-pieces.

and as there are $\frac{A}{\frac{1}{2}t_1}$ strips in the width of the pole, where A is the pole-width in cm., the total loss in the pole for the depth h is

$$\frac{1}{2} v^2 \left(\frac{B_{max} - B_{min}}{2}\right)^2 \frac{A L_f h}{\rho} \times 10^{-16}$$

$A L_f h$ being the volume of the portion under consideration (Fig. 401). Or per sq. cm. of pole-face the loss for the depth h is

$$\frac{1}{2} v^2 \left(\frac{B_{max} - B_{min}}{2}\right)^2 \frac{h}{\rho} \times 10^{-16} \text{ watts}$$

A still closer approximation may be made by again returning to the actual curve of distribution and drawing through it a straight line (shown in Fig. 401, ii) corresponding to the average value of B_g. The actual flux may then be regarded as produced by an alternating flux superposed upon the straight line. This alternating flux may be replaced by an equivalent sine-wave (Fig. 401, iii) having the same virtual value B_v, and giving a certain maximum value B' which is not quite the same as $\frac{1}{2}(B_{max} - B_{min})$; the value $(B')^2$ must then be substituted in the previous expression for $\left(\frac{B_{max} - B_{min}}{2}\right)^2$. Since v may also be expressed as $S\frac{N}{60} t_1$, the loss per sq. cm. of pole-face in the layer of thickness h cm. is thus

$$\frac{1}{2}\left(S.\frac{N}{60}\right)^2 (B' . t_1)^2 \frac{h}{\rho} \times 10^{-16} \text{ watts.}$$

Since the eddy-currents curve round in the mass of the pole, and especially at the ends of the pole-face, they may be partially reduced by axial slits along the pole-face, the thickness of the subdivisions being less than $\frac{1}{2}t_1$; but any such subdivision cannot be so effective as the more usual plan of laminating the pole in a direction at right angles to the axis of rotation just as the armature core is itself laminated. Considering a single thin lamination, from the proportions of the lamination all longitudinal, *i.e.* axial, resistance to the actual circulatory currents as they cross the lamina may be neglected in comparison with the circumferential resistance down one side and up the other. On this assumption the E.M.F.'s generated axially give immediately without deduction the P.D.'s corresponding to the circumferential components of the actual current, *i.e.* to the varying currents *along* the strips, through which alone energy is lost and which alone need to be considered. The central axis of the lamina being an equi-potential line and being regarded as at zero potential,

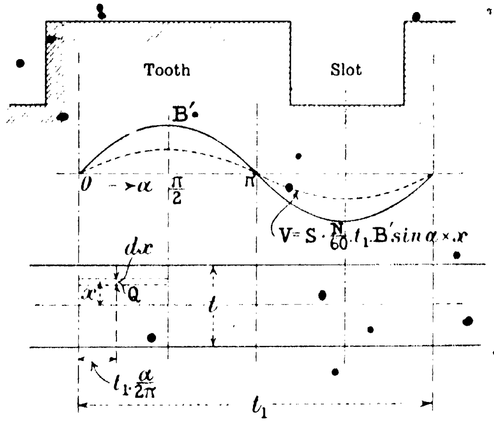


FIG. 402. Calculation of eddy loss in pole piece lamination.

The potential at any point Q (Fig. 402) along a thin axial strip, of length x cm. reckoned from the centre line, acting as the E.M.F. generating element, is proportional to its length x and dependent on its situation circumferentially, *i.e.* relatively to the sinusoidal curve of B' as fixed by the angle α. At any point Q therefore the potential is

$$E = S\frac{N}{60} t_1 x B' \sin\alpha \times 10^{-8}.$$

Now taking a thin lamina within the original lamination, distant x cm. from the centre line, of width dx and with one of its ends located on the line along which the flux-density has its normal value, the spacial rate of change of E in the circumferential direction along the lamina is

$$\frac{dE}{\frac{t_1}{2\pi}\cdot d\alpha} = S\frac{N}{60} B' . 2\pi x . \cos\alpha \times 10^{-8}$$

This is also the product of the circumferential current density Δ at any point $Q \times$ the resistivity ρ, and the watts per unit volume $= \Delta^2\rho$. The watts at the point in question, or $\Delta^2\rho \times$ the infinitesimal volume, are then

$$\left(S\frac{N}{60}\right)^2 (B')^2\, 4\pi^2 x^2 . \cos^2\alpha \times \frac{h}{\rho} \times \frac{t_1}{2\pi} . d\alpha . dx \times 10^{-16}$$

The integral $\int x^2 dx$ between the limits $x = -\frac{t}{2}$ and $x = \frac{t}{2}$, where t is the thickness of one lamination, being $\frac{t^3}{12}$, and the integral $\int \cos^2 a \,.\, da$ between the limits $a = 0$ and $a = \frac{\pi}{2}$ being $\frac{\pi}{4}$, the loss in the thickness of one lamination and in a length corresponding to a quarter of the tooth-pitch is

$$\frac{\pi}{4} \cdot \frac{\pi}{6} \cdot \left(S \frac{N}{60}\right)^2 (B')^2 t_1 \,.\, t^3 \,.\, \frac{h}{\rho} \times 10^{-16}$$

Multiplying by the number of laminae $= \frac{L_f}{t}$, and by the number of lengths $\frac{A}{\frac{1}{4}t_1}$, the total loss in the volume V of a layer h cm. deep is

$$\frac{\pi^2}{6}\left(S \frac{N}{60}\right)^2 (B')^2 \,.\, t^2 \frac{V}{\rho} \times 10^{-16}$$

or the same as the expression of § 19 for the loss in an alternating field; and the loss per sq. cm. of pole-face is

$$1{\cdot}645\left(S \frac{N}{60}\right)^2 (B')^2 \,.\, t^2 \frac{h}{\rho} \times 10^{-16} \text{ watts}$$

so that the effect of the tooth-pitch has disappeared.

But in either the second formula for solid pole-pieces, or the last formula for laminated pole-pieces there remains the great difficulty underlying the value to be given to h. As we pass the outer skin of the pole-piece, and proceed farther into its mass, the flux rapidly becomes more uniform, and the watts, being proportional to the square of the difference of the flux-density, diminish still more rapidly. It is, in fact, very difficult to determine the exact depth of the fluctuations, and the values which are to be assigned to B_{max} and B_{min} for each successive layer. With ordinary speeds and a considerable number of teeth, as in practical cases, the periodicity of the alternating currents set up — from 500 to 2000 cycles per second — is so high that the screening action largely reduces the loss, and has the effect of rendering the general law of the proportionality of the eddy-current E.M.F. to the speed, and of the loss to the square of the speed, no longer true. Especially is this the case with solid pole-pieces, to which the case of a magnetic brake becomes more nearly analogous. In such a brake, in which all the work is expended in producing eddy-currents in a solid mass, it is found[1] that the loss or the energy absorbed is proportional to a very low power of the speed, such as the 1·2 power. Approximate calculations based on the second formula for solid pole-pieces thus give values far exceeding those that are found in practice, and it can only be regarded as illustrating some of the elements of the problem.

§ 29. More accurate formulæ for eddy-loss in pole-pieces with damping.— The complex effects of screening can hardly be taken into account in an elementary treatment of the subject,[2] but more reasonable figures are obtained on the assumption[3] that the induced currents are damped out or extinguished within the mass of metal after the same law as that by which the intensity of an electromagnetic wave of high frequency decreases when meeting the

[1] G. Dettmar, *E.T.Z.*, Vol. 21, pp. 947–8.

[2] See F. Niethammer, *E.T.Z.*, Vol. 20, p. 967, and (1900) p. 549.

[3] Due to R. V. Picou after A. Potier in *Industrie Electrique* (1905), p. 35; and also to R. Rüdenberg, *E.T.Z.* (23rd Feb., 1905), p. 181.

surface of a metal. The resulting formula is then that the loss per square centimetre of a pole-face in watts[1] is

$$\frac{1}{8\pi} \cdot f^{1\cdot5}(B't_1)^2\sqrt{\frac{1}{R \cdot \mu}} \times 10^{-7}$$

$$\text{or } 4\left(S\frac{N}{60}\right)^{1\cdot5}(B't_1)^2\frac{1}{\sqrt{R \cdot \mu}} \times 10^{-9}$$

where B' is reckoned at the surface, f is the frequency, the wave-length or tooth-pitch t_1 is in centimetres, and R is the resistivity of a centimetre cube of the metal in absolute electromagnetic units. The high electrical resistivity of cast iron is therefore decidedly advantageous in reducing the eddy-loss, but since both μ and R appear together in the denominator, the gain is partly counterbalanced by the low permeability when the density of the flux at the pole-face is high. Since the variation of the flux-density is never very high at the surface, μ may be taken as corresponding to the mean air gap density. The thickness of the layer by which the fluctuation is damped to 1 per cent. of its amount at the surface is now $h = 0{\cdot}73\sqrt{\frac{R \cdot t_1}{v \cdot \mu}}$, and is thus very small. Further, the loss becomes proportional to the 1·5th power instead of to the square of the frequency. The experiments of Messrs. T. F. Wall and S. P. Smith[2] confirm the term $\left(S\frac{N}{60}\right)^{1\cdot5}$ which is here adopted.

The chief difficulty remains in the term $(B't_1)^2$, *i.e.* especially in the value to be assigned to the amplitude of the variation of the induction, B', at the surface. Since the formula presupposes a sinusoidal wave, the flux curve at the pole-face must be resolved into its fundamental and harmonics, and the loss from each added together to obtain the total loss, *i.e.* the *equivalent* sinusoidal wave must be found, and the amplitude of this expressed in terms of the normal B_g as kB_g can then be inserted instead of B'. The values of k will depend upon the ratios w_3/t_1 and w_3/l_g as a very complex function, but the actual flux-distribution curves obtained by Dr. T. F. Wall[3] have been analysed by Messrs. Adams, Lanier, Pope, and Schooley,[4] and for a particular value of w_3/t_1, viz. 0·5, which is not greatly different from average practice, the value of k was thus determined for different values of w_3/l_g. The curve of k^2 in relation w_3/l_g so obtained is in its lower part up to $w_3/l_g = 6$ fairly well expressed as $k^2 = (w_3/l_g)^{1\cdot6} \times 10^{-2}$, and experiment approximately confirmed this. For larger values of w_3/t_1, k^2 will be larger, for smaller values smaller. The above average expression for k^2 may be checked in another way, viz. by means of an expression for $\frac{1}{2}(B_{max} - B_{min})$, from which must afterwards be again derived an approximation for B'. According to the formula of Mr. Carter,[5]

$$B_{max} - B_{min} = \mathbf{B}_{g\ max}\left\{1 - \frac{1}{\sqrt{(1 + w_3^2/4l_g^2)}}\right\}$$

so that

$$\frac{B_{max} - B_{min}}{2} = KB_{g\ max}\left(\frac{1}{2} - \frac{1}{\sqrt{4 + \left(\frac{w_3}{l_g}\right)^2}}\right)$$

[1] The same formula in a different form and its establishment are given by Mr. F. W. Carter, "Pole-face Losses," *Journ. I.E.E.*, Vol. 54, p. 170, eq. (20).

[2] *Electr.*, Vol. 57, p. 568; and *Journ. I.E.E.*, Vol. 40, p. 577.

[3] *Journ. I.E.E.*, Vol. 40, p. 555 ff.

[4] *Trans. Amer. I.E.E.*, Vol. 28, Part II, p. 1133.

[5] *Journ. I.E.E.* Vol. 34, p. 49; and Vol. 54, p. 168, *q.v.*

where K has the same meaning as in Chapter XVI, § 7. If a line be drawn from the edge of a tooth to a point on the pole-face opposite the centre of a slot, and if this line makes an angle α with the surface of the pole-face, the same result may be more simply expressed[1] as $\frac{B_{min}}{B_{max}} = \sin \alpha$. This agrees very well with the experimental values which can be deduced from the curves of Dr. T. F. Wall and Mr. Matthews,[2] and in normal cases gives figures of about 0·2 B_g on an average. If the curves of flux distribution on the pole-face were strictly sinusoidal, $\frac{1}{2}(B_{max} - B_{min})$, as above calculated, could be at once substituted for B', and when the ratio of slot opening to air-gap is not very large or very small, the curves of flux distribution are in fact more or less sinusoidal, especially if the slots are slightly overhung.

Fig. 403[3] indicates (*a*) how with a long air-gap (1·125 in.) the density variation gradually dies away as the unslotted pole-face is approached, and (*b*) how for the same air-gap ($\frac{1}{2}$ in.) as the excitation is increased the depth of the slot-depressions, *i.e.* the value of $B_{max} - B_{min}$, increases.

Between the limits of $w_3/l_g = 2·5$ and $= 5$, and with $w_3/t_1 = 0·5$, or $w_3 = w_{t1}$ the values deduced from the above formula for $\left(\frac{B_{max} - B_{min}}{2}\right)^2$ agree closely with the expression $B_{g\,max}^2 \left(\frac{w_3}{l_g}\right)^{1·5} \times 10^{-2}$, and the flux curves are nearly sinusoidal about a value for $w_3/l_g =$ about 3. The same formula may therefore be tentatively used to discover k^2 for other values of w_3/t_1 but with the proviso that $\left(\frac{B_{max} - B_{min}}{2}\right)^2$ for values of w_3/l_g lower than 2·5 is confessedly too low and must be gradually increased until it is doubled for $w_3/l_g = 1$. In the opposite direction when w_3/l_g is large, the flux curve is flatter above each tooth with pointed depressions between the teeth, so that $\frac{1}{2}(B_{max} - B_{min})$ is considerably greater than the arithmetical average value of the pulsating flux. Yet the shape of the curve yields such harmonics that its form factor is very high; in other words, the virtual value is even increased, and the amplitude of the equivalent sinusoidal wave B' is actually again higher than $\frac{1}{2}(B_{max} - B_{min})$. The value $\{\frac{1}{2}(B_{max} - B_{min})\}^2$ must therefore again be increased when $w_3/l_g > 5$ to obtain $k^2B_g^2$, although not so much as in the opposite direction.

When w_3/t_1 is small, the same effect takes place. For a given value of w_3/l_g, as the tooth-pitch t_1 is increased, the zones of iron through which the currents flow past the slots become small as compared with the width of the teeth over which the induction is practically uniform, so that the eddy-current loss over the pole face as a whole might be but small. The formula takes this into account, since for a given armature on increasing t_1 although the product of t_1^2 with K^2 itself increases, yet the number of slots and frequency are reduced. The loss is, however, probably then underestimated, so that when $t_1 > 2w_3$, it is perhaps better to assume the flux-curve to be made up of depressions opposite each slot in length equal to $2w_3$ with the crests joined by straight lines of length $(t_1 - 2w_3)$ over which $B_{g\,max}$ holds. We then have to consider only $\frac{2w_3}{t_1}$ of the total pole-face, but the frequency is increased to $S \cdot \frac{N}{60} \cdot \frac{t_1}{2w_3}$, and in place of the wave-length t_1 we have the shortened wave-

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1 F. M. Roeterink, *Archiv für Elektrotechnik*, Vol. 7, p. 305.

2 *Journ. I.E.E.*, Vol. 40, p. 572.

3 Based on Figs. 10, 6, and 8 of the paper by F. S. Dellenbaugh, Jr., on "A Direct Recording Method of Measuring Magnetic Flux Distribution" (*Journ. Amer. I.E.E.* (June, 1920), Vol. 39, p. 583). The density in the air-gap was measured by a rapidly rotated long thin search coil or armature 0·22 in. diameter with 4 slots and 4-part commutator, which could be inserted in the air-gap. Owing to the circumstances of the test the flux as experimentally determined was not quite symmetrical about the centre of the pole; the want of symmetry has therefore been empirically corrected in Fig. 403.

length $2w_3$. The expression for the loss per sq. cm. of the *total* pole-face then becomes[1] when t_1 exceeds $2w_3$

$$4\left(S\frac{N}{60}2w_3\right)^{1\cdot5}(B')^2\sqrt{\frac{t_1}{R\mu}}\times 10^{-9}$$

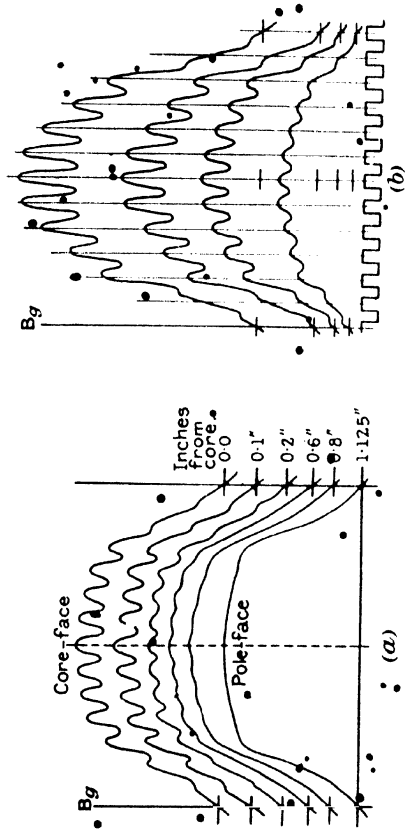

FIG. 403.—Variation of flux-density curve.

Returning to the average case of $w_3/t_1 = 0{\cdot}5$, or $t_1 = 2w_3$, we now have

$$4\left(S\frac{N}{60}\right)^{1\cdot5}B_{g\,max}^2\left(\frac{w_3}{l_g}\right)^{1\cdot85}\frac{t_1^2}{\sqrt{R\mu}}\times 10^{-11}\text{ watts per sq. cm.}$$

This expression in contrast to the expressions first obtained gives actually too low values for the loss with solid pole pieces; broadly speaking, its form is probably based upon correct principles, but the constant must be raised

[1] *Cp.* O. S. Bragstad and A. Fraenckel, *Electr.*, Vol. 62, p. 967.

to as much as 10 if the values given below for R are used, so that for solid pole-pieces we finally have

$$10\left(S\frac{N}{60}\right)^{1\cdot5} B_g{}^2{}_{max}\left(\frac{w_3}{l_g}\right)^{1\cdot85}\frac{t_1{}^2}{\sqrt{R\mu}}\times 10^{-11}$$

$$=\left(S\frac{N}{60}\right)^{1\cdot5}\left(\frac{B_{g\,max}}{1000}\right)^2\left(\frac{w_3}{l_g}\right)^{1\cdot85}\frac{t_1{}^2}{\sqrt{R\mu}}\times 10^{-4}\text{ watts per sq. cm.}\quad . \quad (215)$$

The experimental results which have been published have not contained any measurements of R and μ, although the proportionality or otherwise to $\frac{1}{\sqrt{R\mu}}$ could be more easily checked than the other factors.

There is no reason to suppose that the true loss by eddies in the pole-faces increases faster than the square of the flux-density if the same permeability could be maintained. The higher powers of 2·1 and 2·2 obtained experimentally by Messrs. Wall and Smith are to be ascribed to the variation of μ, while the still higher power of 2·5 obtained experimentally by Messrs. Adams, Lanier, Pope, and Schooley was probably in part due to increase in eddy-currents in the conductors within the slots at the higher densities.

Comparison between the two formulæ given in § 28 shows that the watts in the laminated pole should stand to those of the solid pole[1] as $\frac{1\cdot645\,t^2}{\rho^1}$ to $\frac{1}{2}\frac{t_1{}^2}{\rho}$, *i.e.* the watts of the solid pole must for the same material be multiplied by a reducing factor $\frac{\pi^2}{3}\frac{t^2}{t_1{}^2} = 3\cdot29\frac{t^2}{t_1{}^2}$, and the specific resistance of the laminated pole is as if it were increased to a value $\frac{t_1{}^2}{3\cdot29t^2}$ times its real value. But if the inverse proportionality to the square root of the specific resistance is adopted, the reducing factor for the same material must itself be taken as

$$\sqrt{\frac{3\cdot29t^2}{t_1{}^2}}\text{ or }1\cdot81\frac{t}{t_1}.$$

Experiment, however, clearly shows that even this lower value magnifies far too much the effect of laminating. But, as already mentioned, the pulsating flux penetrates more deeply into the laminated pole, and the screening currents therein have also the secondary effect of increasing the hysteresis loss,[2] to which causes the reduced effectiveness of the laminations is probably to be attributed. A better practical approximation is therefore given by the purely empirical factor $1\cdot6\sqrt{\frac{t}{t_1}}$. A further effect of lamination appears to be a reduction in the exponent of the ratio w_3/l_g, so that according to the experiments of the American investigators it becomes about $(w_3/l_g)^{1\cdot5}$ for laminations 0·152 cm. thick and $(w_3/l_g)^{1\cdot22}$ if 0·0356 cm. thick.

Combining these results with equation (215), the effect of the tooth-pitch does not entirely disappear in the laminated pole, and we have *e.g.* for discs 0·0356 cm. thick

$$1\cdot6\left(S\frac{N}{60}\right)^{1\cdot5}\left(\frac{w_3}{l_g}\right)^{1\cdot22}\left(\frac{B_{g\,max}}{1000}\right)^2\frac{t_1{}^{1\cdot5}}{\sqrt{R\mu}}\times\sqrt{0\cdot0356}\times 10^{-4}\text{ watts per sq. cm.}$$

$$=0\cdot2\left(S\frac{N}{60}\right)^{1\cdot5}\left(\frac{w_3}{l_g}\right)^{1\cdot22}\left(\frac{B_{g\,max}}{1000}\right)^2\frac{t_1{}^{1\cdot5}}{\sqrt{R\mu}}\times 10^{-4}\quad ,,\quad ,,\quad ,,$$

[1] The same ratio also results from the expressions given by Mr. Carter (*Journ. I.E.E.*, Vol. 54, p. 170), viz., that the ratio of the watts per sq. cm. with laminated pole-shoes to those with solid poles is as $\frac{\pi}{24}\frac{t^2}{t_1{}^{3/2}} : \frac{1}{8\pi}t_1{}^{\frac{1}{2}}$

[2] See F. W. Carter, p. 170, *loc. cit.*

In the same way are calculated the other expressions given in the following Table.[1]

Thus the expression here adopted for the loss by eddy-currents in *solid* pole-pieces is

$$\left(S\frac{N}{60}\right)^{1\cdot5}\left(\frac{B_{g\,max}}{1000}\right)^{2}\left(\frac{w_s}{l_g}\right)^{1\cdot85}\frac{t_1^{2}}{\sqrt{R\mu}}\times 10^{-4}\text{ watts per sq. cm. of pole-face} \quad . \quad . \quad . \quad (215)$$

where t_1 is the tooth-pitch in cm., and R is the resistivity of a centimetre cube of the iron or steel in absolute electro magnetic units, *i.e.* $R = \rho \times 10^9$ where ρ is the resistivity in ohms. For wrought-iron, cast-steel, and cast-iron, R may be taken respectively as 1×10^4, 2×10^4 and 10×10^4. The loss increases very considerably when the ratio w_s/l_g exceeds 2, and the usage of practice which calls for laminated poles when this value of the ratio is exceeded is entirely justified by experiment.

With *laminated* poles where the thickness t of the laminations comes into the question,

Thickness of lamination.		*Watts per sq. cm. of pole face.*
$\frac{1}{12}$th in. = 0·212 cm.	.	$7\cdot2\left(S\,.\,\frac{N}{60}\right)^{1\cdot5}\left(\frac{B_{g\,max}}{1000}\right)^{2}\left(\frac{w_s}{l_g}\right)^{1\cdot6}\frac{t_1^{\,1\cdot5}}{\sqrt{R\mu}}\times 10^{-6}$
0·06 in. = 0·152 ,,	.	3·7 ,, ,, $\left(\frac{w_s}{l_g}\right)^{1\cdot5}$,, ,,
$\frac{1}{40}$th in. = 0·0635 ,,	.	0·64 ,, ,, $\left(\frac{w_s}{l_g}\right)^{1\cdot3}$,, ,,
0·014 in. = 0·0356 ,,	.	0·2 ,, ,, $\left(\frac{w_s}{l_g}\right)^{1\cdot22}$,, ,,

where t_1 is in cm. For ordinary armature sheet steel, $R = 1\cdot5 \times 10^4$, and for high-resistivity alloyed sheets such as Stalloy, $R = 5 \times 10^4$ to 6×10^4.

In reality calculation in the case of laminated poles can do little more than suggest reasonable figures obtained in good practice, owing to the indeterminate nature of the additional loss due to contact between the edges of the laminations. The possible effect of this is forcibly illustrated by an experiment of Messrs. Adams, Lanier, Pope, and Schooley in which a loosening of the bolts compressing the laminations together reduced the loss to 70 per cent. of its previous value.

In every case when the field is distorted by armature reaction under full-load, the eddy-current loss is increased, not only in the armature but also in the pole-pieces; the amplitude of the variation of the field is increased in the case of a dynamo at the trailing edge, and decreased at the leading corner, but as the loss is proportional to the square of the amplitude, there must be a net increase in the loss. With a short air-gap and wide open slots the effect is so great that the increased heating of the trailing corners is distinctly marked. Messrs. T. F. Wall and S. P. Smith[2] found an increase in the eddy-currents in the pole-pieces of as much as 50 per cent. between no-load and full armature current.

§ 30. Eddy-currents due to flux-pulsation.—A further difficulty in the scientific calculation of the loss by eddy-currents lies in the fact that the measured results of experiment include not only the eddies set up by the waves of flux sweeping over the pole-face (which are alone considered in the formulæ) but also those due to "flux-pulsation" *i.e.* a pulsation in the magnitude of the total flux throughout the entire magnetic circuit which may also be set up by the teeth of a slotted armature.

With a small air-gap, *i.e.* with a large ratio w_s/l_g, as the teeth and slots occupy different positions relatively to the pole-faces at different times in

[1] *Cp.* also B. G. Lamme, *Trans. Amer. I.E.E.*, Vol. 35, p. 277, where another empirical formula is given for laminations 0·079 cm. thick.

[2] *Journ. I.E.E.*, Vol. 40, pp. 579 and 593.

the period corresponding to the passage of one tooth-pitch, the total permeance of the air-gap may vary with corresponding effect on the value of the total flux.

When the polar arc is an exact multiple of the tooth-pitch (case i, *cp.* Fig. 482 of Chapter XXVI, which shows the relative positions at 4 instants during the period corresponding to one tooth-cycle), there are always at any time the same number of teeth immediately under the polar arc, and also the same number of slots immediately under the polar arc (6 out of 9 in the diagram). The change of the permeance is then mainly due to variation of the permeance presented to the fringe in the interpolar gap between the pole-tips as opposed to the permeance well under the pole-faces which remains constant. When the polar arc is equal to (a whole number $x + \frac{1}{2}$) tooth-pitches (case (ii)) the number of teeth under the polar arc varies from $x + 1$ to x. From this fact it might be inferred that flux-pulsation would be a maximum in case (ii). But further inspection shows that in case (ii) when the number of teeth under the polar arc diminishes from $x + 1$ to x, the number of slots contrariwise increases from x to $x + 1$, and in addition two teeth are added immediately outside the polar edges. The result is to equalize the total permeance more nearly than might at first be expected. On the other hand, in case (i), at the ends of a pole face there are at the beginning of the cycle 2 teeth and at the beginning of the second half of the cycle 2 slots, balanced, it is true, by the nearer approach of the outer teeth to the pole-tips. Illustrative cases will be found worked out in Chapter XXVI. Careful calculation of the total permeance with accurate allowance for fringing and slots fails to indicate with certainty any marked difference as holding generally between cases (i) and (ii) with different proportions of slot-width to tooth-width. In either case and with different degrees of chamfering of the pole-tips the variation with practical dimensions is only of the order of 1 or 2 per cent. Indeed, experiment seems to show that, if there is any general law, flux-pulsation throughout yoke and pole is more likely to be a maximum in case (i) when the polar arc is an exact multiple of the tooth pitch.[1]

It only remains to add that even when the flux-pulsation throughout yoke and pole is a minimum, as the lines alternately spring further out from the edges of the pole-shoes and then contract inwards again, there may be an appreciable flux-pulsation in the face layers of the pole-shoe without pulsation further along the pole or through the yoke, and thence may arise reluctance-pulsation loss in the skin of the pole face, though not in the magnetic circuit as a whole.

Prof. W. M. Thornton has shown that there are also distinct pulsations produced in the field by commutation in a direct-current machine[2]; their frequency corresponds to the speed and number of commutator sectors, and their amplitude is increased by any causes which assist in producing sparking, as by commutation in an unduly strong field when the short-circuit current set up in the loops under commutation powerfully affects by its magnetizing ampere-turns the value of the main flux. An additional cause of pulsation in the value of the flux throughout the entire magnetic circuit is found in the case of an armature core which is not truly cylindrical, or of which the shaft shows a tendency towards whirling; in the former case the eccentricity of the armature causes a double frequency pulsation of the magnetic flux in the main magnetic circuit.[3] But Mr. M. B. Field has shown that in such cases of pulsation throughout the entire magnetic circuit the loss by eddy-currents is not likely to be large,[4] even in a solid pole or yoke; the magnetic effect does not extend to any great depth from the surface of the iron, so that only a thin skin is affected, and the loss is not proportional to the volume and to the square of the frequency, but rather to the area of surface acted upon and to the square root of the frequency.

[1] See G. W. Worrall, *Journ. I.E.E.*, Vol. 39, p. 217; and Vol. 40, p. 413.

[2] *Journ. I.E.E.*, Vol. 33, pp. 547 and 556.

[3] *Journ. I.E.E.*, Vol. 32, pp. 596 and 599; and Vol. 33, p. 547.

[4] *Electrician*, Vol. 52, pp. 598 and 704. *Cp.* also *Journ. I.E.E.*, Vol. 33, pp. 584 and 568, and especially p. 1125.

CHAPTER XXII

THE DESIGN OF CONTINUOUS-CURRENT DYNAMOS

§ 1. Range of speeds in practical use.—The practical art of designing is a matter of striking a balance between a variety of conflicting considerations, all of which are of importance in different degrees, and each of which will vitally affect the entire design of the machine. Thus a dynamo must be efficient, yet at the same time it must not be too costly to manufacture; it must be compact, yet well ventilated; thoroughly strong, yet not too heavy. Any one feature, however desirable in itself, will, if carried to excess, have some disadvantageous consequence in another direction, and he is the best designer who can effect a series of compromises such that, while each consideration is given its proper weight, none are forced into undue prominence, and a design well-balanced as a whole results from his practised judgment.

In the majority of cases, for a given output, the speed of the dynamo may be taken as fixed: either it is directly specified, or it is to a great extent settled by recognized practice or questions of mechanical strength and durability. Thus, to take the case of an ordinary continuous-current dynamo, it may in small sizes be driven by belt or ropes, but more usually in all sizes it is directly coupled to the prime mover. In the latter case the prime mover may be either a steam engine, steam turbine, or less frequently a water turbine, oil or gas engine. When driven directly by a steam engine, the speed of the dynamo may be classified as high or low, according as the engine is of the enclosed type with forced lubrication, or of the open type used for marine or mill work.

Directly-coupled steam turbines in the largest sizes are confined to alternators, but for continuous-current outputs have been used up to 500 kW and occasionally up to 1000 or even 2000 kW if the voltage is not less than 500–600. They have, however, so largely been replaced by steam turbine sets in which the dynamo is driven, still at a high speed, through reduction gearing, that only the latter combination is here tabulated. With modern helical reduction gearing the most economical speeds of both the driving and driven unit can be combined with but little loss in efficiency and a saving in total cost. The speeds of dynamos driven directly by large Diesel engines may be taken approximately as being somewhat higher than those given for open engines in Table XVI. Thus, although the speeds selected by makers for different outputs vary considerably, yet the measure of agreement is sufficient to enable a table to be

drawn up indicating average values of the speeds for the different classes of prime movers.

TABLE XVI

Output in kilowatts.	Speed in revs. per min. of continuous-current generators. Engine-driven: Open.		Enclosed.	Larger sizes geared to steam turbine. Small sizes belt-driven.	Revs. per min. of steam turbine.
10	300		750	1500	
20	250		650	1200	
50	300	Two-crank	600	1000	7000
100	250		475	850	6000
200	200		400	750	6000
300	150	Three-crank	350	650	6000
500	120		300	600 (500 volts) 500 (250 ,,)	5000
1000	90		250	550 (500 ,,) 450 (250 ,,)	4000
1500	85		200	500 (500 ,,) 350 (250 ,,)	3000

Standard voltages for continuous-current dynamos are 115, 230, 460, and 525.

§ 2. Determination of necessary D^2L of armature.—It has earlier been stated that the utility, cost, and leading dimensions of a machine, whether generator or motor, are in the main determined by the torque which it has to absorb or develop. If, therefore, the torque is made the basis of the design for the purpose of settling approximately the diameter and length of the rotating member, the same method of procedure can, and should, be employed for all classes of rotating machinery.

The useful torque in terms of the special unit of "watts per rev. per min." (Chapter IV, § 2) is quickly obtained from the useful electrical output of the generator, or useful mechanical output of the motor, both expressed in watts and divided by the number of revs. per min. But to obtain the quantity corresponding to the total induced watts of the armature, there must in the case of the generator be added to the terminal volts the volts lost over the resistances of armature, brushes, series field winding and commutating-pole winding, and to the external current there must be added the shunt current; in the case of the motor either these quantities must be subtracted from the volts and amperes of the electrical input, if this is given as a datum, or the same result is reached by taking the brake horse-power divided by the mechanical

efficiency (including in this also an allowance for hysteresis and eddy-currents, both of which will make a call upon the armature conductors for increased torque). In each case for a given armature current, the rate of conversion of energy from a mechanical to an electrical form or *vice versa* is proportional to the induced E.M.F.

In the continuous-current machine, when shunt-wound, the loss of volts over the resistance of armature, brushes, and commutating-pole winding decreases from about 10 per cent. of the terminal voltage in small low-speed machines to 3 per cent. in large machines, and averages about 4 per cent., while at the same time the shunt current decreases similarly from 8 to 2 per cent. of the external current I_e and averages about 3 per cent. Hence $1{\cdot}04\,V_e \times 1{\cdot}03\,I_e = 1{\cdot}07\,V_e I_e = E_a\,I_a$. In the compound-wound machine the additional loss of volts over the series winding counterbalances the reduction in the shunt current, so that in general in the continuous-current generator the rate of development of electrical energy or the induced watts = approximately[1] $1{\cdot}07\,V_e I_e = E_a I_a$. In the case of the motor, the rate of conversion of electrical into mechanical energy is $\dfrac{B.H.P. \times 746}{\eta_m} = E_a I_a$, where η_m is something less than the true mechanical efficiency for the reason mentioned above, but exceeds the over-all or net efficiency which also includes true electrical losses. Now a second expression for the total torque, $E_a I_a/N$ in watts per rev. per min., has already been found in equations (5*e*) and (4*a*) of Chapter IV, § 7. Hence the total torque

$$T = \frac{E_a I_a}{N} = k_f \, . \, k_d \, k \, B_{g\,max} \, JZ \, \frac{\pi}{60} \, DL \times 10^{-8}$$

$$= k \, B_{g\,max} \, JZ \, \frac{\pi}{60} \, DL \times 10^{-8}$$

when, as is permissible in the continuous-current machine, both k_f and k_d are identified with unity.

Now for JZ may be substituted $ac.\,\pi D$, where ac = the ampere-conductors per unit length of the armature periphery, or per cm. when D and L are reckoned in cm., so that

$$\frac{E_a I_a}{N} = k \frac{\pi^2}{60} B_{g\,max} \, . \, ac \, . \, D^2 L \times 10^{-8} \qquad . \quad . \quad . \quad (216)$$

In the continuous-current machine, k is practically $= \beta$, the ratio of pole-arc to pole-pitch, and is a constant, say with commutating poles $= 0{\cdot}675$. Thence, $k\dfrac{\pi^2}{60} = 0{\cdot}111$, and the possible torque

[1] Only average figures covering a wide range are here given, so as to illustrate the process of designing *ab initio* from first principles.

from a machine of given D^2L becomes proportional to the product of the two terms,[1] $B_{g\,max}$ and ac, the one magnetic and the other electric, or

$$\frac{E_a I_a}{N} = 0{\cdot}111\, B_{g\,max} \,.\, ac \,.\, D^2L \times 10^{-8} \quad . \quad . \quad . \quad . \quad (217)$$

$$= G\, D^2L \text{ watts per rev. per min.}$$

and G is the "specific torque coefficient"

$$= \frac{E_a I_a}{D^2LN} = 0{\cdot}111\, B_{g\,max} \,.\, ac \times 10^{-8} \quad . \quad . \quad . \quad . \quad (218)$$

If D and L are measured in inches and $ac_{''}$ is given in ampere-conductors per inch length of armature periphery, while $B_{g\,max}$ is retained in C.G.S. lines per sq. cm., approximately we have $G = 0{\cdot}72\, B_{g\,max} \,.\, ac_{''} \times 10^{-8}$. But if $B_g{''}_{max}$ is also expressed in lines per sq. inch, we return to equation (218).

§ 3. The "specific torque coefficient" and its reciprocal.—Thus G is the specific torque (in terms of "watts per rev. per min.") that can be obtained per cubic cm. or per cubic inch (not of the true volume of the armature, but) of the product of the square of the armature diameter and the axial length of its core. Other forms for the same quantity may often be found, employing $B_{g\,av}$ or revs. per sec., or useful output instead of total induced watts, but all come back in the end to the same fundamental relation. The reciprocal of the specific torque coefficient, or $\frac{1}{G} = \frac{D^2LN}{\text{watts}}$, is also often used, and may be described as the "size coefficient" of a machine in relation to its specific torque, since it is the number of cubic cm. or cubic ins. that the D^2L of the armature must give for each watt per rev. per min. For accurate comparison of machines when, as in a turbo-alternator, the air-gap may be large, the diameter to the centre of the air-gap should be considered as one of the two crucial dimensions, but practically, in continuous-current machines, it suffices to take immediately the diameter of the toothed armature.

§ 4. The importance of D^2L.—Every armature core, therefore, of given diameter and length is to be regarded as being able to develop a certain torque, either resisting or driving, which in the former or generator case must be overcome by the prime mover and in the latter case will drive it as a motor. The values of B_g and ac will be pushed up to the maxima that experience has shown to be advisable for its diameter from considerations of heating, sparking, tooth saturation, and reasonable cost of magnet iron and field copper. But this done, there is a certain specific torque

[1] Sometimes referred to as "the specific magnetic and electric loadings" of a machine.

procurable from each cubic cm. or inch of its D^2L. The importance of this quantity simply arises from the facts that with the assumed values of $B_{g\ max}$ and ac

$$\text{total flux} \propto \pi DL$$
$$\text{total ampere-conductors} \propto \pi D.$$

The torque, being proportional to the product, flux × ampere-conductors, is therefore proportional to $\pi^2 D^2L$, so long as the same limiting values of $B_{g\ max}$ and ac hold good. It will be noted that there is no reference to the number of poles, the nature of the armature winding, the voltage, or the speed, all of which only introduce secondary effects that do not affect the validity of the relation

$$\text{watts per rev. per min.} \propto D^2L$$

as a preliminary generalization for all machines. If by an increase in the diameter the number of ampere-conductors per pole exceeds the permissible limit or the pole-pitch becomes too great, it is assumed that the number of poles is correspondingly increased.

§ 5. The specific values of $B_{g\ max}$ and ac.—The specific values of $B_{g\ max}$ and of ac, as already explained in Chapters XIII, § 39, and XIX, § 16, each tend towards a certain maximum value that cannot be exceeded even in large machines, and each falls off greatly in machines of small diameter, but over any moderate range of output may be treated as constant. In the toothed armature of the continuous-current machine, internal to the poles, the flux-density at the roots of the highly tapered teeth in small machines limits the possible value of $B_{g\ max}$, since for each value of B_g there must be a corresponding width of tooth to carry the flux of a tooth-pitch. Further, for the same reason, there is a limit to the possible increase in the depth of the slots to receive more ampere-conductors or to increase their area of copper and so reduce their heating. The two values of $B_{g\ max}$ and ac thus become in the smaller machines mutually dependent, and one can only be increased at the expense of the other.[1]

In place of the specific quantities $B_{g\ max}$ and ac, we may alternatively consider $kB_{g\ max} \cdot \pi DL = 2p \cdot \Phi_a$ or the total flux, and $ac \cdot \pi D =$ the total ampere-conductors. When curves are plotted from many actual machines for each of these quantities in relation to watts per rev. per min., they are found to possess a very considerable degree of uniformity.[2] In practice, the total flux and the total ampere-conductors both increase more or less similarly, as might naturally be expected, i.e. each increases as the square root of the torque. When plotted as ordinates against

[1] See J. C. Macfarlane and H. Burge on "Output and Economy Limits of Dynamo-electric Machinery," *Journ. I.E.E.*, Vol. 49, p. 235.

[2] For such curves, see R. Goldschmidt, *Journ. I.E.E.*, Vol. 40, p. 457.

the machine torque as abscissa, each curve would then be a parabola with its axis on the abscissa axis,[1] and in fact such is roughly their shape. By the use of such curves a provisional settlement of Φ_a and of $Z = ac \cdot \pi D/J$ could be made, but the values of $B_{g\ max}$ and ac are even more reliable guides at the outset.

Approximately the average limits of these in centimetre units may be given as, say,

Armature diam.	$B_{g\ max}$	$\times\ ac$
5″	5550	$\times 180 = 1 \quad \times 10^6$
50″ or over	9500	$\times 330 = 3{\cdot}14 \times 10^6$

Or with $B_{g\ max}$ retained in C.G.S. lines per sq. cm. and $ac_{''}$ reckoned per inch of periphery, the values of the two limits are $2{\cdot}54 \times 10^6$ and 8×10^6. If $B_g''_{max}$ is itself in C.G.S. lines per sq. inch, the values are $16{\cdot}4 \times 10^6$ and $51{\cdot}5 \times 10^6$.

§ 6. The value of G.—The corresponding values of G are then in centimetre units $1{\cdot}11 \times 10^{-3}$ and $3{\cdot}5 \times 10^{-3}$ and with $ac_{''}$ per inch, $1{\cdot}83 \times 10^{-2}$ and $5{\cdot}75 \times 10^{-2}$. Thence

the watts per rev. per min. = from 0·00111 to 0·0035 per cm.^{3} of D^2L
= from 0·0183 to 0·0575 per cubic inch of $D_{''}^2L_{''}$

Or conversely

cubic cm. of D^2L = 900 to 285 per watt per rev. per min.
cubic ins. of $D^2_{''}L_{''}$ = 55 to 17.5 „ „ „ „ „ „

But though the value of G thus increases and that of its reciprocal diminishes as the size of dynamo increases owing to the better utilization of space possible in large machines, they quickly approach their maximum and minimum value respectively when the size of armature and the watts per rev. per min. rise from quite small values. An average value of 8500 for $B_{g\ max}$ and of 300 ampere-conductors per cm. of armature periphery thus holds for a large range of moderately large toothed armatures with commutating poles.[2]

With good design, under favourable conditions, such values as are shown in Fig. 404 should be reached, commutating poles being presupposed and thoroughly good ventilation in the larger machines. Component values for the separate factors in the product $B_{g\ max}\ ac_{''}$

[1] *Electr. World*, Vol. 51, p. 419.

[2] For values of the product $ac \cdot B_g$ in relation to D^2L, see the curves in the above-quoted paper of Messrs. J. C. Macfarlane and H. Burge. For average machines of large size, they give $ac \cdot B_g = 3{\cdot}2 \times 10^6$ in centimetre units, whence the dimensional torque = 0·058 watts per rev. per min., or with $ac_{''}$ reckoned per inch, = $8{\cdot}1 \times 10^6$, which may as an aid to memory be conveniently divided into $B_g = 9000$ and $ac = 900$ ampere-conductors per inch.

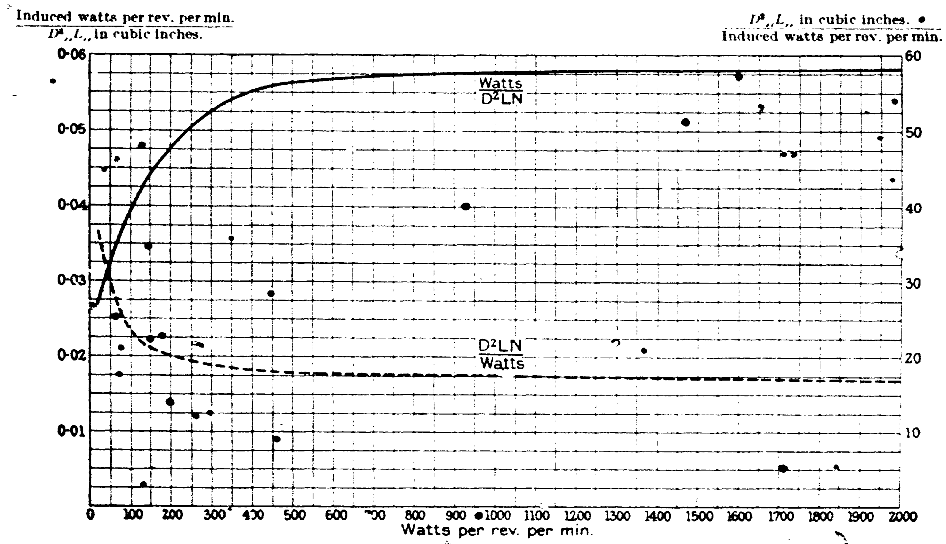

FIG. 404.—Specific torque coefficient and size coefficient of continuous-current dynamos

corresponding to Fig. 404 are indicated in Fig. 405.[1] Further, if experience suggests the taking of a lower or a higher value of G, the values for the ordinates on Fig. 405 for the same abscissa as on Fig. 404 serve to indicate suitable starting points for calculation.

§ 7. Division of the product D^2L.—Although by the use of such a curve as Fig. 404, an approximate estimate of the requisite D^2L for any given output and speed can quickly be obtained as a starting point for design, the division of the product into its two components, D and L, is not prescribed. To make this division it is necessary that the number of poles should be settled.

A limiting value of 300 amperes per brush arm will only be exceeded when other conditions render a long and expensive commutator imperatively necessary (Chapter XII, § 17). Adopting this value then as a normal maximum, the number of brush arms of one sign, or of pole-pairs will be $\dfrac{I_a}{300}$, and trial should first be made[2] with $p = I_a/300$, the next greater whole number being taken when the quotient of $I_a/300$ has a fairly large fractional remainder.

From considerations of an economical section for the pole, an average value for L in the continuous-current machine has already been laid down in equation (107) Chapter XV, § 17, as $L = 0{\cdot}75\,\pi D/2p$. Inserting this value

$$D^3 = \frac{2p}{0{\cdot}75\pi}(D^2L) = 0{\cdot}85p(D^2L) \quad . \quad . \quad . \quad . \quad (219)$$

and from the known value of D^2L, the diameter and thence the length are both immediately determined.

Thus with the minimum of labour a provisional starting point is obtained from first principles and without reference to other designs or machines previously built, but the dimensions will still require to be checked by simultaneous consideration of two important factors in their mutual relation. The first of these is the peripheral velocity, and the second the length of core. The former in the presence of a commutator should preferably not exceed 3500 to 4100 feet per min.; if therefore it works out too high, the core must be lengthened. If, on the other hand, it is low, and the length of core exceeds 15¾ in. or 40 cm. (Chapter XV, § 17), the diameter must be increased, and this increase, if considerable, should be accompanied by an increase in the number of poles.

But though the procedure as first described will have secured a more or less square section for the magnet pole-core—reasonably

[1] Under the favourable conditions assumed, $B_{g\,max}$ and ac_a, and especially the former, can be made to approach their limiting values more rapidly than was indicated in the more general cases of Figs. 203 and 331 in relation to armature diameter, and the present curves are therefore more square-cornered.

[2] Or to eliminate the 2-pole machine which for reasons explained in Chap. XII, § 17 is now seldom built, we might say $p = (I_a/400) + 1$.

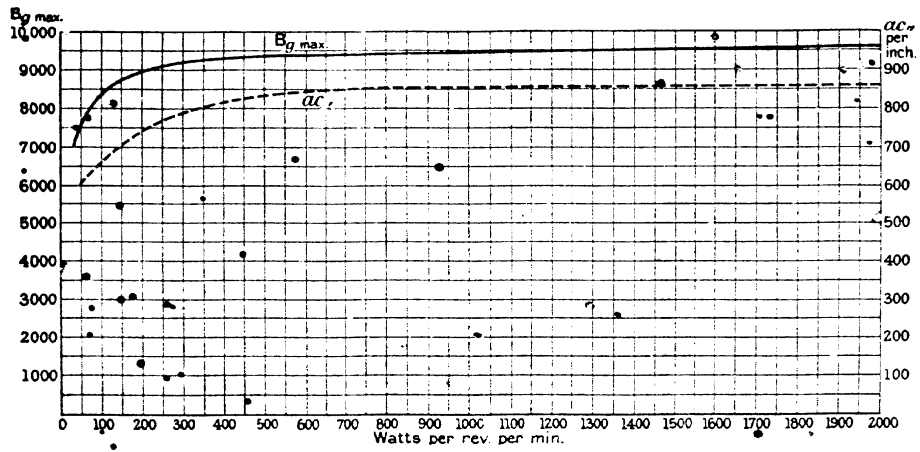

FIG. 405.—Values of $B_{g\ max}$ and ac relatively to torque in continuous-current dynamos.

economical in field copper—the number of poles obtained by it has been dictated by values of commutator length and of J, such as are not usually exceeded in practice. Further, the simplex parallel-connected lap armature winding with $a = p$, as recommended in Chapter XII, § 17 (1), has been mainly in view. Thus, although the division of D^2L which results may succeed at the first trial for large machines at fairly high speeds the number of poles will often require to be modified for other reasons, and will then in turn dictate the best armature winding to employ.

In the first place, as explained in Chapter XII, § 17, when I_a does not exceed about 300 amperes, the 4-pole magnet with its appropriate ratio of L to D will be adopted, and will probably lead to a simplex wave-winding. Next, apart from this very usual case, machines are often required in which with a comparatively small number of amperes the watts per rev. per min. and the D^2L are large owing to the speed being low. In such cases, the previous rule for p based on 300 amperes per brush arm will lead to less than the permissible minimum number of poles for the diameter, with too great a pole-pitch making the magnet unduly heavy and too great a length of armature core. Both must then be reduced by an increase in the number of poles and a consequent increase in the diameter (assuming this to be admissible), until such limiting values as 20 ins. for the pole-pitch and 15 ins. for the length of the armature core (Chapter XV, § 17, *end*) are reached. The values of J and of the commutator sector-pitch (Chapter XII, § 17 (3)) may then become too much reduced if the single-turn lap winding be retained; wave-winding will therefore be resorted to in preference to lap coils of two turns, and if the reduction of the number of pairs of armature paths from p to $a = 1$ is too drastic, an intermediate value is found by the employment of a multiplex wave-winding. The value of J can thus again be brought back to some average value between, say, 100–150 amperes, with an intermediate and reasonable length of commutator. Lastly, it may be that a magnet-frame of appropriate D^2L but with more poles than are necessarily required by the current per brush arm is alone available (Chapter XII, § 17 (*b*)); the division of D^2L is then prescribed to the designer, and the case is again met in a similar manner by the use of a multiplex wave-winding.

But in all cases when the final choice of the two dimensions for diameter and length of armature core remains open to the designer, considerable latitude may be permitted to the exercise of his judgment. The assumed values of $B_{g\ max}$ and ac, *i.e.* of G, must give the required product of D^2L, but within certain limits the relation between D and L may be varied without greatly affecting the cost or efficiency of the machine, although, as a general rule, as small a diameter as is practicable is to be preferred. Hence in the technical

office of a dynamo factory reference will be made to a list of standard armatures, or at least of standard diameters of core discs, from which armatures of different lengths may be built up. After a number of machines have been standardized, a table can be drawn out showing the maximum value which the torque expressed as $\frac{\text{watts}}{\text{revs. per min.}}$ has for each of the standard armatures in continuous working under average conditions of voltage and speed.

§ 8. Secondary considerations affecting the watts per rev. per min. for each size of armature core.—But whether in the use of any such table or in the value taken for G from such a curve as that of Fig. 404, allowance must be made for the disturbing effect of certain secondary conditions which prevent the watts per rev. per min. of any size of core from being strictly constant, when the same permissible rise of temperature is assumed. These are mainly the speed and the voltage. With regard to the first, it is evident that if the revolutions are exceptionally high, the eddy loss will assume such large proportions as to limit decisively the possible current that can be carried by a given winding on the armature. Although this is to a certain extent counterbalanced by the increased cooling due to the high peripheral speed, yet on the whole if the speed be high the watts per rev. per min. for a given D^2L or size of armature core must be slightly reduced below the normal value for medium speeds, or conversely with very low speeds may be high. But most important of all is the influence of the E.M.F. A high voltage implies a large number of active conductors, with an increased thickness of insulation, the percentage loss of space in insulation is therefore much greater than in low-voltage machines, and the watts per rev. per min. are reduced. If the amperage is small, round wires may be necessitated, by which the ratio of the copper to the available area is very largely reduced as compared with the same armature wound with rectangular bars. In a small multipolar toothed machine with round wires the ratio of the copper area to the area of the slot may sink to as low as 0·25. In a low-voltage machine, say, for 110 volts with rectangular bars, the ratio rises from 0·35 for small outputs at low speeds to 0·55 for large outputs; but here again, if the bars become very thick, it may be necessary to limit the number of bars per slot to two only in order that the width of opening may not be too great, when the ratio again sinks to 0·45. At 250 volts the ratio for normal speeds and outputs of 40 to 200 kilowatts ranges from 0·4 to 0·52; while at 500 volts it rises from 0·25 in very low-speed small machines with round wire to 0·3 in small machines of 50 to 80 kilowatts at moderate speeds, and to 0·5 in machines of 500 to 1500 kilowatts. The combination of a low speed and a high voltage is therefore unfavourable, and the loss may more

than counterbalance any gain derived from the first-mentioned cause, so that the value of G or of the watts per rev. per min. will be on the low side.

§ 9. Examples of design.—Good practical designs being now available to the reader in many excellent books,[1] it will only here be necessary to consider in detail the design of two dynamos, one a small machine with discs keyed directly to the shaft, the other a larger machine with hub construction,[2] for the purpose of illustrating the application of the principles and methods described in the preceding chapters.

The calculations are arranged schematically in the natural order which the designer can follow, so as to reduce to the minimum any need for cross-reference, checking, and subsequent correction. Provided that it is known that the chosen values of $B_{g\ max}$ and ac are such as are suitable for the given output, the armature winding can be completed together with the calculation of the ampere-turns for the magnetic circuit as far as the pole-faces. The heating of the armature can then be checked, leaving the completion of the calculation of the field ampere-turns and field winding to be subsequently resumed. Many of the calculations here carried through would not be necessary in the light of previous practical experience or data of previous designs: they are, however, added for the sake of completeness in our typical examples.

Lastly, although ordinary slide-rule accuracy is presumed in the calculations, it must be understood that the data and methods employed often do not warrant even so much accuracy. The reason for assuming such definiteness is that in a large measure when the results of one calculation are again used for a further calculation, the slide rule itself automatically takes care of the degree of accuracy that may reasonably be expected. When the last significant figures are omitted or rounded off, there is a great tendency for the omissions or rounding off to lead the designer almost unconsciously into a comparatively large cumulative error on one side or the other. Any margin that safety may require is far better added with the avowed purpose of securing a margin at the end of a rigid calculation rather than piecemeal at any intermediate stage.

§ 10. Design of dynamo for 55 kW at 400 revs. per min.—Suppose that it is required to design a shunt-wound dynamo with toothed drum armature and commutating poles to give 55 kW at 230 volts

[1] See especially Prof. Miles Walker, *The Specification and Design of Dynamo-electric Machinery*; and Dr. S. P. Smith *Notes on Theory and Design of Continuous-current Machines*, p. 36.

[2] Both based on drawings kindly supplied by Messrs. W. H. Allen, Sons & Co., Ltd., Bedford. But for the calculations here set forth, including outputs, coefficients employed, etc., the writer is solely responsible.

when directly coupled to a steam engine running at 400 revs. per min. Then proceeding from first principles[1]

Volts	230
Amperes	239
Kilowatts	55
Revs. per min.	400
Induced watts per rev. per min. $= \dfrac{1{\cdot}07 \times 55{,}000}{400}$ approx. $=$	147
$G = 0{\cdot}72\, B_{g\ max}\, ac_{,,} \times 10^{-8}$	0·043
$B_{g\ max}$ from Fig. 405	8650
$ac_{,,}$,, ,,	690
$D_{,,}^2 L_{,,}$ in cubic inches $= 147/0{\cdot}043 =$	3420
No. of pole-pairs $= (I_a/300)+1$, so by Chap. XXII, § 7, say,	2

$L = 0{\cdot}75\, \pi D/2p = 0{\cdot}589 D$

$D_{,,}^3 = 3420/0{\cdot}589 = 5800$

$D_{,,} = 18$ ins. $L_{,,} = 10{\cdot}6$ ins.

Peripheral velocity $= 1885$ ft. per min.

The peripheral velocity is low, so that it will be assumed that the nearest standard 4-pole frame has dimensions as under—

Diameter of armature	19 ins.
Over-all length of armature core	10 ins.

Armature Winding.

With simplex lap-winding, $Z = \dfrac{ac_{,,} \times \pi D_{,,}}{I_a/2p} = \dfrac{690 \times 59{\cdot}6}{243/4}$

$= 676,$

a provisional addition of 4 amperes being made to the external current to allow for the shunt current.

No. of commutator sectors, C,
with single-turn loops $= 338$

Commutator diameter, say, $0{\cdot}75 \times 19 = 14{\cdot}25$ ins.

Commutator sector-pitch, $\dfrac{3{\cdot}14 \times 14{\cdot}25}{338} = 0{\cdot}132$. This is less than the minimum permissible, so that 2-turn lap coils would be necessary with C = about 169. In preference to this, by Chap. XII, p. 254, will be chosen single-turn coils wave-connected, and $a = 1$.

With 6 conductors per slot, 3 abreast, ac per slot $= 6 \times 243/2 = 730$. Hence, by Fig. 373 c can be made 3. For $a = 1$, $C/p = cS/p$ must have a remainder of 1/2, since by equation (53).

$$y_c = \frac{cS}{p} \pm \frac{a}{p} = \frac{3S \pm 1}{2}$$

[1] Even though the watts per rev. per min. are less than in the 21″ × 11″ machine of Chap. XVI, § 9, the presence of the commutating poles and consequent shorter air-gap will enable the values of $B_{g\ max}$ and ac to be appreciably higher in the present machine.

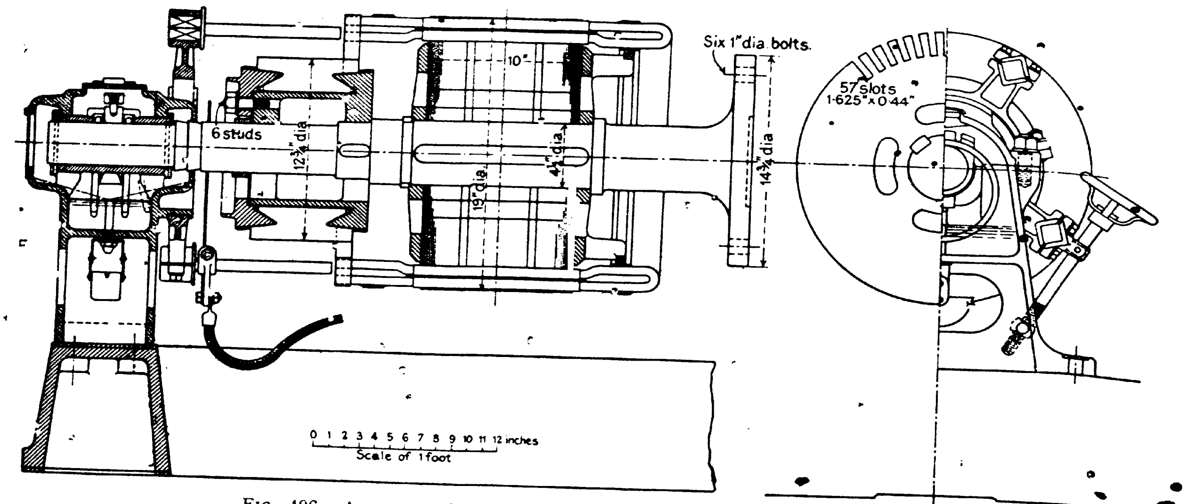

FIG. 406.—Armature of continuous-current dynamo, 19″ dia. × 10″ long.

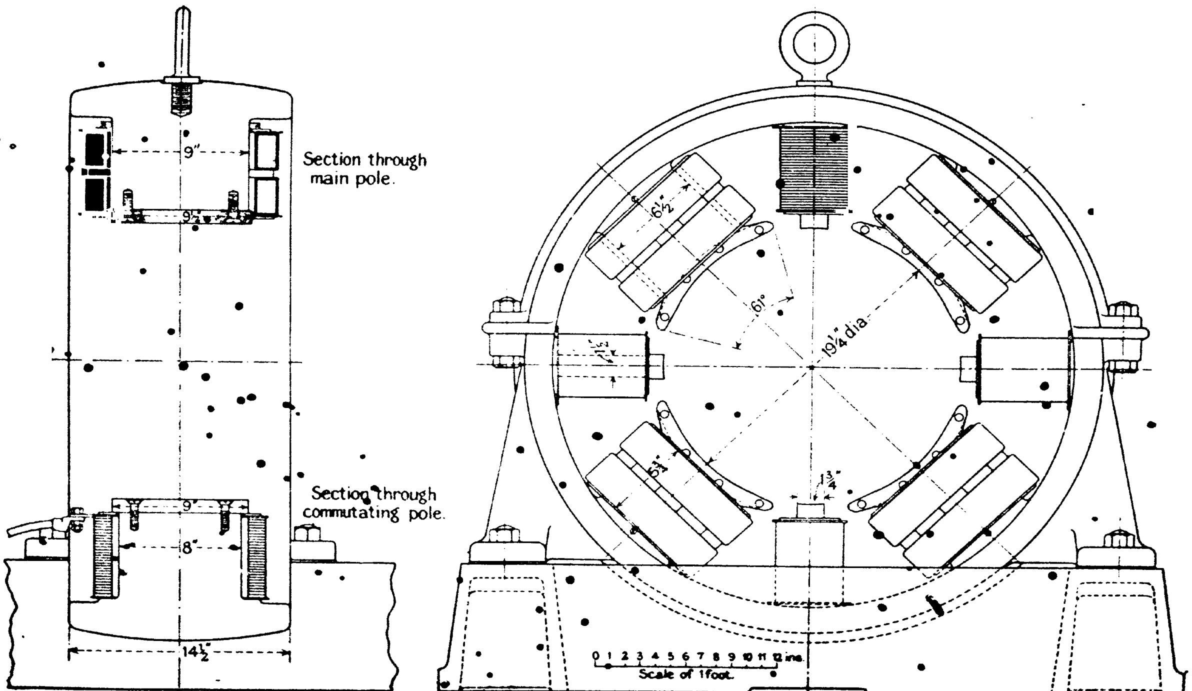

Fig. 407.—Four-pole field magnet of 19" × 10" dynamo.

16—(5665A)

must be a whole number; further, $3S$ must be about 169. The nearest number to 169 which is a multiple of 3 and leaves as remainder 1/2 is then $3 \times 57 = 171$. Hence

$y_c =$	85
$y_B =$	85
$y_F =$	85
$S =$	57
$C =$	171
$Z = U = 4y_c + 2 =$	342
$S/2p = 57/4 = 14{\cdot}25$, and $y_B{}^1 = (y_B - 1)/6 =$	14
Average volts per sector, $2p \cdot V_b/C = 4 \times 230/171 =$	5·4

Slot-pitch, $t_1 = (3{\cdot}14 \times 19)/57 = 1{\cdot}045$ ins.

Slot dimensions, 1·625 ins. deep $\times$ 0·44 in. wide.

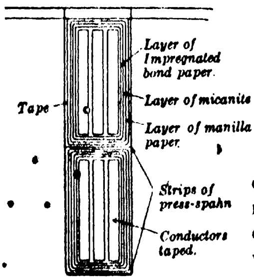

FIG. 408.—Section of slot.

	Pitch of teeth.	*Width of tooth.*
at top	1·045	0·605
at centre	0·957	0·517
at root	0·870	0·430

The voltage being low, out of three conductors abreast only the central one need be taped with a half lap, the other two within the slot being taped without overlapping; on the average, therefore, it will suffice to deduct 0·020 in., or, with some additional margin per bar, 0·025 in. in equation (81), and the same amount in equation (82). Hence—

Thickness of conductor by equation (81)

$$t = \tfrac{1}{3}(0{\cdot}44 - 0{\cdot}075) - 0{\cdot}025 = 0{\cdot}095 \text{ in.}$$

Binding wire sunk into shallow grooves 0·065 in. deep in the core, no wooden wedges; thin press-spahn strips, 0·02 in. and 0·01 in. thick. Total deduction from $h_s = 0{\cdot}160 + 0{\cdot}065 + 0{\cdot}03 = 0{\cdot}255$ in. Hence—

Height of conductor by equation (82)

$$h = \tfrac{1}{2}(1{\cdot}625 - 0{\cdot}255) - 0{\cdot}025 = 0{\cdot}66 \text{ in.}$$

Area $= 0{\cdot}66 \times 0{\cdot}095$ with slightly rounded edges

$=$ say, 0·0615 sq. in.

Resistance of 1000 yds. at 20° C. (68° F.) $= 0{\cdot}02445/0{\cdot}0615$
$= 0{\cdot}398$ ohm

Length of axial projection at either end by equation (83)

$$l_e = \frac{14(0{\cdot}87)\,(0{\cdot}45)}{2\sqrt{0{\cdot}87^2 - 0{\cdot}45^2}} + \frac{3}{8} + 1{\cdot}25 \times 0{\cdot}44 + \frac{1{\cdot}625}{2}$$

$$= 3{\cdot}68 + 1{\cdot}74 \quad = \quad 5\tfrac{7}{16} \text{ ins.}$$

$m_{l_e} = \frac{1}{2} \times 14\ (0{\cdot}957) = 6{\cdot}7$ ins.

Length of a complete end-connexion by equation (84)

$l' = 2(\sqrt{6{\cdot}7^2 + 3{\cdot}68^2} + 1{\cdot}74) \quad = 18{\cdot}78$ ins.

Length of a half loop $= 10 + 18{\cdot}78 = 28{\cdot}78$ ins. $= 0{\cdot}8$ yd.

R_a by equation (94) $= \frac{1}{4}\,(342 \times 0{\cdot}8 \times 0{\cdot}000398) = 0{\cdot}0272$ ohm

hot, with 39° C. rise $= 1{\cdot}16 \times 0{\cdot}0272 = 0{\cdot}0315$ ohm

$I_aR_a = 243 \times 0{\cdot}0315 \quad = 7{\cdot}66$ volts

Magnetic circuit and interpolar ampere-turns.

Effective radial depth of armature core below slots $3\frac{3}{8}$ ins.
Two ventilating ducts, each $\frac{1}{2}$ in. wide.

Double section of core, $2h_cL_i = 2 \times 3{\cdot}375 \times 0{\cdot}9(10 - 1)$
$= 54{\cdot}6$ sq. ins. $= 353$ sq. cm.

Pole-shoes opening out towards their tips.

Single air-gap, increasing from $\frac{1}{8}$ in. to $\frac{1}{4}$ in.,
mean length $l_g = \frac{3}{16}$ in. $= 0{\cdot}1875 = 0{\cdot}477$ cm.

Polar angle $= 61°$. Ratio to pole-pitch, $\beta = 0{\cdot}677$.
Polar arc at centre of gap, $A' = \frac{1}{4}\pi(19{\cdot}1875) \times 0{\cdot}677 = 10{\cdot}2$ ins
Pole-pitch on armature surface $= \frac{1}{4}\pi \times 19 = 14{\cdot}9$ ins.
Polar arc „ „ „ $= 10{\cdot}1$ „

	4·8 „
Commutating pole-face width	1·5 „
Two interpolar gaps	3·3

Half of one interpolar gap between main and commutating pole of opposite sign, $c = 0{\cdot}825$ in.

$c/l_g = 0{\cdot}825/0{\cdot}182 = 4{\cdot}53$. $\gamma = 100°$. By Fig. 253, $K_1 = 2{\cdot}2$.

Between main and commutating pole of the same sign, using Fig. 254, $K_1 = 2{\cdot}8$. The mean value for both strips may therefore be taken as $K_1 = \frac{1}{2}(2{\cdot}2 + 2{\cdot}8) = 2{\cdot}5$.

$A' + K_1l_g = 10{\cdot}2 + 2{\cdot}5 \times 0{\cdot}1875 = \quad 10{\cdot}67$ ins.
Axial length of pole-face, $L_f = 9{\cdot}5$. $\quad a = \frac{1}{4}$ in.
$a/l_g \ = 0{\cdot}25/0{\cdot}1875 = 1{\cdot}33$. By Fig. 254, $K_2 = 1{\cdot}5$
$w_d/l_g \ = 0{\cdot}5\ /0{\cdot}1875 = 2{\cdot}67$. By Fig. 256, $K_3 = 0{\cdot}36$
$L_f + K_2l_g - K_3\,w_d\,n_d = 9{\cdot}5 + 1{\cdot}5 \times 0{\cdot}1875 - 0{\cdot}36 = \quad 9{\cdot}42$ ins.
Effective area of air-gap, by equation (113)
$= 10{\cdot}67 \times 9{\cdot}42 \times 6{\cdot}45 = 648$ sq. cm.
$w_s/l_g \ = 0{\cdot}44/0{\cdot}1875 = 2{\cdot}34$
$w_{t1}/w_s = 0{\cdot}605/0{\cdot}44 = 1{\cdot}375$. By Fig. 262, $K = 1{\cdot}155$

$$AT_g = 0{\cdot}8\, B_{g\ max} \times 1{\cdot}155 \times 0{\cdot}477 = 0{\cdot}44\, B_{g\ max}$$

$$\Phi_a = \frac{a}{p} \times \frac{6000}{ZN} \times 10^6 \times E_a = \frac{1}{2} \times \frac{6000\, E_a}{342\, N} \times 10^6 = 8{\cdot}77 \frac{E_a}{N} \times 10^6$$

To allow for the governor range, say

no-load speed = 405 revs. per min.

full-load speed = 395 „ „ „

At no-load, $\Phi_a = 8{\cdot}77 \times 230/405 = 4{\cdot}99$ megalines

Allowing 2 volts over brushes and 3·3 volts over commutating-pole winding, $E_a = 230 + 7{\cdot}66 + 2 + 3{\cdot}3 = 242{\cdot}86$.

Hence at full-load, $\Phi_a = 8{\cdot}77 \times 242{\cdot}86/395 \doteq 5{\cdot}4$ megalines.

Interpolar ampere-turns on main magnetic circuit.

		No-load.	*Full-load.*
Φ_a in megalines =		4·99	5·4
$B_{g\ max} = \Phi_a/648 =$		7700	8350
$0{\cdot}44\, B_{g\ max}$	= **AT$_g$** =	**3385**	**3670**
Q (p. 501) = $1{\cdot}045 \times 9{\cdot}42/8{\cdot}1 = 1{\cdot}22$			
Q' (p. 502) = $10/8{\cdot}1 = 1{\cdot}235$			
$Q \times B_{g\ max} =$		9400	10,180
Uncorrected density at crown of tooth, B_{t1}'			
$= Q \,.\, B_{g\ max}/w'_{t1} =$		15,500	16,800
Corrected, say, $0{\cdot}85 B_{t1}'$		13,150	14,300
$at_1 =$		8·4	13
Uncorrected density at centre, B_{tc}'			
$= Q \,.\, B_{g\ max}/w_{tc} =$		18,200	19,650
Slot-ratio, K_{sc} (equation 117)			
$= 0{\cdot}957 \times 1{\cdot}235/0{\cdot}517 = 2{\cdot}285$			
Corrected density at centre, B_{tc} =		18,000	19,400
at_{tc}		100	188
$a_c \times 10^{-6}$		0·2	0·4
Uncorrected density at root, B_{t2}'			
$= Q \,.\, B_{g\ max}/w_{t2}$		21,850	23,700
Slot-ratio, $K_{s2} = 0{\cdot}87 \times 1{\cdot}235/0{\cdot}43 = 2{\cdot}5$			
Corrected density at root, B_{t2} =		21,100	22,200
$a_2 \times 10^{-6}$		0·85	1·5
$at_{av} = (a_2 - a_c)/(B_{t2} - B_{tc}) =$		210	393
$\frac{1}{4}(at_1 + at_{tc})$		54	100
$l_t/2 = 1{\cdot}625 \times 2{\cdot}54 \times 0{\cdot}5 = 2{\cdot}06$ cm.		264	493
By equation (119)	**AT$_t$** =	**545**	**1015**
$B_c = \Phi_a/353$		14,100	15,300
$at =$		12·4	23·6
$l_c/2 = 3\frac{1}{2}$ ins. $\times 2{\cdot}54 = 8{\cdot}9$ cm.			
	AT$_c$ =	**110**	**205**
$AT_g + AT_t + AT_c =$	**AT$_r$** =	**4040**	**4890**

Heating of armature.

$I_a^2 R_a = 7{\cdot}66 \times 243 \qquad = 1860$ watts

Volume of iron in teeth $= 0.517 \times 1{\cdot}625 \times 57$
$\times\ 0{\cdot}9(10 - 1) = 388$ cub. ins. $= 6{,}350$ cm^3.

Mean diameter of core below slots $= 15{\cdot}75 - 3{\cdot}375$
$= 12{\cdot}375$ ins.

Volume of iron in core $= 12{\cdot}375 \times \pi \times 2{\cdot}54 \times 353/2$
$= 17{,}420$ cm^3.

Frequency, $f = pN/60 = 2 \times 400/60 = 13{\cdot}33$ cycles per sec.

For $B_c = 15{,}300$ and $B_{t1} = 14{,}300$,
joules per cm^3 per cycle by Fig. 215 may be taken for both core and teeth as $h = 0{\cdot}00147$

Hysteresis loss by equation (97)
$= 0{\cdot}00147 \times 13{\cdot}33 \times (17{,}420 + 6350) = 465$ „

By equation (210), $19^3(5 \times 8{\cdot}35^2 \times 10$
$+\ 12 \times 4{\cdot}685^2) \times 10^{-10} = 0{\cdot}00259$

$$80\left(\frac{23{,}700 - 17{,}000}{1000}\right)^2 \times 57^2 \times \frac{0{\cdot}44 \times 1{\cdot}625}{342 \times}$$

$\times\ 10 \times 19^{\frac{1}{2}} \times 0{\cdot}1875^{\frac{1}{2}} \times 2^{\frac{1}{2}} \times 10^{-10} = 0{\cdot}000345$

$F = 0{\cdot}00259 + 0{\cdot}000345$

$FN^2 = 0{\cdot}00293 \times 400^2 \qquad = 470$ „

Total loss in armature $\qquad = 2795$ „

Cooling surface, outer, $3{\cdot}14 \times 19 \times (10 + 11{\cdot}5) = 1280$ sq. ins.

„ „ inner, corrected, $3{\cdot}14 \times 12{\cdot}8 \times 11{\cdot}5 = 460$ „ „

1740 „ „

At peripheral velocity 1990 ft. per min. $k = t°$ C. $\times S_c/W$
$= 21{\cdot}5$ by Fig. 399.

Surface rise of temperature by thermometer,
$t° = 21{\cdot}5 \times 2795/1740 = 34{\cdot}5°$ C.

Completion of magnetic circuit and field ampere-turns.

Cast-steel pole-core 6·5 ins. wide $\times$ 9 ins. parallel to shaft with corners rounded to 1 in. radius $= 57{\cdot}64$ sq. ins. $= 372$ sq. cm.

Double section of cast-steel yoke $= 61{\cdot}5$ sq. ins. $= 396$ sq. cm.

Laminated pole-shoes, 1 in. deep.

By equation (123)—

$\mathfrak{F}_t = 8 \times 2{\cdot}1 + 9 + 2{\cdot}25 \times 10 + 0{\cdot}66 \times 19 \quad = \qquad 61$

To check this—

(1) Between the tips of the pole-shoes of a main and a commutating pole

$$\mathfrak{P}_1 = 2{\cdot}54\,\frac{9{\cdot}5 \times 1}{1{\cdot}8} \times 2 \times \frac{1}{2} = 13{\cdot}4 \text{ in relation to } 2AT_p$$

(2) Between the flanks of the pole-shoes, partly for flux from a main into a commutating pole and partly for flux from a main pole-shoe to a main pole-shoe of opposite sign

$$\mathfrak{P}_2 = 1{\cdot}86 \times 1 \times \log\frac{\pi \times 1 + 1{\cdot}8}{1{\cdot}8} \times 4 \times \frac{1}{2}$$

$$1{\cdot}86 \times 4 \times \log\frac{\pi \times 1 + 5}{5} \times 4 = 1{\cdot}63 + 6{\cdot}25 = 7{\cdot}88$$

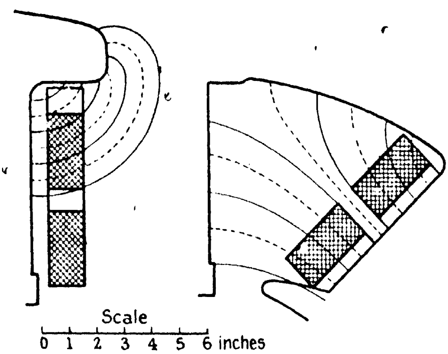

FIG. 409.—Calculation of leakage permeance from pole core.

(3) Between side of pole core and yoke or commutating pole by Fig. 409

$$\mathfrak{P}_3 = 2{\cdot}54\left(\frac{9 \times 2{\cdot}06}{4{\cdot}22} \times \frac{4{\cdot}8}{5{\cdot}25} + \frac{9 \times 1{\cdot}83}{5{\cdot}7} \times \frac{3{\cdot}25}{5{\cdot}25} + \frac{9 \times 1{\cdot}59}{5{\cdot}4} \times \frac{2{\cdot}625}{5{\cdot}25} + \frac{9 \times 1{\cdot}75}{3{\cdot}5} \times \frac{1{\cdot}5}{5{\cdot}25} + \frac{9 \times 1{\cdot}27}{1{\cdot}59} \times \frac{0{\cdot}3}{5{\cdot}25}\right)$$

$$= 10{\cdot}2 + 4{\cdot}55 + 3{\cdot}37 + 3{\cdot}26 + 1{\cdot}04 = 22{\cdot}42$$

(4) From the flanks of a main pole core

(*a*) into the flanks of a main pole of opposite sign,

$$1{\cdot}86 \times 3 \times \log \frac{\pi \times 3{\cdot}25 + 12{\cdot}7}{12{\cdot}7} \times 4 \times \frac{5{\cdot}25}{5{\cdot}75} = 5{\cdot}2$$

(*b*) into yoke (Fig. 409),

by Fig. 271, $\sin \alpha = \dfrac{2{\cdot}7}{\sqrt{1{\cdot}91^2 + 2{\cdot}7^2}} = 0{\cdot}818$

α in circular measure $= 2{\cdot}19$

$$2{\cdot}54 \times \frac{6{\cdot}5}{2{\cdot}19} \times \frac{3}{5{\cdot}25} = 4{\cdot}3$$

$$\mathfrak{F}_4 = 9{\cdot}5$$

$\mathfrak{F} = 13{\cdot}4 + 7{\cdot}88 + 22{\cdot}42 + 9{\cdot}5 = 53{\cdot}2$. Adopt 61, as above assumed.

Φ_a in megalines		4·99	5·4
$\phi_l = 1{\cdot}257 \times 2.4\,T_r \times 61 =$ in megalines		0·62	0·75
$\Phi_m = \Phi_a + \phi_l =$		5·61	6·15
$B_m = \Phi_m/372 =$		15,100	16,500
at_m by Fig. 207		16·5	32
$l_m = 7{\cdot}5$ ins. $= 19$ cm.	$AT_m =$	**313**	**610**
$B_y = \Phi_m/396$		14,200	15,500
at_y by Fig. 207		12	21·8
$l_y/2 = 12\frac{1}{2}$ ins. $= 31{\cdot}8$ cm.	$AT_y =$	**382**	**695**
	$AT_r =$	**4040**	**4890**
	AT per pole	**4735**	**6195**
	say	**4750**	**6200**

Field magnet winding. Shunt. Sectional ventilated.

Length of bobbin between flanges 5¾ ins.

Two sections, divided by ½ in. air-gap, and $\frac{3}{16}$ in. air-way between coil and pole.

Net winding length 5¼ ins.

Depth of winding with insulating wrapping, 1½ ins., net 1⅜ ins.

Length of mean turn, $2(4{\cdot}5 + 7) + 2\pi \times 2{\cdot}2$
$= 36{\cdot}8$ ins. $= 1{\cdot}02$ yd.

Length of outer turn, $2(4{\cdot}5 + 7) + 2\pi \times 2{\cdot}9 = 41{\cdot}2$ ins.

Retaining 3 volts in rheostat on full-load, as a precautionary margin in case of need, exciting voltage per pole
$= \frac{1}{4}(230 - 3) = 56{\cdot}75$

By equation (124) $\omega' = \dfrac{56{\cdot}75 \times 1000}{6200 \times 1{\cdot}02 \times 1{\cdot}16} = 7{\cdot}74$ ohms at 20° C.

By equation (126) $d = 0{\cdot}176/\sqrt{7{\cdot}74} = 0{\cdot}0634$ in. This is so close to No. 16 S.W.G. $= 0{\cdot}064$, that that gauge will be chosen,

increasing slightly the margin of volts: when single-cotton-covered, $d_1 = 0{\cdot}07$ *in.*, $\omega' = 7{\cdot}6$ *ohms.*

Turns per layer, $5{\cdot}25/0{\cdot}07 =$	75
No. of layers, $1{\cdot}375/(0{\cdot}9 \times 0{\cdot}07) =$ say,	22
Turns per pole $= 75 \times 22 =$	1650
Total yds. $= 4 \times 1650 \times 1{\cdot}02 =$	6740

R_s at 20° C. $= 6{\cdot}74 \times 7{\cdot}6 = 51{\cdot}1$, hot $= 51{\cdot}1 \times 1{\cdot}16 = 59{\cdot}4$ ohms.

$I_s = 3{\cdot}76$ amperes. Exciting volts $= 223$: 7 volts to be absorbed in rheostat.

$I_s^2 R_s = 840$, per pole $=$ 210 watts

Cooling surface of one coil

$$= 41{\cdot}2 \times 5{\cdot}25 + 36{\cdot}8 \times 1{\cdot}375 \times 2 = 316 \text{ sq. ins.}$$

$k = t°\text{ C.} \times S_c/W$ at peripheral speed of 1990 ft. per min. by Fig. 398 $= 49$.

Surface rise, $t° = 49 \times 210/316 =$ 32·6° C.

On no-load, I_s must be $4750/1650 = 2{\cdot}88$, and the resistance of the shunt winding and rheostat, $230/2{\cdot}88 = 80$ ohms. When the shunt is cold, the resistance of the rheostat must be $80 - 51{\cdot}1 = 28{\cdot}9$, say, 30 ohms.

Commutation.

Carbon brushes, 3 per brush arm, each $1\frac{1}{2}$ ins. $\times$ $\frac{7}{8}$ in.

Contact area per arm $= 3 \times 1{\cdot}5 \times 0{\cdot}875 = 3{\cdot}94$ sq. ins.

Current-density, $243/(3{\cdot}94 \times 2) = 31$ amperes per sq. in.

With the wave winding now adopted, the commutator diameter can be reduced to $12\frac{3}{4}$ ins.

Pitch of sectors, $(12{\cdot}75 \times 3{\cdot}14)/171 = 0{\cdot}234$ in.

$$\left(\frac{b_1 - b_m}{b}\right)_+ = \left(\frac{0{\cdot}875 - 0{\cdot}03}{0{\cdot}234}\right)_+ = (3{\cdot}6)_+ = 4$$

By equation (190), $$T = \left\{\frac{0{\cdot}875 - 0{\cdot}03}{0{\cdot}234} + \left(1 - \frac{1}{2}\right)\right\} \times \frac{60}{171 \times 400}$$

$$= (3{\cdot}6 + 0{\cdot}5) \times 0{\cdot}000876$$
$$= 0{\cdot}00359 \text{ sec.}$$

Placing brush 1 centrally over commutator sector 1 (Fig. 410), brushes 2 and 4 just reach over the mica strips to sectors 46 and 127 at trailing and leading edge respectively, making 5 sectors touched by each of these brushes, and 9 short-circuited coil-sides in each of two zones corresponding to slots 14, 15, 16, and 28, 29, 30. If 0·013 inch is cut off the trailing edge of each brush, conductors 91 and

176 are removed from short-circuit, and a similar amount cut off each leading edge removes conductors 83 and 168. $\frac{b_1 - b_m}{b}$ is then exactly 3·5, and when the commutator moves slightly forward the picture of slots 14, 15, 16, and 29, 30 becomes that of Fig. 364 for a remainder of $\frac{1}{2}$ with conductors 85 and 170 as the considered coil-sides. The time of commutation will, however, then be reduced,

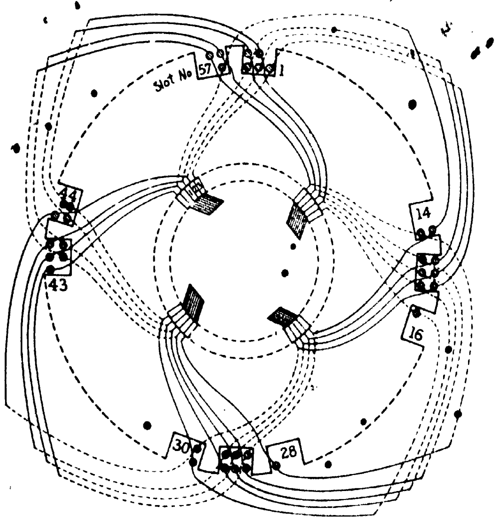

Fig. 410*a*.—Short-circuited loops on 4-pole wave wound armature.

and the full brush-width will be here retained with conductors 85 and 170 each in a slot fully filled, as giving the greatest possible value of $\mathcal{S} + \Sigma . \mathcal{A}$. Thence

for conductor 85 in slot 15, $k_1' = 3\left(\frac{2\pi}{3} + 2\pi + \pi\right) = 34{\cdot}6$

for conductor 170 in slot 29, $k_2' = 3\left(\frac{2\pi}{3} + \pi\right) = 15{\cdot}7$

a_1'' or a_2'' by equation (188) $= 12{\cdot}57\left(\frac{0{\cdot}14}{0{\cdot}44} + 0{\cdot}2\right) = 6{\cdot}5$

$D/2p\, w_3 = 19/(4 \times 0{\cdot}44) = 10{\cdot}8$

By Fig. 361, $b_1 = 17{\cdot}3 = b_2$

$k_1'h_w/w_s + j_{a1} \times a_1'' + j_{b1}b_1$

$= 34{\cdot}6(1{\cdot}485/0{\cdot}44) + 6 \times 6{\cdot}5 + 9 \times 17{\cdot}3 = 117 + 39 + 155{\cdot}5 = 311{\cdot}5$

$k_2'h_w/w_3 + j_{a2} \times a_2'' + j_{b2}b_2$

$= 15{\cdot}7 \times 3{\cdot}38 + 6 \times 6{\cdot}5 + 9 \times 17{\cdot}3 = 53 + 39 + 155{\cdot}5 = 247.5$

559

Iron length of core 9 ins. $\times$ 2·54 = 22·8 cm.

$$22{\cdot}8 \times 559 = 12{,}720$$

$$\lambda' = j_c e_c = 5 \times 3 \log \frac{18{\cdot}78 + 1}{0{\cdot}66 + 5 \times 0{\cdot}115}$$

$$= 15 \log 16 = 18$$

$$2l'\lambda' = 2 \times 19{\cdot}78 \times 2{\cdot}54 \times 18 \qquad = \quad 1{,}810$$

$$\mathcal{L} + \Sigma . \mathcal{M} \text{ in henrys} \qquad = 14{,}530 \times 10^{-9}$$

$$(\mathcal{L} + \Sigma . \mathcal{M}) 2J/T = \frac{0{\cdot}00001453 \times 243}{0{\cdot}00359} = \qquad 0{\cdot}99 \text{ volt}$$

Commutating pole cast solid with yoke 8 ins. $\times$ 1½ ins. wide, with steel pole-shoe 9 ins. $\times$ 1¾ ins. breadth screwed on.

w_c by equation (205) should be $= (0{\cdot}875 - \frac{1}{2} \times 0{\cdot}234) \frac{19}{12{\cdot}57}$

$+ 1{\cdot}045 - 0{\cdot}115 \times 2 = 1{\cdot}945$, but the above will be near enough.

$v = 19 \times 3{\cdot}14 \times 2{\cdot}54 \times 400/60 = 1000$ cm. per sec.

Average B_{rc} by equation (193) $= \dfrac{0{\cdot}495 \times 10^8}{9 \times 2{\cdot}54 \times 1000} = 2165$

$l_{gr} = 0{\cdot}150$ in. $= 0{\cdot}381$ cm.

$AT_{gr} = 0{\cdot}8\, B_{gr} K_r l_{gr} = 0{\cdot}8 \times 2165 \times 1{\cdot}16 \times 0{\cdot}381 =$ 770

$JC/4p = (122 \times 342)/8 = 5210$

Approximate $AT_r = 5210 \times 1{\cdot}3 = 6780$. But a detailed calculation is added below.

$ac\left(\dfrac{Y}{2} - \dfrac{w_c}{4}\right) = \dfrac{7{\cdot}02}{7{\cdot}45} \times 5210 =$ 4910

Effective polar arc of commutating pole

$= w_o + 2{\cdot}25\, l_{gr} = 2{\cdot}08$ ins.

$a/l_g = 0{\cdot}5/0{\cdot}15 = 3{\cdot}33$. K_2 by Fig. 254 $= 2{\cdot}5$.

$K_2 l_{gr} = 0{\cdot}375$

Effective area of reversing field

$= 2{\cdot}08 \times 9{\cdot}375 \times 6{\cdot}45$

$= 125$ sq. cm.

$$\phi_r = 125 \times 2165 = 271{,}000$$
$$\phi_{lr} = \tfrac{5}{8} \times 750{,}000 = 469{,}000$$
$$\phi_{mr} = \overline{740{,}000}$$

To check assumed ϕ_{lr}, it may be reckoned that permeances $\mathcal{P}_1$ + first part of $\mathcal{P}_2$ + first 2 terms of $\mathcal{P}_3 = 13{\cdot}4 + 1{\cdot}63 + 14{\cdot}75 = 29{\cdot}78$ will be acted on now by a M.M.F. of $1{\cdot}257 \times (4750 + 6000)$, giving a leakage flux of 403,000, and this will in fact be increased

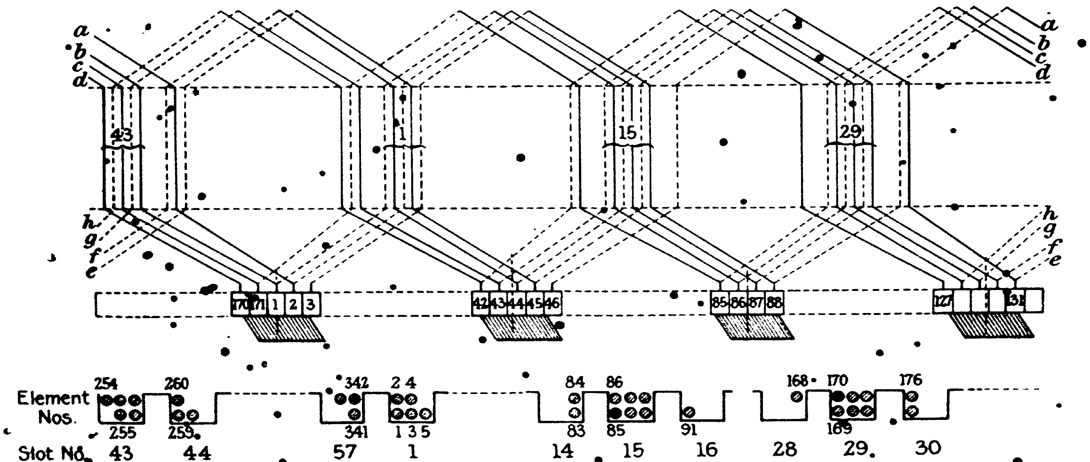

FIG. 410b.—Short-circuited loops developed on the flat.

by flux being drawn into the commutating pole on the one side, instead of passing directly into the yoke in the upper half of the main pole.

Area of pole-core, 8 ins. × $1\frac{1}{2}$ ins., with ends rounded to $\frac{7}{8}$ in. radius = 6·25 × 1·75 + π × 0·875² = 13·3 sq. ins. = 85·7 sq. cm.

B_{mr} = 740,000/85·7 = 8650

at by Fig. 207 = 5. l_{cp} = 20·2. AT_{mr} = 5 × 20·2 = 100

$$\frac{\Phi_m + \phi_{mr}}{a_y} = \frac{(6\cdot15 + 0\cdot74)\times10^6}{396} = 17{,}400 \qquad \frac{\Phi_m - \phi_{mr}}{a_y} = \frac{(6\cdot15 - 0\cdot74)\times10^6}{396} = 13{,}650$$

at = 70 = 11

Difference = 59

$(l_y/4)$ × 59 = 15·9 × 59 = 940

The calculation illustrates the importance of the term dealing with the difference of densities in the yoke sections in a 4-pole machine wherein the length of path in the yoke bears a high ratio to the pole-core length.

By equation (173), AT_r = 770 + 80 + 4910 + 100 + 940 = 6800

AT_r say, 28 turns × 243 amps. = 6804, all coils in series.

Length of coil = $5\frac{7}{8}$ ins.

5·875/29 = 0·203 in.
0·135 „
·068 „ clearance between turns.

Single spiral of bare copper, 1·125 in. × 0·135 in. Area 0·152 sq. in.

ω' per 1000 yds., at 20° C. = 0·02445/0·152 = 0·161 ohm.

$\frac{1}{2}$ in. air-way at ends of each coil

Mean length of turn = 2 × 7·25 + 2π × 1·4375
= 23·55 ins. = 0·655 yd.

Total length of 4 coils = 0·655 × 28 × 4 = 73·5 yds.

Resistance = 0·161 × 0·0735 × 1·14 = 0·01355 ohms.

Loss of volts = 243 × 0·01355 = 3·29

Watts = 3·29 × 243 = 800 = 200 per coil.

Cooling surface of one coil = (2 × 7·25 + 2π × 2) × 5·875
= 158 sq. ins. = 0·79 sq. in. per watt.

Weight of 1 yd. = 11·55 × 0·152 = 1·76 lb.
„ „ 73·5 yds. = 130 lb.

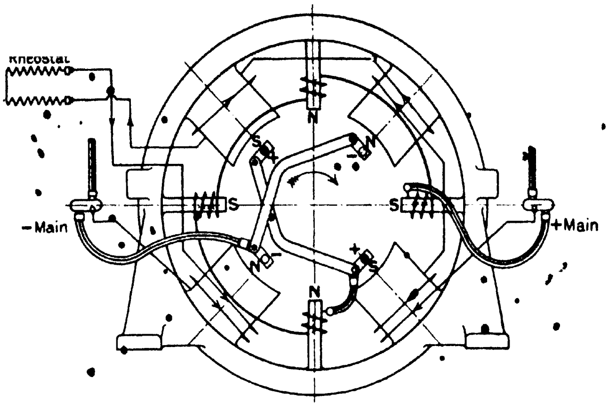

FIG. 411.—Field-magnet and commutating-pole connexions for shunt-wound dynamo.

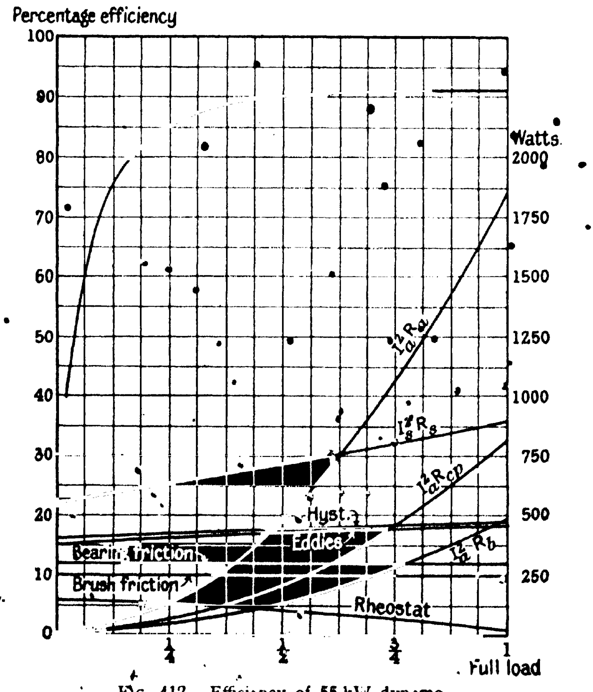

FIG. 412.—Efficiency of 55 kW dynamo.

Heating of commutator.

$I_a^2 R_b = 2 \times 243 =$ 486 watts

Peripheral speed of commutator, $\pi \times 12{\cdot}75/12 \times 400$
$= 1335$ ft. per min.

Total brush pressure at $1\frac{1}{2}$ lb. per sq. in.
$= 1{\cdot}5 \times 3{\cdot}94 \times 4 = 23{\cdot}65$ lb.

Taking $\mu = 0{\cdot}35$, by equation (214) brush friction
$= 0{\cdot}35 \times 23{\cdot}65 \times 1{\cdot}335 \times 22{\cdot}6 =$ 250 „

736 „

External cylindrical surface, $\pi \times 12{\cdot}75 \times 5{\cdot}75 = 230$ sq. ins.
One face of lugs, 1 in. $\times$ 3 ins. $\times$ 171 $= 513$ „ „

$S_c = 743$ „ „

$S_c\{1 + 0{\cdot}3 \times 1{\cdot}335^{1{\cdot}3}\} = 743 \times 1{\cdot}437 = 1065$

By equation (Chap. XXI, § 27), $t°$ C. $= 55 \times 736/1065 = 38{\cdot}2°$ C.

Efficiency (Fig. 412).

Losses in watts.	¼	½	¾	Full-load
$I_a^2 R_a$	124	473	1042	1860
Hysteresis	425	435	455	465
Eddies	400	425	450	470
$I_a^2 R_b$	50	125	275	486
$I_a^2 R_{cp}$	53	204	450	800
$I_s^2 R_s$	565	680	760	840
Rheostat	146	102	80	24
Brush friction	250	250	250	250
One bearing and air friction	300	300	300	300
Total losses	2313	2994	4,062	5495
Output	13,750	27,500	41,250	55,000
Input	16,063	30,494	45,312	60,495
Efficiency	85·6%	90%	91%	91·1%

Binding wire.

By equation (93),

$f_c = 7{\cdot}5 \times 342 \times 0{\cdot}055 \times 10 \times 17{\cdot}375 \times 400^2 \times 10^{-7} = 392$ lb.

On the core.—Three bands, each $\frac{3}{4}$ in. wide, of non-magnetic Eureka wire, 0·040 in. diameter with ultimate breaking strength of 75,000 lb. per sq. in.

No. of wires, $3 \times 0{\cdot}75/0{\cdot}04 = 56$

Total sectional area, $56 \times 0{\cdot}00125 = 0{\cdot}07$.

Apparent factor of safety, $75,000 \times 0{\cdot}07/392 = 13{\cdot}4$.

On the end-winding—

$(\sqrt{m_1^2 + l^2})/l = (\sqrt{6{\cdot}7^2 + 3{\cdot}68^2})/3{\cdot}68 = 2{\cdot}07.$

At each end, $f_e = 392 \times 2{\cdot}07 \times 5{\cdot}75/10 = 467$ lb.

One band, $1\frac{1}{8}''$ wide, of steel wire, 0·040 in. diameter, with ultimate breaking strength of 200,000 lb. per sq. inch.

No. of wires, $1{\cdot}125/0{\cdot}04 = 28$.

Sectional area, $28 \times 0{\cdot}00125 = 0{\cdot}035$ sq. ins.

Apparent factor of safety, $200{,}000 \times 0{\cdot}035/467 = 15$.

In each case the real factor of safety is less by an amount depending on the initial tension under which the bands are put on, and the stress of bending the wire round the armature.[1]

§ 11. Design of dynamo for 450 kW at 400 revs. per min.—The next design is that for a dynamo of 450 kW output at 400 revs. per min., which is to give its full-load at 500 volts when shunt-wound and also is to be over-compounded, so that its voltage rises from 500 volts at no-load to 525 volts at full-load. Two conditions have, therefore, to be considered, the full-load current in the two cases being 900 or 855 amperes respectively, and the change from the latter condition to the former will be made by short-circuiting the series winding and by an alteration of the setting of the shunt rheostat. It will therefore be reasonable to anticipate that when normal values of $B_{g\ max}$ and ac have been selected, a higher value of $B_{g\ max}$ and a lower value of ac, or *vice versa*, will actually hold according to whether the generator is giving its full-load as a compound-wound or as a shunt-wound machine.

Kilowatts	450
Revs. per min.	400
Induced watts per rev. per min. approx. $= \dfrac{1{\cdot}07 \times 450{,}000}{400} =$	1200
$G = 0{\cdot}72\, B_{g\ max}\, ac_u \times 10^{-8}$ by Fig. 404	0·054
$B_{g\ max}$ by Fig. 405	9250
ac_u by Fig. 405	810
$D^2_u L_u$ in cub. inches $= \dfrac{1200}{0{\cdot}054}$	22,200
Volts	500–525
Amperes	900–855
No. of pole-pairs $= I_a/300 = 905/300$ — say,	3

$L = 0{\cdot}75\, \pi D/2p = 0{\cdot}392 D$

$D_u^3 = 22{,}200/0{\cdot}392 = 56{,}700$

$D_u = 38{\cdot}4$ ins. $L_u = 15{\cdot}1$ ins.

Peripheral velocity $= 4010$ ft. per min.

The armature core is rather long and the peripheral velocity

[1] See L. Burstow, *El. Review*, Vol. 89, p. 774.

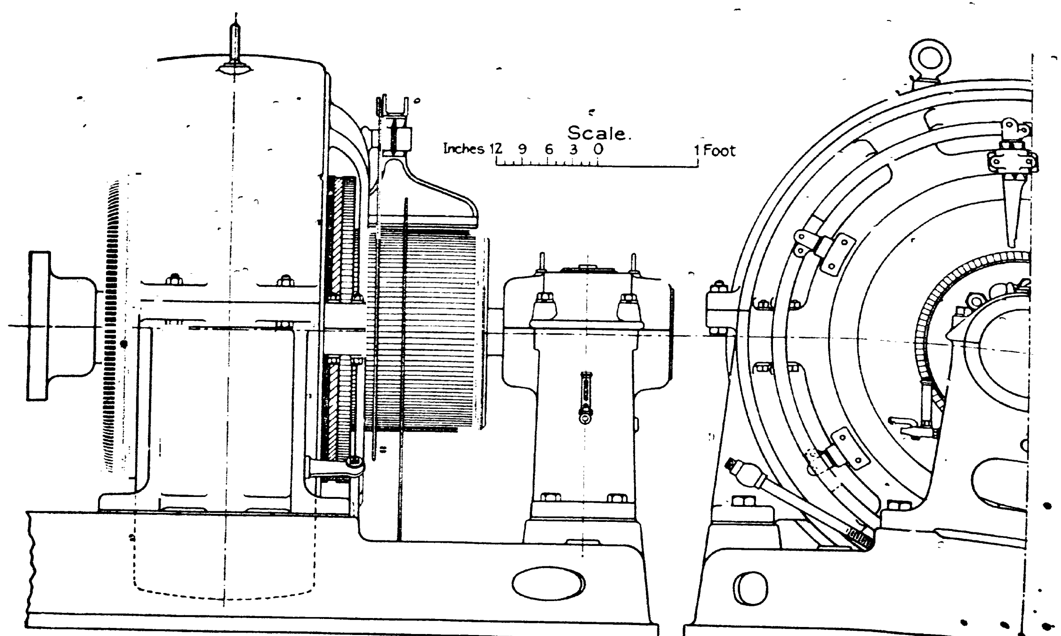

FIG. 413.—General arrangement of 450 kW dynamo.

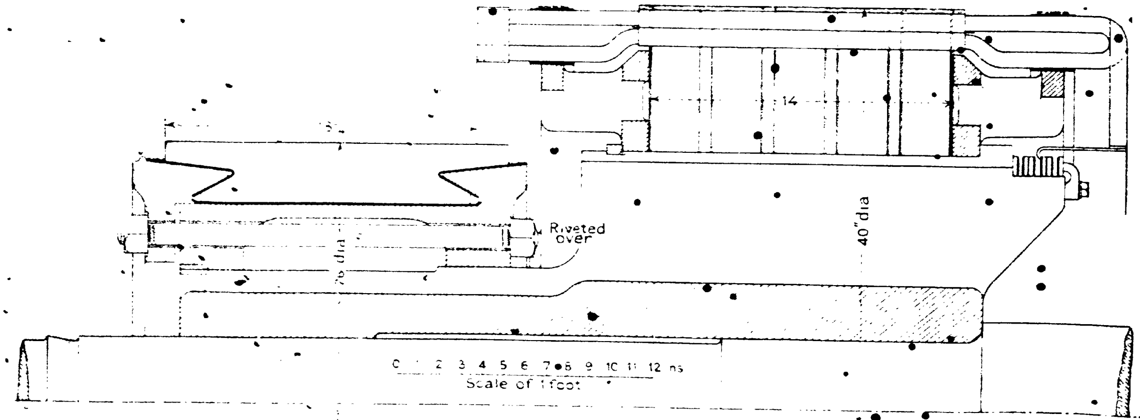

FIG. 414a.—Armature of 450 kW dynamo.

moderate, so that a shorter core will be preferable, and it will be assumed that there is a standard 6-pole frame available (Fig. 413) for an armature core (Fig. 414) having the following dimensions—

D_a =	40 ins.
L_a =	14 ins.
Peripheral velocity =	4190 ft. per min.

Armature winding.

With simplex lap winding, $Z = \dfrac{ac_a \times \pi D_a}{I_a/2p} = \dfrac{810 \times 125{\cdot}5}{859/6 \text{ or } 905/6}$

$= 710$ or 674

6 conductors per slot, 3 abreast.

ac per slot $= 6 \times 905/6 = 905$ maximum, which is permissible by Fig. 373.

To make S divisible by p and permit of equalizing connexions, choose $S = 114$ slots and $Z =$ 684

No. of commutator sectors, C, with single-turn loops 342

Commutator diameter, say, $0{\cdot}6 \times 40$ ins. $=$ 24 ins.

Commutator sector-pitch, $\dfrac{3{\cdot}14 \times 24}{342} = 0{\cdot}220$ in., rather near the limiting value (Chapter XII, § 17 (3)) : hence make

Commutator diameter = 26 ins.

Average volts per sector $= 2p \,.\, V_b/C = 6 \times 527/342 = 9{\cdot}25$ volts

Slot dimensions (Chapter XIII, § 18) 1·75 ins. deep $\times$ 0·45 in. wide (Fig. 414*b*).

	Pitch of teeth.	*Width of tooth.*
at top	1·1 in.	0·65 in.
at centre	1·052 ,,	0·602 ,,
at root	1·005 ,,	0·555 ,,

Thickness of conductor, by equation (81),

$t = \frac{1}{3}(0{\cdot}45 - 0{\cdot}075) - 0{\cdot}035 = 0{\cdot}09$ in.

Height of conductor, by equation (82),

$h = \frac{1}{2}(1{\cdot}75 - 0{\cdot}36) - 0{\cdot}03 = 0{\cdot}665$ in.

Area $= 0{\cdot}665 \times 0{\cdot}09 =$ 0·06 sq. in.

Resistance of 1000 yds. at 20° C. (68° F.)

$= 0{\cdot}02445/0{\cdot}06 = 0{\cdot}407$ ohm

$S/2p = 114/6 = 19$. Winding to be long-chord, *i.e.* span of loop at back short of a full pole-pitch by one slot-pitch, or $y_s{}^1 = 18$.

y_s by equation (49) $= 6 \times 18 + 1 = 109$. $y_r = -107$

Six equalizing rings of strip 0·77 in. $\times$ 0·09 in. joined to coils shown in Fig. 415.

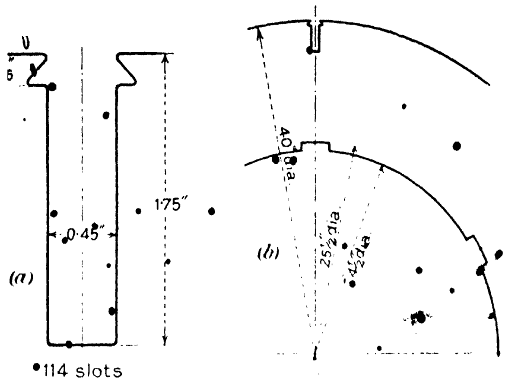

FIG. 414b. (a) Slot dimensions of 450 kW dynamo.
(b) Portion of core-disc " "

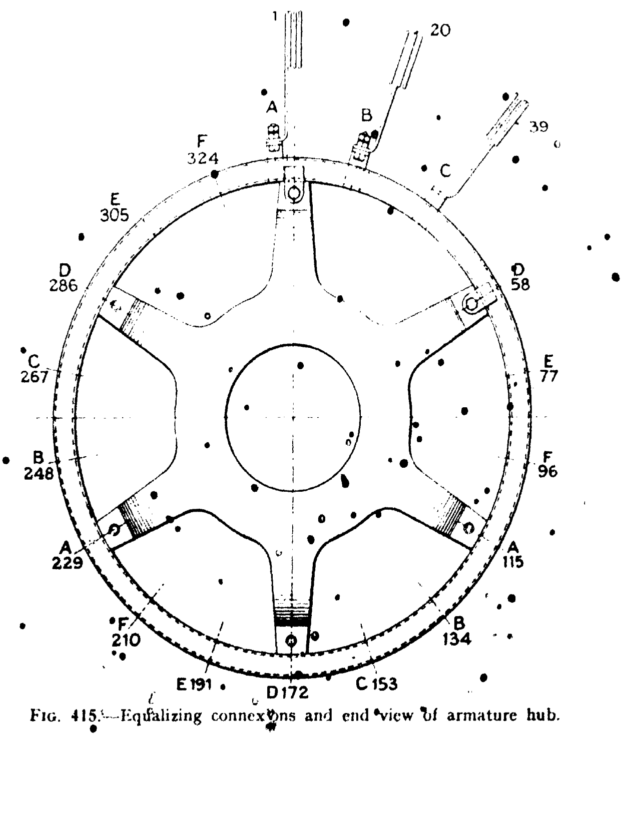

FIG. 415.—Equalizing connexions and end view of armature hub.

Ring.	A	B	C	D	E	F
No. of coil, or of bar in the upper layer.	1	20	39	58	77	96
" " " " " "	115	134	153	172	191	210
" " " " " "	229	248	267	286	305	324

The radial connecting pieces are riveted and sweated to the curved ends of the loops; the axial connecting pieces to the front of rings A, C, E, or to the back of rings B, D, F, the rings themselves being insulated with micanite, taped and compressed by the small clamps against the back of the arms of the hub (*cp.* Figs. 414*a* and 415).

Length of axial projection at either end by equation (83)

$$l_e = \frac{18(1{\cdot}005)\ (0{\cdot}515)}{2\sqrt{1{\cdot}005^2 - 0{\cdot}515^2}} + \frac{1}{2} + 1{\cdot}25 \times 0{\cdot}45 + 1{\cdot}75/2$$

$$= 5{\cdot}4 + 1{\cdot}938 = \text{say } 7\tfrac{3}{8} \text{ ins.}$$

$m_1 = \frac{1}{2} \times 18(1{\cdot}052) = 9{\cdot}45$ ins.

Length of a complete end-connexion, by equation (84)

$l' = 2\{\sqrt{9{\cdot}45^2 + 5{\cdot}4^2} + 1{\cdot}938\} = 25{\cdot}58$ ins.

Length of a half loop $= 14 + 25{\cdot}58 = 39{\cdot}6$ ins. $=$ 1·1 yd.

R_a, by equation (94) $= \dfrac{684 \times 1{\cdot}1 \times 0{\cdot}000407}{36} = 0{\cdot}0085$ ohm

at 20° C.; hot with 39° C. rise $= 1{\cdot}16 \times 0{\cdot}0085 = 0{\cdot}00985$ ohm.

$I_a R_a = \left.\begin{matrix} 859 \\ \text{or } 905 \end{matrix}\right\} \times 0{\cdot}00985 = 8{\cdot}46$ or $8{\cdot}9$ volts.

Magnetic circuit and field ampere-turns up to the pole-faces.

Radial depth of armature core below slots (Fig. 414*b*) $5\frac{1}{2}$ ins.

Four ventilating ducts, each $\frac{1}{2}$ in. wide.

Double section of core, $2h_c L_i = 2 \times 5{\cdot}5 \times 0{\cdot}9(14-2)$

$= 118{\cdot}8$ sq. ins. $= 766$ sq. cm.

Single air-gap, mean length $l_g = 0{\cdot}185$ in. $=$ 0·47 cm.

Polar angle $= 42°$. Ratio to pole-pitch, $\beta = 0{\cdot}7$

Polar arc at centre of gap, $A' = \frac{1}{6}\pi(40{\cdot}185) \times 0{\cdot}7 =$ 14·7 ins.

Pole-pitch on armature surface	= 20·92 ins.
Polar arc " " "	= 14·62 "
	6·3 "
Commutating pole-face width	2·5 "
Two interpolar gaps	3·8 "

Half of one interpolar gap between a main and a commutating pole of opposite sign, $c = 0{\cdot}95$ in.

$c/l_g = 0{\cdot}95/0{\cdot}185 = 5{\cdot}13$. $\gamma = 100°$. By Fig. 253, $K_1 = 2{\cdot}3$

Between main and commutating pole of the same sign, using Fig. 254, $K_1 = 3{\cdot}1$. Mean value for both strips may therefore be taken as $K_1 = \frac{1}{2}(2{\cdot}3 + 3{\cdot}1) = 2{\cdot}7$

$A' + K_1 l_g = 14{\cdot}7 + 2{\cdot}7 \times 0{\cdot}185 =$ 15·2 ins.

Axial length of pole-face, $L_f = 13\frac{1}{4}$ ins. $a = \frac{3}{8}$ in.

$a/l_g = 0{\cdot}375/0{\cdot}185 = 2{\cdot}03$. By Fig. 254, $K_2 = 1{\cdot}9$

$w_d/l_g = 0{\cdot}5/0{\cdot}185 = 2{\cdot}7$. By Fig. 256, $K_3 = 0{\cdot}35$

$L_f + K_2 l_g - K_3 w_d n_d = 13{\cdot}25 + 1{\cdot}9 \times 0{\cdot}185 - 0{\cdot}35 \times 2 = 12{\cdot}9$ ins.

Effective area of air-gap by equation (113)

$= 15{\cdot}2 \times 12{\cdot}9 \times 6{\cdot}45 = 1265$ sq. cm.

$w_s/l_g = 0{\cdot}45/0{\cdot}185 = 2{\cdot}43$. $w_{t1}/w_s = 0{\cdot}65/0{\cdot}45 = 1{\cdot}44$

K by Fig. 262 $= 1{\cdot}155$.

$AT_g = 0{\cdot}8\, B_{g\ max} \times 1{\cdot}155 \times 0{\cdot}47 = 0{\cdot}435\, B_{g\ max}$

$$\Phi_a = \frac{6000}{684} \times \frac{E_a}{N} \times 10^6 = 8{\cdot}77 \frac{E_a}{N} \times 10^6$$

At 500 volts no-load and 400 revs. per min., $\Phi_a = 10{\cdot}97$ megalines

Allowing 2 volts loss over brushes, $\frac{1}{2}$ volt over series winding with diverter and 1 volt without diverter, and 2·1 volts over commutating-pole winding

E_a at 500 volts full-load and, say, 396 revs. per min. $= 504{\cdot}35 + 8{\cdot}9 = 513{\cdot}3$ $\Phi_a = 11{\cdot}385$

E_a at 525 volts full-load and, say, 396 revs. per min. $= 530{\cdot}1 + 8{\cdot}5 = 538{\cdot}6$ $\Phi_a = 11{\cdot}95$

	No-load.	Full-load.	
Volts	500	500	525
Φ_a = in megalines	10·97	11·385	11·95
$B_{g\ max} = \Phi_a/1265$	8670	9000	9450
AT_g =	**3775**	**3915**	**4100**
$B_c = \Phi_a/766 =$	14,300	14,850	15,600
$at =$	14	18	25
$l_c/2 = 5$ ins. $\times 2{\cdot}54 = 12{\cdot}7$ cm. $AT_c =$	**178**	**228**	**317**
$Q = 1{\cdot}1 \times 12{\cdot}71/10{\cdot}8 = 1{\cdot}295$			
$Q' = 14/10{\cdot}8 = 1{\cdot}295$			
$Q \times B_{g\ max} =$	11,250	11,650	12,250
Uncorrected density at crown of tooth,			
$B_{t1}' = Q \,.\, B_{g\ max}/w_{t1}$	17,300	17,920	18,850
Corrected = say, $0{\cdot}85 B_{t1}'$	14,700	15,250	16,000
$at_1 =$	18	23	36
Uncorrected density at centre,			
$B_{tc}' = Q \,.\, B_{g\ max}/w_{tc}$	18,700	19,350	20,380

	No-load.	Full-load.	
Slot-ratio, $K_{sc} = 1{\cdot}295 \times 1{\cdot}052/0{\cdot}602 = 2{\cdot}27$			
Corrected density at centre, B_{tc}	18,600	19,000	20,000
at_{tc}	140	160	240
$a_c \times 10^{-6}$	0·3	0·31	0·5
Uncorrected density at root, $B_{t2}' = Q \cdot B_{g\ max}/w_{t2}$	20,300	21,000	22,100
Slot-ratio, $K_{s2} = 1{\cdot}295 \times 1{\cdot}005/0{\cdot}555 = 2{\cdot}34$			
Corrected density at root, B_{t2}	20,000	20,500	21,270
$a_2 \times 10^{-6}$	0·5	0·65	1·0
at_{av} by equation (118)	154	226	394
$\frac{1}{2}\ (at_1 + at_{tc})$	79	92	138
	233	318	532

$l_t/2 = 1{\cdot}75 \times 2{\cdot}54 \times 0{\cdot}5 = 2{\cdot}22$ cm.

		No-load.	Full-load.	
By equation (119)	$AT_t =$	518	706	1180
$AT_g + AT_c + AT_t =$	$AT_p =$	4471	4849	5597
	say $=$	4475	4850	5600

Heating of armature.

When compound-wound for 525 volts, $I_a{}^2R_a$
$= 8{\cdot}46 \times 859 = 7250$ watts

Volume of iron in teeth,
$0{\cdot}602 \times 1{\cdot}75 \times 114 \times 0{\cdot}9(14-2) \times 16{\cdot}38 = 21{\cdot}300$ cm³.

Mean diam. of core below slots $= 36{\cdot}5 - 5{\cdot}5 = 31$ ins.

Volume of iron in core $= 31 \times \pi \times 2{\cdot}54 \times 383$
$= 95{,}000$ cm³.

Frequency, $f = 3 \times 400/60 = 20$ cycles per sec.

Joules per cm.³ per cycle = say, 0·0015

Hysteresis loss $= 0{\cdot}0015 \times 20 \times 116{,}300 =$ 3500

By equation (210) $80\left(\dfrac{22{,}100 - 17{,}000}{1000}\right)^2 \times 114^2$

$$\times \frac{0{\cdot}45 \times 1{\cdot}75}{684} \times 14 \times 40^{\frac{1}{2}} \times 0{\cdot}185^{\frac{1}{2}} \times 3^{3} \times 10^{-10}$$

$= 0{\cdot}001785$

$40^3\,(5 \times 9{\cdot}45^2 \times 14 + 12 \times 5{\cdot}28^2) \times 10^{-10} = 0{\cdot}042$

$F =$ 0·0438

$FN^2 = 0{\cdot}0438 \times 400^2 =$ 7000 ,,

Total loss in armature = 17,750 ,,

Cooling surface, outer $= 3{\cdot}14 \times 40 \times (14 + 14{\cdot}75) =$ 3610 sq.ins.

,, ,, inner, corrected $= 3{\cdot}14 \times 34 \times 14{\cdot}75 =$ 1575 ,,

5185

At peripheral velocity of 4190, $k = t^{\circ}\text{C.} \times S_c/W$

$= 10{\cdot}5$ by Fig. 399.

Surface rise of temperature by thermometer,

$t^{\circ} = 10{\cdot}5 \times 17{,}750/5185 = 36^{\circ}$ C.

A similar calculation when shunt-wound and $I_a^2R_a = 8{\cdot}9 \times 905 = 8050$ watts but with lower flux-densities will show little variation, the two changes practically balancing.

Completion of ampere-turns and field winding.

A cast-steel pole core, 10 ins. wide $\times$ 13 ins. parallel to shaft, with rounded corners, $2\frac{1}{2}$ ins. radius (Fig. 416):

area $= 5 \times 13 + 5 \times 8 + \pi \times 2\frac{1}{2}^2 = 124$ sq. ins. $= 800$ sq. cm.

Double section of cast-steel yoke $= 74{\cdot}5$ sq. ins. $\times\ 2 = 960$ sq. cm.

By equation (123)

$\mathcal{S}_1 = 10 \times 2{\cdot}1 + 11 + 2{\cdot}25 \times 14 + 0{\cdot}6 \times 40 = 87{\cdot}5$, say 90.

To check this (*cp.* p. 246 for divisions) –

$$(1)\quad \mathcal{S}_1 = 2{\cdot}54 \times \frac{13{\cdot}25 \times 1{\cdot}25}{2{\cdot}82} = \qquad 15$$

$$(2)\quad \mathcal{S}_2 = 1{\cdot}86 \times 1{\cdot}25 \times \log \frac{\pi \times 1{\cdot}25 + 2{\cdot}82}{2{\cdot}82} \times 2$$

$$+ 1{\cdot}86 \times 6{\cdot}25 \times \log \frac{\pi \times 1{\cdot}25 + 10}{10} \times 4 = 1{\cdot}8 + 6{\cdot}68 = 8{\cdot}48$$

$$(3)\quad \mathcal{S}_3 = 2{\cdot}54 \times 13 \times \left(\frac{1{\cdot}5}{4{\cdot}875} \times \frac{7{\cdot}5}{8} + \frac{1{\cdot}4}{5{\cdot}5} \times \frac{6{\cdot}5}{8} + \frac{1{\cdot}5}{6{\cdot}15} \times \frac{5{\cdot}5}{8}\right.$$

$$\left. + \frac{1{\cdot}5}{7{\cdot}5} \times \frac{4{\cdot}5}{8} + \frac{1{\cdot}5}{6{\cdot}9} \times \frac{6{\cdot}5}{8} + \frac{1{\cdot}4}{5} \times \frac{2{\cdot}5}{8} + \frac{1{\cdot}25}{3{\cdot}125} \times \frac{1{\cdot}5}{8} + \frac{1}{1{\cdot}5} \times \frac{0{\cdot}5}{8}\right)$$

$$= 35{\cdot}8$$

$$(4)\quad \mathcal{S}_4 = 1{\cdot}86 \times 6 \times \log \frac{\pi \times 1 + 6}{6} \times 2$$

$$+ 1{\cdot}86 \times 6 \times \log \frac{\pi \times 4 + 8}{8} \times 4 \times \frac{5{\cdot}5}{8}$$

$$\left(\sin a = \frac{4{\cdot}5}{\sqrt{3{\cdot}5^2 + 4{\cdot}5^2}}\right) \qquad + 2{\cdot}54 \times \frac{10}{2{\cdot}24} \times \frac{2{\cdot}5}{8}$$

$$a = 2{\cdot}24 \qquad\qquad = 20{\cdot}19$$

$\mathcal{S} = 15 + 8{\cdot}48 + 35{\cdot}8 + 20{\cdot}19 = 79{\cdot}47$. Adopt 90, as assumed.

Φ_a in megalines	10·97	11·385	11·95
$\phi_l = 1{\cdot}257 \times 2AT_r \times 90$	1·015	1·095	1·266
$\Phi_m =$	11·985	12·48	13·216

$B_m = \Phi_m/800$		14,980	15,600	16,520
at_m by Fig. 207		16	20	40
$l_m = 11$ ins. $= 28$ cm.	$\mathrm{AT}_m =$	**450**	**560**	**1120**
$B_y = \Phi_m/960$		12,500	13,000	13,800
at_y by Fig. 207		8	9	12
$l_y/2 = 19$ ins. $= 48{\cdot}3$ cm.	$\mathrm{AT}_y =$	**387**	**435**	**580**
	$\mathrm{AT}_p =$	**4475**	**4850**	**5600**
	AT per pole $=$	**5312**	**5845**	**7300**

Main pole winding.

Length of coil, $8\frac{1}{4}$ ins. divided into three sections by two $\frac{3}{8}$ in. ventilating spaces, and with $\frac{3}{8}$ in. air channel between coil and pole-core. Net winding length $= 7\frac{1}{2}$ ins.

Depth of winding, $1\frac{1}{8}$ in. approx.

Length of mean turn $= 2(5 + 8) + 2\pi \times 3{\cdot}4375$

$= 47{\cdot}6$ ins. $= 1{\cdot}325$ yd.

,, ,, outer ,, $= 2(5 + 8) + 2\pi \times 4 = 51{\cdot}1$ ins.

AT_{s1} at 500 volts $= 5312$

AT_{s2} ,, 526 ,, $= 5312 \times 526/500 = 5600$

Series $AT_m = 7300 - 5600 = 1700$.

$\dfrac{L_s}{L_m}$ should $\therefore$ be $\dfrac{5600}{1700} \times 1{\cdot}1 =$ about $3{\cdot}6$.

Two shunt sections, each $2\frac{7}{8}$ ins. deep axially, and one series section $1\frac{3}{4}$ ins. deep.

Shunt winding.—Retaining 2 volts in rheostat, exciting voltage at full-load and 500 volts $= \frac{1}{6} \times 498 = 83$ per pole.

By eq. (124) $\omega' = \dfrac{83 \times 1000}{5845 \times 1{\cdot}325 \times 1{\cdot}16} = 9{\cdot}24$ ohms at 20° C.

By equation (126) $d = 0{\cdot}176/\sqrt{9{\cdot}24} = 0{\cdot}058$, s.c.c. to $0{\cdot}064$ in.

$5\frac{3}{4}/0{\cdot}064 = 90$ turns per layer

$1{\cdot}125$ in. $\div (0{\cdot}9 \times 0{\cdot}064) =$ say, 19 layers

Turns per coil $= 90 \times 19 =$	1710
Total yds. $= 1710 \times 6 \times 1{\cdot}325 =$	13,600
$R_s = 13{\cdot}6 \times 9{\cdot}24 = 125{\cdot}5$ ohms at 20° C.	
hot $= 125{\cdot}5 \times 1{\cdot}16 =$	145·5 ohms
$I_s = 498/145{\cdot}5 =$	3·42 amps.
$I_s^2 R_s/6 =$	283 watts

Cooling surface of shunt sections with one end-flange included,

$S_c = 51{\cdot}1 \times 5\frac{3}{4} + 47{\cdot}6 \times 1{\cdot}125 =$	347 sq. ins.
$k = t°\,\mathrm{C.} \times S_c/W = 36$ by Fig. 398	
Surface rise, $t°$ C. $= 36 \times 283/347 =$ say,	30° C

When running compound, $I_s = 3{\cdot}11$ at no-load which will rise to 3·28 with the compounding: the rheostat will therefore leave a margin of volts under the compound condition.

Weight of 1 yd. $= 11{\cdot}55 \times 0{\cdot}02445/9{\cdot}24 = 0{\cdot}0306$ lb.

„ „ shunt wire $= 13{,}600 \times 0{\cdot}0306 = 416$ lb., say, 3¾ cwt.

Series winding in two parallels of 3 bobbins.

No. of turns at 525 volts $= 1700/427 = 4$, nearly.

$1{\cdot}75/5 = 0{\cdot}35$ in. Single spiral of bare copper on edge, 1 in $\times$ 0·27 in.

ω' per 1000 yds. $= 0{\cdot}02445/0{\cdot}27 = 0{\cdot}0907$ ohm.

No. of yds. in all the series coils $= 1{\cdot}325 \times 4 \times 6 = 31{\cdot}8$.

Resistance of the 2 parallels

$$= \tfrac{1}{4}(0{\cdot}0318 \times 0{\cdot}0907) \times 1{\cdot}15 = 0{\cdot}00083 \text{ ohm.}$$

Loss of volts $= 855 \times 0{\cdot}00083 = 0{\cdot}71$

„ „ watts $= 0{\cdot}71 \times 855 = 606 \div 101$ per coil.

$S_c = 51{\cdot}1 \times 1\tfrac{3}{4} + 47{\cdot}6 \times 1 = 137$ sq. ins.

$t°$ C. $= 35 \times 101/137 = 25{\cdot}8°$ C.

Commutation.

Carbon brushes, 7 per brush arm, each 1½ ins. × ¾ in.

Brush contact area = 7·875 sq. ins. per arm.

Current-density $= I_a/(7{\cdot}875 \times 3)$ amps. per sq. in. = 38·25 or 36·25.

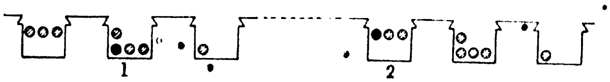

FIG. 417.—Conductors simultaneously short-circuited in two interpolar zones.

Pitch of sectors $= (26 \times 3{\cdot}14)/342 = 0{\cdot}238$ in.

$$\frac{b_1 - b_m}{b} = \left(\frac{0{\cdot}75 - 0{\cdot}03}{0{\cdot}238}\right) = \text{barely over 3, but say 4}$$

Two zones of short-circuited coil-sides, as shown in Fig. 417.

$$k_1' = 3\left(\frac{2\pi}{3} + 2\pi\right) + \pi = 28{\cdot}25$$

$$k_2' = 3 \times \frac{2\pi}{3} = 6{\cdot}28$$

$$a_1'' \text{ or } a_2'' \text{ by eq. (188)} = 12{\cdot}57\left(\frac{2 \times 0{\cdot}36}{0{\cdot}65 + 0{\cdot}45} + 0{\cdot}2\right) = 10{\cdot}75$$

$D/2pw_s = 40/(6 \times 0{\cdot}45) = 14{\cdot}8$. By Fig. 361, $b_1 = 17$, $b_2 = 14$

$k_1' h_w/w_s + j_{a1} \times a_1'' + j_{b1} b_1$

$= 28{\cdot}25\ (1{\cdot}39/0{\cdot}45) + 4 \times 10{\cdot}75 + 8 \times 17$

$= 87{\cdot}2 + 43 + 136 = \qquad 266{\cdot}2$

$k_2' h_w/w_s + j_{a2} \times a_2'' + j_{b2} b_2$

$= 6{\cdot}28(1{\cdot}39/0{\cdot}45) + 3 \times 10{\cdot}75 + 8 \times 14$

$= 19{\cdot}4 + 32{\cdot}2 + 112 = \qquad 163{\cdot}6$

$\qquad 429{\cdot}8$

Iron length of core, $(14 - 2) \times 2{\cdot}54 = 30{\cdot}5$ cm.

$429{\cdot}8 \times 30{\cdot}5 = 13{,}100$

$$\lambda' = j_c e_c = 4 \times 3 \log \frac{25{\cdot}58 + 2}{0{\cdot}665 + 3 \times 0{\cdot}105}$$

by eq. (189) $= 12 \log 28{\cdot}1 = 17{\cdot}4$

$2l'\lambda' = 2 \times 27{\cdot}58 \times 2{\cdot}54 \times 17{\cdot}4 \qquad = 2{,}440$

$\mathcal{L} + \Sigma . \mathcal{M} = 15{,}540 \times 10^{-9}$ henrys

$$T \text{ by eq. (190)} = \frac{0{\cdot}75 - 0{\cdot}03}{26 \times 3{\cdot}14 \times 400/60} = \qquad 0{\cdot}001325 \text{ sec.}$$

$$(\mathcal{L} + \Sigma . \mathcal{M})\, 2J/T = \frac{2 \times 0{\cdot}00001554 \times 150}{0{\cdot}001325} = \qquad 3{\cdot}52 \text{ volts}$$

$$JZ/4p = \frac{150 \times 684}{12} = 8550$$

$$B_g = \frac{1{\cdot}257\, m\, JZ/4p}{\xi c + Kl_g} = \frac{1{\cdot}257 \times 0{\cdot}9 \times 8550}{1{\cdot}7 \times 3{\cdot}15 \times 2{\cdot}54 + 0{\cdot}537} = 685$$

$L_r = 12$ ins.

$v = 40 \times 3{\cdot}14 \times 2{\cdot}54 \times 400/60 \quad 2125$ cm. per sec.

Average B_{rc} by eq. (194) $=$

$$\frac{150}{12 \times 2{\cdot}54 \times 2125 \times 10^{-8}}(0{\cdot}0116) + 685 \frac{2}{12} = 2680 + 114 = 2794$$

Commutating pole cast solid with yoke.

$l_{gr} = 0{\cdot}150$ in. $= 0{\cdot}381$ cm.

$AT_{gr} = 0{\cdot}8\, B_{gr}\, K_r\, l_{gr} = 0{\cdot}8 \times 2794 \times 1{\cdot}16 \times 0{\cdot}381 = \qquad 990$

Breadth of commutating pole-face by equation (205)

$$w_c = (0{\cdot}75 - 0{\cdot}239)\frac{40}{26} + 2{\cdot}2 - 0{\cdot}12 \times 2 = 2{\cdot}74 \text{ in., say } 2\tfrac{3}{4} \text{ ins.}$$

Effective polar arc of commutating pole $= w_c + 2{\cdot}25\, l_{gr} = 2{\cdot}84$ ins.

Area of pole-shoe $= 2{\cdot}84 \times 12 \times 6{\cdot}45 = 220$ sq. cm.

$\phi_r \ = 220 \times 2794 \qquad = 615{,}000$

$\phi_{lr} = \frac{3}{4} \times 1{\cdot}266 \times 10^6 = \underline{950{,}000}$

$\phi_{mr} = \qquad\qquad 1{,}565{,}000$

Commutating-pole core $9\frac{1}{2}$ ins. $\times$ 2 ins., with corners rounded to in. radius. Area, $7{\cdot}5 \times 2 + \pi \times 1^2 = 18{\cdot}14$ sq. ins. $= 117$ sq. cm.

$B_{mr} = 1{,}565{,}000/117 = 13{,}400$

at by Fig. 207 $= 10$. $\quad l_{cp} = 28$ cm.

$AT_{mr} = 10 \times 28 = \qquad 280$

$ac\ (Y/2 - w_c/4) = \dfrac{8550}{10{\cdot}46}(10{\cdot}46 - 0{\cdot}625) = \qquad 8050$

$\dfrac{\Phi_m + \phi_{mr}}{a_y} = \dfrac{(13{\cdot}216 + 1{\cdot}565) \times 10^6}{960} = 15{,}400 \qquad \dfrac{\Phi_m - \phi_{mr}}{a_y} = \dfrac{(13{\cdot}216 - 1{\cdot}565) \times 10^6}{960} = 12{,}150$

$at = 18 \qquad\qquad = 9$

Difference $= 9$

$(l_y/4) \times 9 = 24{\cdot}15 \times 9 = \qquad 217$

By equation (173), $AT_r = 990 + 80 + 8050 + 280 + 217 = 9617$

Commutating-pole coils, 2 sets in parallel, each of 3 in series.

Turns per coil, $9617/450 =$ say, $21\frac{1}{2}$.

Length of coil, $8\frac{3}{4}$ in.

$8{\cdot}75/22 = 0{\cdot}398$ in.

$\underline{0{\cdot}128}$,, clearance between turns

$0{\cdot}27$,,

Single open spiral of bare copper, 1 in. $\times$ 0·27 in. as for series inding.

Mean length of turn $= 2 \times 7{\cdot}5 + 2\pi \times 1{\cdot}875 = 26{\cdot}75$ ins. $= 0{\cdot}743$ yd.

Outer ,, ,, ,, $= 2 \times 7{\cdot}5 + 2\pi \times 2{\cdot}375 = 29{\cdot}8$ ins.

No. of yds. in all the commutating coils $= 0{\cdot}743 \times 21\frac{1}{2} \times 6 = 96$

Resistance of the 2 parallels, hot

$= \frac{1}{4}(0{\cdot}096 \times 0{\cdot}0907) \times 1{\cdot}13 = 0{\cdot}00246$ ohm.

Loss of volts $= 2{\cdot}21$ or $2{\cdot}1$.

Watts $= 1990 = 332$ per pole.

Cooling surface $= 29{\cdot}8 \times 8\frac{3}{4} + 26{\cdot}75 \times 2 = \qquad 313{\cdot}5$ sq. ins.

For open spiral, $k =$ say, 29.

$t°$ C. $= 29 \times 332/313{\cdot}5 = \qquad 30{\cdot}7°$ C.

Weight of 1 yd. $= 11{\cdot}55 \times 0{\cdot}27 = 3{\cdot}12$ lb.

,, ,, series and comm. pole coils $= 3{\cdot}12(96 + 31{\cdot}8) = 400$ lb.

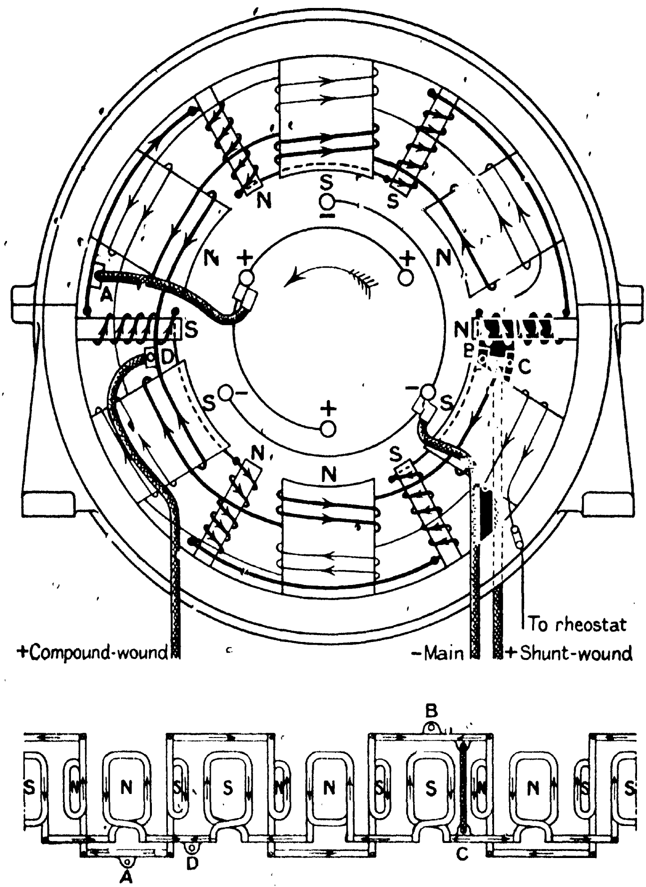

FIG. 418.—Field-magnet connexions of 450 kW dynamo, with development when the uppermost pole is divided and the yoke-ring opened out from the top.

Heating of commutator.

Peripheral speed of commutator, $\pi \times 400 \times 26/12$
$= 2720$ ft. per min.

Total brush pressure at $1\frac{1}{2}$ lb. per sq. in.
$= 1{\cdot}5 \times 7{\cdot}875 \times 6 = 71$ lb.

Taking $\mu = 0{\cdot}3$, by equation (214) brush friction
$= 0{\cdot}3 \times 71 \times 2{\cdot}72 \times 22{\cdot}6 = 1310$ watts

$I_a^2R_b = 2 \times 903$ when running shunt-wound	=	1806 watts
Brush friction loss		1310 ,,
		3116 ,,

External cylindrical surface, $\pi \times 26 \times 16 = 1305$
One face of lugs $1\frac{1}{2} \times 3 \times 842 = 1540$ $\Big\} 2845 = S_c$

$S_c\{1 + 0{\cdot}3 \times 2{\cdot}72^{1{\cdot}3}\} = 2845 \times 2{\cdot}1 =$ 5960

By equation (Chap. XXI, § 27). $t°$ C. $=55 \times 3116/5960 =$ 28·7° C.

Efficiency, at full-load, when running compound

$I_a^2R_a$	7250 watts
Hysteresis loss	3500 ,,
Eddy loss	7000 ,,
$I_a^2R_b$	1715 ,,
$I_a^2R_{cp}$	1990 ,,
$I_s^2R_s$	1560 ,,
Rheostat	160 ,,
Brush friction	1310 ,,
One bearing and air friction	2000 ,,
Total losses	26,485 ,,
Output	450,000 ,,
Input	476,485 ,,
Efficiency	94·4%

§ 12. Further illustrations of machines.—Fig. 419 shows a 500 kW generator, 250 volts, 2000 amperes, for direct driving at 300 revs. per min., and as further examples of the design of larger continuous-current generators there are added photographs[1] of two machines built by the Allis-Chalmers Manufacturing Co. of Milwaukee, Wisconsin, U.S.A., and of a third manufactured by the Oerlikon Co., Switzerland.

Fig. 420 shows a large slow-speed machine for 1500 kW, 240-250 volts, 90 revs. per min., built by the Allis-Chalmers Co. for direct coupling to a Diesel oil engine. On the other hand, Figs. 421 (*a*, *b*, *c*) show a machine for 3000 kW, 300 volts, 10,000 amperes, 300 revs. per min., supplied by the Allis-Chalmers Co. to the Norsk Aluminium Co., Norway, and driven by a water turbine. Fig. 421*b* shows the armature core with commutator of shrink-ring

[1] For which the author is indebted to the courtesy of the firms named.

construction; one of the main points calling for care in the design of hydro-electric direct-coupled units is the high speed that may be attained by the water turbine or water wheel, and in the present case an overspeed of 540 revs. per min. has been taken into account in the design. A novel feature is that by means of a hollow brush yoke support, ring and hollow brush forks, air is blown on to the

FIG. 419. 500 kW dynamo, 250 volts, 2000 amperes, 300 revs. per min.
(Messrs. W. H. Allen, Sons & Co., Ltd.)

commutator, the brush forks having holes drilled in them so as to deflect the air on to the commutator directly in front of each brush. The auxiliary blower furnishing the air is seen in Fig. 421*a*. The bracing of the end-connexions of the compensating field winding is well seen in Fig. 421*c*.

Another large continuous-current dynamo, driven by a water turbine at 300 revs. per min., is shown in Fig. 422. Three such machines were constructed by the Oerlikon Co. for the Chippis

(Valais) works of the Neuhausen Aluminium Co. The normal output of the machine is 2650 kW, 340 volts, 7800 amperes, but it is capable of a continuous load of 3000 kW, 375 volts, 8000 amperes. Further, it is capable of overloads of 25 per cent. for

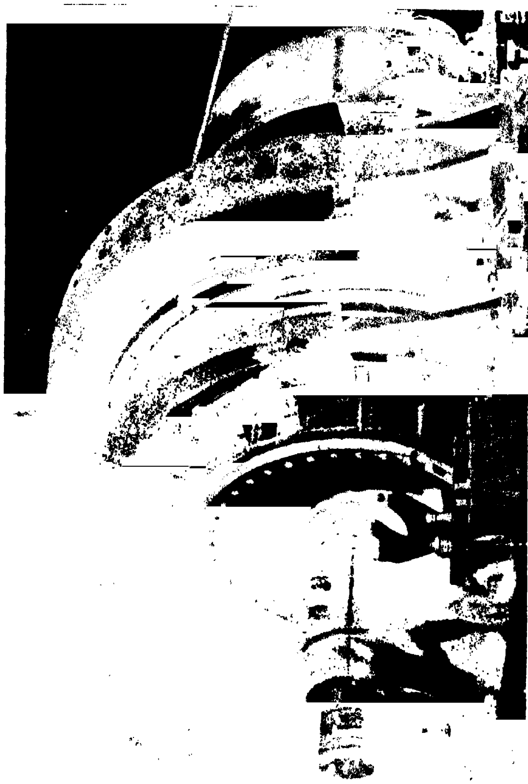

FIG. 420.—1500 kW, 240-250 volt, 90 r.p.m. dynamo. (Allis-Chalmers Mfg. Co.)

2 hours (10 per cent. increase in voltage, 15 per cent. in current) and of 50 per cent. for half an hour (10–15 per cent. increase in voltage, 40–35 per cent. in current). Its efficiency, as calculated from measured losses, is 93 per cent. at half-load, 94·5 per cent. at full-load, and the maximum temperature rise after 24 hours' run 45° C. The commutator has 3 shrink-rings, and carbon brushes

FIG. 421a.—Compensated generator, 3000 kW, 300 volt, 10,000 amperes, driven by water turbine at 300 r.p.m., overspeed 540 r.p.m.
(Allis-Chalmers Mfg. Co.)

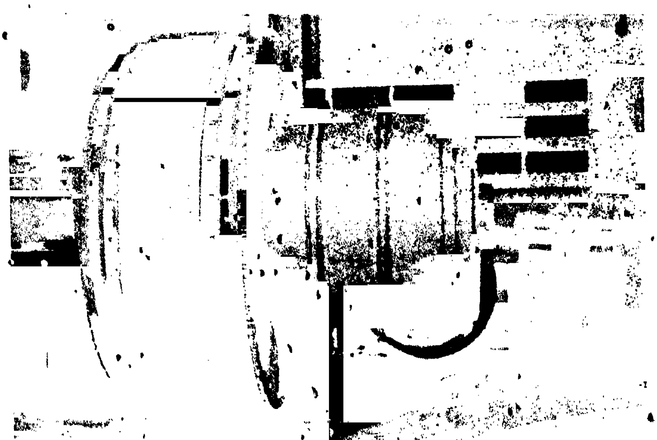

FIG. 421b.—Armature core and commutator for 3000 kW dynamo of Fig. 421a.

are employed. The shaft is carried on high pedestals to render the lowest brushes easily accessible; while a ladder, insulated from the baseplate, enables the highest brushes to be attended to easily. The number of poles is 20, with the same number of commutating poles.

The continuous-current turbo-dynamo, coupled directly to the steam-turbine, being now less frequently built, it suffices to state that in the larger sizes for central station work, the field-magnet has usually been totally enclosed by cast-iron casings at the ends,

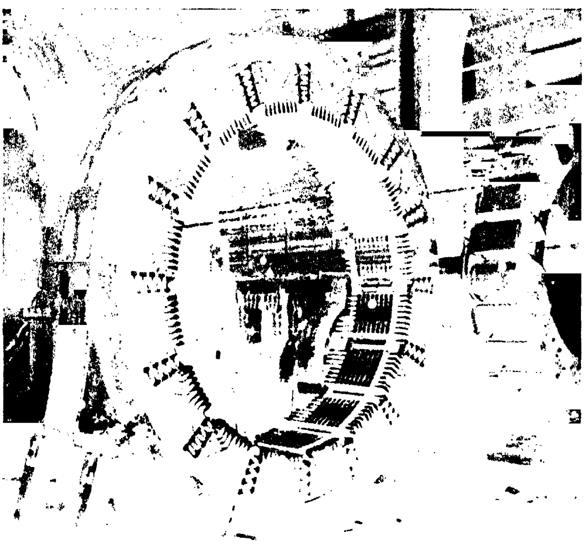

FIG. 421c.—Field-magnet of 3000 kW dynamo of Fig. 421a.

so that only the commutator projects into the open, and the ventilation is then assisted by a fan mounted on the shaft at the rear end of the armature where the air enters. On leaving the fan chamber, the stream of air divides, part passing through the bore and past the compensating winding ends to an outlet at the top of the machine, and part passing by axial canals into the armature core outwards through radial ducts into the bore to join the former stream and also partly onwards to the front end of the armature past the winding, whence it is driven out by the commutator lugs.

A non-salient-pole construction of field-magnet has often been

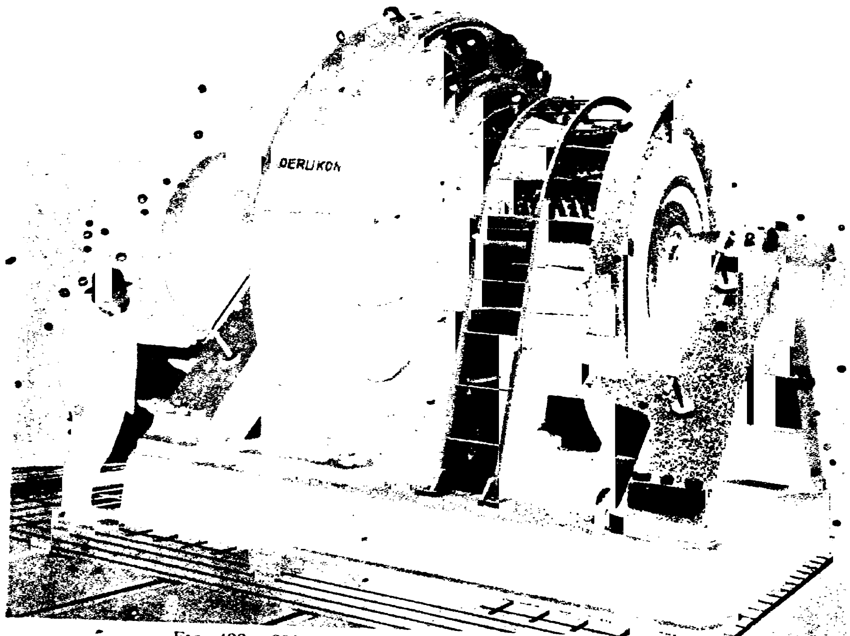

FIG. 422.—3000 kW, 375 volt, 300 r.p.m. dynamo. (Oerlikon Co.)

employed, built up of discs slotted on the inner periphery to receive the exciting coils and also in quadrature with them the compensating winding, so that when finished the interior is practically smooth. The scheme of the compensating winding as employed by Messrs. Brown-Boveri & Co. is illustrated in Fig. 423 for a 4-pole machine; each small inner loop embraces a broad commutating tooth lying between two larger slots which hold the main field-coils: each pole-face then has several slots to take the compensating turns of copper strip placed edgewise in the slots and locked therein by wedges.

§ 13. Measurement of losses by calibrated motor.—The three principal methods of measuring the losses which occur in continuous-current dynamos may here be introduced as bearing on their design.

Given a machine whose losses as a motor at different horse-powers and speeds have been very carefully determined with its brushes adjusted to the best position for minimum loss, it can then be

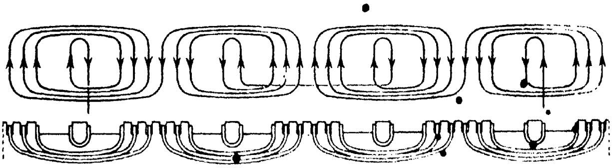

FIG. 423.—Developed diagram of compensating field-winding for turbo dynamos.

rigidly coupled to another machine, and the losses in the latter determined from the horse-power transmitted through the coupling from the motor, *i.e.* from the additional watts taken by the motor so far as these actually correspond to useful horse-power. In this way an unwound core unexcited but with its wooden wedges in place or its slots temporarily filled with wood strips can be tested for friction and windage, for brush friction, and, when excited, for its additional no-load loss due to hysteresis and eddy-currents in its core. But in any such test the accurate alignment of the shafts within their bearings is very important, and the frictional losses should be specially checked and compared with those which may be debited to each machine under normal conditions. Since the friction loss is inversely proportional to the temperature of the oil, the machine must be run for a sufficient time to allow the bearings to reach a constant temperature. Further slight changes in the pressure on the shafts due to unbalanced magnetic pull when the field of the driven machine is excited may cause a slightly different alignment sufficient to alter the friction appreciably.

§ 14. Measurement of losses by motor-current method.—The

second method[1] consists in running the machine as a motor without load at various speeds, and noting the current through its armature, the voltage applied to it, and the speed, the excitation being kept constant at the desired value for each reading of these quantities.

The mechanical power developed in the armature of a continuous-current motor is $E_a \times I_a = (V_e - I_aR_a - I_aR_b)\, I_a$, where V_e = the potential difference impressed on the motor armature at the brushes, I_a = the current through it, and R_a, R_b are the resistances of the armature and of the two sets of brushes of opposite sign. Since the motor is now assumed to be running light, I_a is small, and with a fairly large armature of low resistance I_aR_a is practically negligible as compared with V_e; the loss of volts over the resistance of carbon brushes should, however, be taken into account even at no-load. Subject to this deduction, the back E.M.F. of the motor is closely equal to the voltage impressed upon it, and with a fixed excitation the speed is easily varied throughout a wide range by increasing or decreasing the applied voltage. Since the field Φ_a during the test is kept constant, the back E.M.F. of the armature $E_a = \frac{p}{a} \Phi_a \times Z \cdot \frac{N}{60} \times 10^{-8}$ volts is proportional to the speed, and when plotted in relation thereto gives an inclined straight line passing through the origin (Fig. 425).

Again, since Φ_a is constant, the total torque rotating the armature is proportional to the current I_a. The latter may therefore be mentally split up into three portions, the first supplying the torque to overcome the resisting force from friction of the bearings and windage, the second corresponding to the torque from hysteresis, and the third to that from eddy-currents. For the moment let it be supposed that the first item has been deducted from I_a, and that the remainder when plotted in relation to speed gives the line AB (Fig. 424). Now, the losses from hysteresis and eddy-currents being proportional respectively to the speed and to the square of the speed (Chapter XXI, §§ 21–23), or $W_H + W_E = HN + FN^2$ (where H and F are two constants for the machine with a given excitation), the torque from hysteresis is $\propto H$ and is a constant, while the torque from eddies is $\propto FN$ and is proportional to the speed. The required division of the current AB is at once obtained as shown in Fig. 424; for it must be made up of a constant portion of height OA and a portion rising in an inclined straight line OD from the origin in proportion to the speed. If we experimentally determine a number of points on the line AB, by producing it backwards to cut the vertical at A the constant H can be determined, and from the differences, say $PQ - QG$, the constant F can be determined. The rate of loss in watts from both hysteresis and

[1] *Electrician*, Vol. 26, pp. 699, 700; and Vol. 27, p. 162.

eddies at any particular speed N is then obtained by multiplying together the corresponding ordinates of the current and voltage $= QP \times E_a$; the watts absorbed by hysteresis are $W_H = QG \times E_a$, and by eddies are $W_E = GP \times E_a$. Thence $H = W_H/N$ and $F = W_E/N^2$. Or if the iron losses at two speeds N_1 and N_2, the former high and the latter low, are respectively w_1 and w_2, the two coefficients[1] are

$$H = \frac{N_1^2 . w_2 - N_2^2 . w_1}{N_1^2 . N_2 - N_2^2 . N_1}$$

$$F = \frac{N_2 . w_1 - N_1 . w_2}{N_1^2 . N_2 - N_2^2 . N_1}$$

So far the loss from friction of the bearings and from windage has been supposed to be previously known and eliminated. For a constant intensity of pressure per square inch of bearing surface and for a constant temperature the law that the coefficient of friction, and therefore the torque, varies as the square root of the velocity of the shaft, holds as approximately true for all ordinary peripheral speeds ranging between 150 and 500 feet per minute (Chapter XIII, § 12). Hence in ordinary running the friction torque and the component of the no-load motor current which is proportional thereto, when plotted in relation to speed, should be slightly bowed or concave to the horizontal axis.

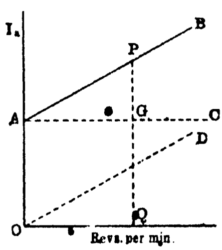

FIG. 424.—Separation of eddy-current from hysteresis and friction losses.

On the other hand, the torque from the air resistance is approximately proportional to the square of the speed, especially when there are ventilating air-ducts with numerous blades interspersed along the armature core[2]; and since the aim of the designer is to produce the maximum of cooling action, by disposing the armature winding and arms of the end-plates to act as an effective fan, the proportion of the air-friction to that of the bearings may be quite appreciable. The loss from bearing friction and windage is thus $w_f = w_{(b+a)} = f_b N^{1.5} + f_a N^3$, and the effect is to raise the friction current with increasing speeds more nearly to a constant quantity. There is thus some justification for regarding the loss from friction and windage as more or less proportional to the speed, or as equal to the speed multiplied by a coefficient analogous to H for hysteresis.

[1] For certain cautions as to the application of these values in designing armatures, *vide* Chap. XXI, § 18.

[2] The true law of the windage loss has, however, been shown by Prof. W. M. Thornton (*Journ. I.E.E.*, Vol. 50, p. 498) to be very complex.

On this assumption, for $W_H = HN$ may be substituted $W_{(H+f)} = (H+f)N$, and the total current is again an inclined straight line as in Fig. 424, but divisible into the two portions, viz. a constant amount proportional to the torque from hysteresis, friction, and windage, and a portion increasing with the speed, so that

$$H + f = \frac{W_{(H+f)}}{N}, \text{ and } F \text{ as before} = \frac{W_F}{N^2}$$

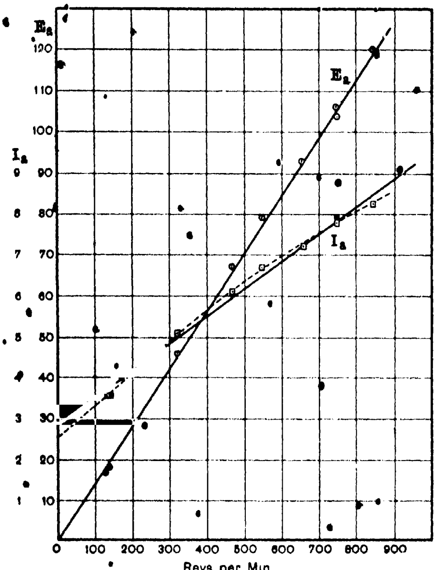

FIG. 425. Motor-current test of dynamo with armature 12″ diam. × 7″ long.

Experiment shows that in most cases the total armature current is not far from a straight line, although often slightly bowed. At high speeds the torque from air-friction increasing faster than the speed may render the current-curve even convex to the abscissa axis.[1]

Fig. 425 shows the observed results of a test on a small machine, the values of the induced E.M.F. being given by the inclined straight line passing through the origin, and the corresponding values of the armature current by the inclined straight line which, when produced backwards, gives the initial or starting value of 2·85 amperes. The true curve is probably shown by the dotted line to which the straight

[1] *Cp.* the experimentally obtained figures, *Electrician*, Vol. 52, p. 831.

line is a fair approximation. At 600 revs. per min., with an induced E.M.F. of 85 volts, the total current taken is 6·85 amperes; since the current required to balance the torque from hysteresis and friction is 2·85 amperes, the friction, windage, and hysteresis loss

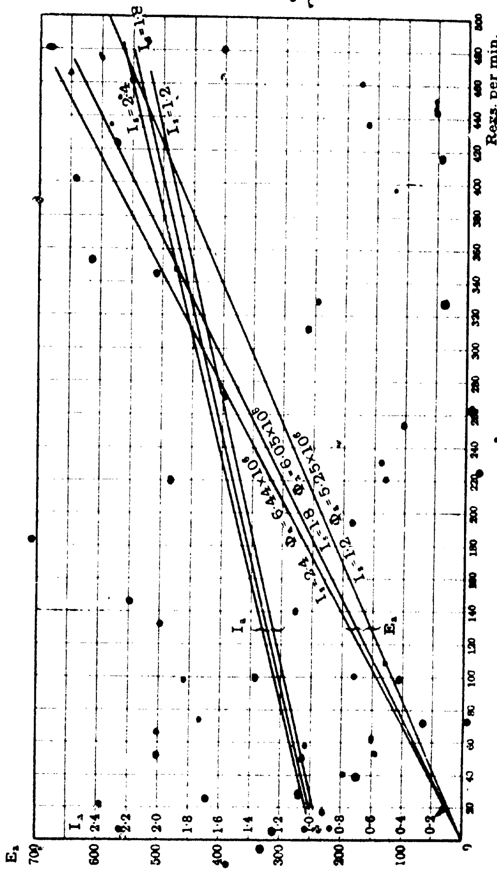

FIG. 426.—Motor-current test of armature 21" × 11".

is 85 × 2·85 = 242 watts, and the difference of 4 amperes multiplied by 85 volts gives the loss by eddy-currents in the armature, namely, 340 watts.

Fig. 426 gives the corresponding values of E_a and I_a for a larger 4-pole machine with toothed armature 21" diam. × 11" long under three different degrees of excitation; the ampere-turns per pole

were respectively 4850, 7250, and 9675, and the air-gap densities 6200, 7140, and 7600. At the normal speed of 400 revs. per min., and the intermediate excitation, the losses by friction, windage and hysteresis and by eddies are respectively 545 volts × 0·945 amperes = 515 watts, and 545 × 1·035 = 565 watts, making a total of 1080 watts in all.

Owing to the objection that the loss from the mechanical friction of the bearings and by windage is not strictly proportional to the speed, the *motor-current test* is inferior in accuracy to the retardation method to be described in the next section; yet it is simpler and on the whole yields much useful information, so long as the current readings do not diverge greatly from an inclined straight line. It is essential with carbon brushes that their position should be adjusted to give the minimum loss, since by an incorrect setting a considerable additional loss can result from unequal current-density over the contact surfaces of the brushes. There remains, too, the objection that it is difficult in practice to keep the bearings in a steady normal state under the changes of speed which are a necessary part of the test.[1] Usually a run of three or four hours is required to attain to a steady temperature of the bearings, without which all motor-current tests are open to considerable inaccuracy. Even then a change from a high to a low speed temporarily reduces the friction loss to a value below that which would be obtained in steady running, or *vice versa* for the reverse change, so that some minutes must be allowed to elapse before reading the current after a change of speed. It is best to take two sets of readings, the one with ascending and the other with descending speeds, and to note their exact sequence.[2] At very low speeds the friction may increase owing to the lesser amount of oil swept into the bearing by the rotating shaft, or owing to the oil rings failing to act steadily and satisfactorily.

§ 15. Retardation or "running-down" method of measuring losses.—The third or *retardation method*, first applied by M. Routin,[3] necessitates a knowledge of the moment of inertia of the rotating armature, but possesses several incidental advantages. No assumption is made as to the law which the friction obeys, and its actual amount can be very accurately measured, since the changes of speed are automatically made in a definite sequence and with

[1] *Cp.* Finzi, *E.T.Z.* (1903) p. 917.

[2] *Electrician*, Vol. 44, p. 323.

[3] *L'Eclairage Electrique*, Vol. 9, p. 169; Dr. Alfred Hay, *Electr. Review*, Vol. 47, p. 287; Chas. F. Smith, "The Experimental Determination of the Losses in Motors," *Journ. I.E.E.*, Vol. 39, p. 437. The reader is especially referred to Prof. D. Robertson's paper on "The Separation of the No-load Stray Losses in Continuous-current Machines by Stroboscopic Running Down Methods," *Journ. I.E.E.*, Vol. 53, p. 308, where other references are also given and many points of detail discussed.

perfect regularity in each repetition of the test. The armature is first run up to or above full speed, and the driving power is cut off; it is then allowed to come to rest, while at small intervals of time the speed is measured either directly or indirectly. If the speeds are converted into angular velocities ($\omega = 2\pi N/60$) in radians per sec. and plotted in relation to time in seconds, at any given moment let ω be the angular velocity, and let a tangent be drawn to the curve at the point corresponding to the moment in question. The tangent then measures the time-rate of change of the angular velocity, or $d\omega/dt$. Let $I = Mk^2$ be the moment of inertia of the revolving part in C.G.S. units, *i.e.* in grammes of mass × (centimetres)^2. At any point of time as the rotor comes to rest the rate of change of its angular momentum is equal to the retarding torque acting on it in C.G.S. units; or

$$-T = I\frac{d\omega}{dt} \text{ dyne-centimetres,}$$

and the rate at which the stored kinetic energy is expended in overcoming the retarding torque is

$$T\omega = -I\omega\frac{d\omega}{dt} \text{ ergs per second,}$$

or

$$= -I\omega\frac{d\omega}{dt} \times 10^{-7} \text{ watts}$$

Since a kilogramme-(metre)^2 is 10^{-7} C.G.S. units, the moment of inertia I is best expressed in kilogramme-(metres)^2, when the factor 10^{-7} cancels out, and the expression reduces to

$$\text{watts} = -I\omega\frac{d\omega}{dt}$$

Since the velocity is usually measured in revs. per min., and plotted in relation to seconds of time, this may also conveniently be expressed as

$$\text{watts} = -I\frac{2\pi N}{60}\cdot\frac{2\pi}{60}\cdot\frac{dN}{dt} = -I\frac{4\pi^2}{3600}N\frac{dN}{dt}$$

$$= -0{\cdot}01095 IN\frac{dN}{dt} \quad . \quad . \quad . \quad . \quad (220)$$

where I is in kilogramme-(metres)^2. But the tangent to the curve must then be read off in terms of the scales to which the curve is plotted.

The watts thus obtained are at any moment equal to the rate at which energy is dissipated in friction of bearings and brushes, windage, hysteresis, and eddy-currents. Thus if the retardation

curves be determined and plotted, first with the field unexcited and without brushes, then with brushes down, and lastly with the field excited to various strengths, a complete analysis of the various

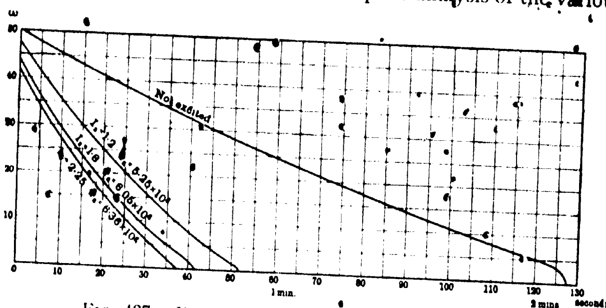

FIG. 427.—Retardation curve of 21" × 11" armature.

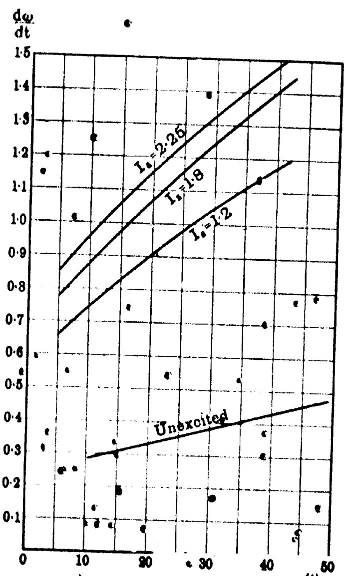

FIG. 428.—Derived curves of rate of change of angular velocity.

losses may be made. After the experimental curves have been taken, the derived curves of $d\omega/dt$, or of dN/dt, should be plotted in relation to ω or N, the tangents at a number of points being taken so as to check the correctness with which they have been deduced.

The torque is then proportional to the dN/dt curves, and the watt curves are obtained by multiplying dN/dt by N and by the constant $0{\cdot}01095I$.

Fig. 427 gives the retardation curves of the same dynamo as that for which the motor-current readings are given in Fig. 426; thence are obtained the derived curves of Fig. 428, and finally the total and the friction *plus* windage losses of Fig. 429. By deduction of

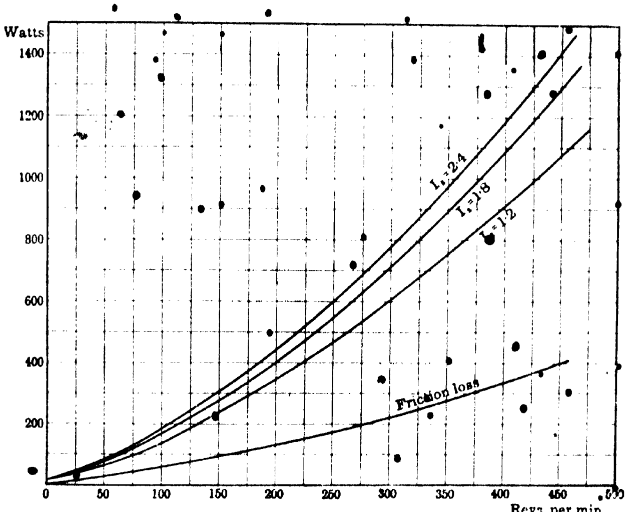

FIG. 429.—Losses in 21″ × 11″ armature.

the friction and windage loss from the total the curves of Fig. 430, for the losses by hysteresis and eddy-currents, are reached. The moment of inertia of the armature was $I_a = 17{\cdot}875$ kilogramme-(metres)².

In order to determine the bearing friction and windage, the retardation curve when the field is not excited is best taken by the following method. The machine is coupled by belt to a small motor, and by its means run up to a little above its full speed; the revolutions are taken on a speed-counter, and the voltage due simply to residual magnetism is read on a low-reading voltmeter with a pair only of small brushes resting on the commutator; the driving belt is then thrown off, and at intervals of, say, three seconds the voltage is read as it gradually dies away. The speed of the armature as it comes to rest is then simply proportional to its voltage, and by means of the residual magnetism can be measured indirectly, and more accurately than by a tachometer, which in itself adds an

indeterminate amount of friction and inertia comparable in small machines with that which is to be measured. Next, the field is excited, and the machine is run at its full speed as a motor. The armature circuit is broken while the voltmeter leads are left attached to the brushes; the same readings are taken of the voltage as it dies away, and a second curve is obtained. If the time of coming to rest is too short to allow of accurate readings being taken at successive intervals of a few seconds, the process must be continually repeated, a stop-watch being started each time that the driving

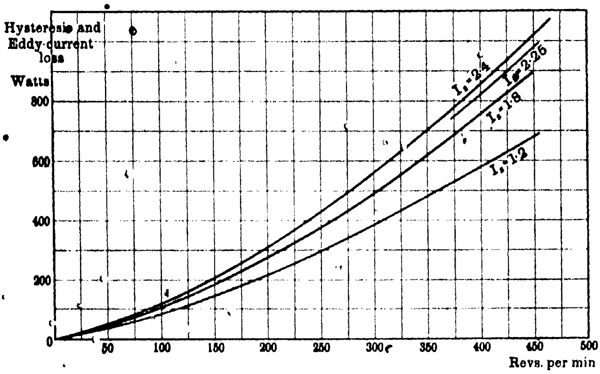

FIG. 430.—Loss by hysteresis and eddy-currents in 21″ × 11″ armature.

power is cut off, and again stopped when the voltmeter needle passes a prearranged point.[1] The difference between the two values of dN/dt on the two curves for the same speed N of the armature, when multiplied by $0{\cdot}01095IN$, measures the power absorbed by the hysteresis and eddy-currents in the excited field, and the latter may be given any desired value in order to test the effect of the flux in a given armature. The subsequent separation of the hysteresis loss from that by eddy-currents must be made on the approximate assumption that the former loss is proportional to the speed, and the latter to the square of the speed. Or graphically, by the principles of § 14, if the watts due to hysteresis and eddies are divided by the corresponding voltage, and the current so derived is plotted with the voltage as abscissæ, an inclined straight line is obtained, of which the intersection with the axis of ordinates measures the current required to overcome the torque due to hysteresis, while the line drawn parallel to the straight line of total current but

[1] For the accurate measurement of the speed, Prof. D. Robertson has devised a special stroboscopic disc, which is fully described in the paper referred to in the preceding foot-note.

passing through zero measures the current required to overcome the torque due to eddy-currents.

If a heavy fly-wheel be attached to the armature, the great advantage is gained that the time of slowing down is extended, and especially when the field is excited and the retarding torque is considerable, the measurements can be much more accurately taken. Further, since the moment of inertia of a fly-wheel with heavy rim can, owing to its symmetrical shape, be easily and accurately

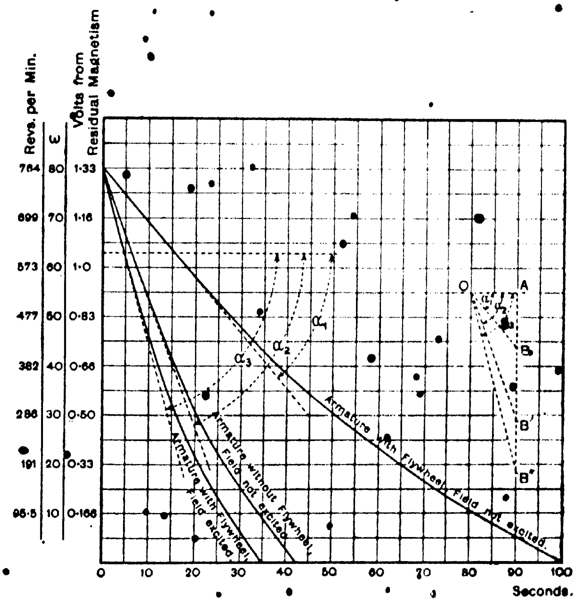

FIG. 431.—Retardation test of 12″ × 7″ dynamo.

calculated as I_w, an approximate calculation of the moment of inertia I_a of the armature alone will lead to but little error, since the total value $(I_w + I_a)$ has now to be substituted for I in equation (220). Care must, however, be taken that the bearing friction is not seriously altered by the addition of too heavy a fly-wheel, causing deflection of the shaft. This again may be checked by taking two no-load retardation curves, with and without the fly-wheel; taking the calculated value of I_a, the friction loss deduced from the two curves should coincide, or *vice versa*, the value of the moment of inertia obtained for the armature alone from the formula,

$$I_a = I_w \cdot \frac{d\omega_1/dt}{d\omega_2/dt - d\omega_1/dt} \quad . \quad . \quad . \quad . \quad (221)$$

(where $d\omega_1/dt$ and $d\omega_2/dt$ are respectively the rates of change with and without fly-wheel for the same value of ω) should agree with the calculated I_a. Thus in Fig. 431 a fly-wheel of known moment of inertia $I_w = 1{\cdot}8$ kilogramme-(metres)^2 was substituted for the usual pulley of the same dynamo for which the curves of Fig. 425 were found. The upper retardation curve was then obtained with the field unexcited, the speed being measured by means of the volts from residual magnetism as given in the third scale of ordinates. At 600 revs. per min. or $\omega = 62{\cdot}8$, $d\omega_1/dt = -1{\cdot}1$, angular velocities and seconds of time being plotted to the same scale and the angle α_1 being 47·75°; at the same speed and under the same conditions but without the fly-wheel, $d\omega_2/dt$ was $-2{\cdot}62$, the angle α_2 being 69·1°. Hence

$$I_a = 1{\cdot}8 \times \frac{-1{\cdot}1}{-2{\cdot}62 + 1{\cdot}1} = 1{\cdot}3$$

and the total moment of inertia of armature and fly-wheel = 3·1 kilogramme-(metres)^2. In any such method, since the denominator is the difference between two quantities, accurate readings are necessary to avoid considerable error.

The moment of inertia of an armature of moderate size can be conveniently determined by direct measurement of the periodic time of a complete oscillation when it is hung vertically by a bifilar suspension. If the two parallel wires, each of length l, are at equal distances of a from the vertical axis (Fig. 432), the radius of gyration k is found from the formula for the periodic time[1]

$$T_p = 2\pi\sqrt{\frac{l}{g} \times \frac{k^2}{a^2}}, \qquad \text{i.e. } k^2 = \frac{a^2 g T_p}{4\pi^2 l}$$

whence if M_1 and m are the masses of the armature and clip and of the clip alone, and T_{p1} and T_{p2} are the corresponding periodic times,

$$I_a = \frac{a^2 g}{4\pi^2 l}(MT_{p1}^2 - mT_{p2}^2)$$

To give I_a in kilogramme-(metres)^2, a and l must be expressed in metres, $g = 9{\cdot}81$ and M and m are in kilogrammes of mass; or if a' and l' are in feet, and W and w are the weights in lb.,

$$I_a = 0{\cdot}0344\frac{(a')^2}{l'}(WT_{p1}^2 - wT_{p2}^2) \text{ kilogramme-(metres)}^2$$

The length of l should be great as compared with $2a$.

In the case of a large armature, if its shaft of radius a is supported on knife-edges and it is set swinging with a small weight attached at radius l from the axis of the shaft (Fig. 433), the radius of gyration

[1] Cp. Journ. I.E.E., Vol. 31, p. 664, "The Breaking of Shafts in Direct-coupled Units," by Messrs. Frith and Lamb.

of the small weight itself being negligible as compared with l and k, then for small oscillations the square of the radius of gyration of the armature alone is

$$k^2 = \frac{m}{M}\left\{gl\frac{T_p^2}{4\pi^2} - (l-a)^2\right\} - a^2$$

where m is the mass of the attached weight, M is that of the armature, and T_p is the time in seconds of a complete swing to and fro. Neglecting a^2 as small in comparison with k^2, we then have

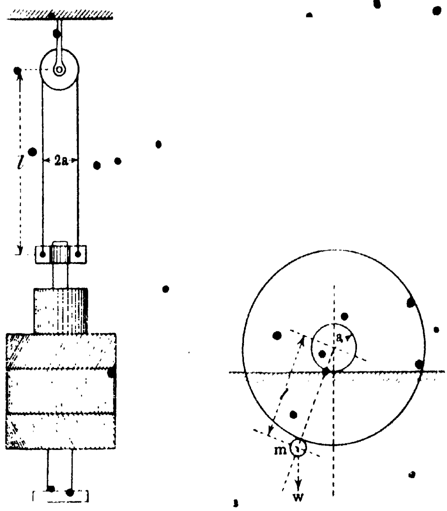

FIG. 432. FIG. 433.

$$I_a = Mk^2 = m\left\{gl\frac{T_p^2}{4\pi^2} - (l-a)^2\right\}$$

and if w is the weight in lb. of the small attached mass, and l' and a' are in feet

$$= 0{\cdot}0344\, wl'\left\{T_p^2 - 1{\cdot}22\frac{(l'-a')^2}{l'}\right\}\text{ kilogramme-(metres)}^2$$

The armature can alternatively be set rolling on curved knife-edges, the radius of which is accurately known; the periodic time of an oscillation thus obtained supplies a third means for calculating its moment of inertia.[1]

With still larger armatures having considerable moment of inertia, a retarding torque may be applied by means of a mechanical brake formed by a band or cord passed over the shaft or pulley, with one

[1] H. Cotton, *Beama*, Vol. 10 (1922), p. 132.

end carrying a weight and the other end anchored to a spring balance, or by means of an electrical load.[1] Thus the second intermediate curve of Fig. 431 might equally well have been obtained by an additional mechanical torque T_0; if the difference between the reading of the spring balance and the weight had been 4·16 lb. = 1·885 kilogrammes weight at a speed of 600 revs. per min. and the radius of the fly-wheel or pulley to which the brake was applied was 10 in. = 0·254 metre

$$T_0 = -1·885 \times 9·81 \times 0·254 = -4·7 \text{ metre-hectokilodynes.}$$

The moment of inertia of the armature and fly-wheel is then

$$I_0 = \frac{T_0}{d\omega_2/dt - d\omega_1/dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (222)$$

$$= \frac{-4·7}{-2·62 + 1·1} = 3·1 \text{ kilogramme-(metres)}^2.$$

When electrically loaded with an output at the rate of w watts, for T_0 will be substituted w/ω.

When fully excited and with the fly-wheel fitted in order to prolong the time of coming to rest, the lowest curve of Fig. 431 was obtained, whence at 600 revs. per min. $d\omega_3/dt = -3·5$, the angle α_3 being 74·1°. The watts expended in overcoming all losses are at 600 revs. per min. $3·1 \times 62·8 \times 3·5 = 680$, while the rate of loss by friction and windage is $3·1 \times 62·8 \times 1·1 = 214$, the difference of 466 watts being the loss by eddy-currents and hysteresis, which may be compared with the figure of 340 obtained for the eddy-loss alone from Fig. 425. Graphically, as shown at the side of Fig. 431, if the watts absorbed by friction are w_f, the watts absorbed by eddies and hysteresis are $w_f \times BB''/AB$, or if with the fly-wheel in position a brake had been applied to give the intermediate curve and the watts absorbed by the brake were $w_1 = T_0\omega$, the watts absorbed by eddies and hysteresis are $w_1 \times BB''/BB'$. The curves, such as Figs. 425 and 426, may also themselves be used to determine the moment of inertia of the armature; if at any speed and excitation the watts taken to drive the armature as a motor are w, and the rate of change of the revolutions at the same speed and excitation is dN/dt from the retardation curve, $\frac{w}{NdN/dt} = C$ should give in all cases consistent values of the constant $C = 0·01095I$. Indeed the two methods of the preceding and the present sections when combined afford a useful check upon the readings taken by the motor-current method, since for all speeds the determination of I from the retardation curve must give the same result if the readings are correct.

[1] Cp. Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 75, 76.

The moment of inertia of a continuous-current armature may also be directly determined electrically by a method due to Dr. G. Kapp,[1] in which current is supplied to the armature first so as to accelerate it, and then so as to retard it or to keep it running at constant speed.

For retardation as for motor-current tests it is essential that the machine should be run beforehand for a sufficient number of hours to allow of the bearings reaching a steady temperature, since upon this depends so largely the friction loss.

§ 16. The friction loss.—Experiment shows that, as the speed increases from rest, the coefficient of friction at first falls rapidly, is then nearly constant at its minimum value within a small range of speed, and lastly rises, giving a loss after the 1·5th power law. Correspondingly each retardation curve when very accurately taken shows towards its end an inversion point and sharp bend (*cp.* Fig. 427). The exact location of this point at which a kind of seizure takes place is chiefly dependent upon the speed at which the coefficient of friction reaches its minimum, but it also depends upon the proportion of the other retarding causes, whether hysteresis or eddies, at this speed; it occurs, in fact, at that speed for which the combined retarding torque from all causes is a minimum. Thus it is reached at a relatively higher speed when the friction of bearings and of the air is alone acting; but when the field is excited and the braking action of eddy-currents is added, since their torque is proportional to the speed, the minimum combined torque occurs at lower speeds. At very low speeds, the coefficient of friction is also affected by the rapidity of the speed-changes. There is a certain time-lag of the oil-film in point of thickness both when the machine is slowing down and also when it is being run up; but in the former case the coefficient of friction is smaller than would be the case if a steady state was reached, while in the latter case the opposite holds. The coefficient of friction at the end of the retardation curve therefore depends upon the time during which the process lasts, and its steady value cannot be thence obtained for very low speeds.[2]

Given the values of the friction watts w_{f1} and w_{f2} at two widely different speeds, N_1 and N_2, the former high and the latter low, on the assumption above mentioned that the loss from friction of the bearings increases as $N^{1.5}$ and that by windage as N^3, the two coefficients pertaining respectively to the bearings and air may be obtained in the manner suggested by Dr Finzi[3] as

$$f_b = \frac{N_1^3 \cdot w_{f2} - N_2^3 \cdot w_{f1}}{N_1^3 \cdot N_2^{1.5} - N_2^3 \cdot N_1^{1.5}} \qquad f_a = \frac{N_2^{1.5} \cdot w_{f1} - N_1^{1.5} \cdot w_{f2}}{N_1^3 \cdot N_2^{1.5} - N_2^3 \cdot N_1^{1.5}}$$

But the true law of the windage loss, as already mentioned (foot-note, p. 277) has been shown to be very complex.

A rough-and-ready empirical formula which gives reasonable values for the total watts from the friction of two bearings and from air resistance is

$$\text{watts} = 0{\cdot}0005\ W_a N$$

where W_a = weight of armature in lb. and N = revs. per min.

§ 17. Additional losses under load.—The sum of the losses, measured at no-load, with the addition of the calculated $I^2(R_a + R_b)$ loss, when used to calculate the efficiency under load, gives an efficiency which is recognized as "conventional" since it is usually

[1] *Journ. I.E.E.*, Vol. 44, p. 248. See also Dr. Sumpner, "The Testing of Motor Losses," *Journ. I.E.E.*, Vol. 31, p. 632, where the same principle is described and employed in a slightly different form.

[2] *E.T.Z.* (1903), p. 916 (Finzi). [3] *Electr. Eng.*, Vol. 32, p. 318. See also F. Honsu, *E.T.Z.*, Vol. 39 (1918), p. 435.

higher than the true figure. The difference between the actual losses under load and the calculated separate losses is an additional "stray loss," or as it is often called "load loss," which in continuous-current machines is to be attributed to—

(1) the increased hysteresis and eddy-current losses in the core and pole-shoes due to the distortion of the field by armature reaction under load, which raises the maximum flux-density in the teeth and alters the cycle of changes in the iron,

(2) the increased eddy current loss in the winding and slots at the pole-tips, also due to field distortion as explained in Chapter XXI, § 21 ; and

(3) secondary commutation losses, due to additional currents in the brushes, to non-uniform distribution of the current over the cross-section of large solid conductors under the action of inductance, and to losses in the iron from pulsation of the field set up by irregular commutation.

The amount of this extra "load loss" can be measured directly by methods given by Messrs. Erben and Page and by Messrs. Olin and Henderson.[1] The latter observers found in several machines with commutating poles that the actual losses at $\frac{1}{2}$, $\frac{3}{4}$, full-load, and 25 per cent. over-load exceeded the no-load core loss + the calculated I^2R_a loss by about 10, 20, 30, and 40 per cent. respectively. The difference which is thereby made in the calculated efficiencies ranges from 1 to $1\frac{1}{2}$ per cent., which must be taken into account when close calculations are necessary.

§ 18. Efficiency test by the Kapp-Hopkinson method.—The *efficiency* of a dynamo, or the ratio—

$$\frac{\text{power supplied from the terminals of the dynamo to the external circuit}}{\text{power given to the shaft of dynamo by the prime mover (engine or belt, etc.)}}$$

is often calculated on a "conventional" basis, as in §§ 10 and 11, by adding the various losses in field, armature, and friction to the output, and dividing the output by the sum of output + losses. But, for the reasons given in the preceding section, more reliable results are only to be obtained by direct measurement of the efficiency.[2] In cases where the dynamo is coupled directly to the engine, the power indicated in the engine cylinders, less the portion of this power which is wasted in the engine itself, gives the brake horse-power supplied to the dynamo ; the results, however, of such a method of calculation cannot be regarded as very accurate, owing to the difficulty of ascertaining the exact value of the waste in the engine. Another method is to transmit the power from the prime mover to the dynamo through a transmission dynamometer which

[1] *Trans. A.I.E.E.*, Vol. 32, Part I, pp. 524 and 480.

[2] *Cp.* especially Dr. C. V. Drysdale, *Engineering*, Vol. 80, p. 679, where a method of testing giving very accurate results is described.

registers the power passing through it; unfortunately, such a dynamometer when of large size is both costly and difficult to manage, and, further, does not admit of very accurate readings being obtained from it. A much more exact method is that due to Dr. Hopkinson,[1] by which two similar machines are so coupled together that the one acts as a motor driving the other as a dynamo. The output from the latter, being returned to the armature of the motor, supplies the greater part of the power required to drive it; and it is thus only necessary to supply to the motor from some external source the amount of power expended in the losses within the two machines. As this is but a small fraction of the total power developed, it is more easily measured on a transmission dynamometer, and even a large error in its determination, since it only affects the comparatively small item of the waste power, produces but slight error in the result. The only objection to the method is that it requires two machines of exactly similar size and output. These are driven at their normal speed and approximately at their normal voltage; the field of one, M, is, however, slightly weakened by a rheostat in its magnet circuit, so that its internal E.M.F. (E_2) is less than the terminal voltage (V_1) of the other machine D; hence D sends a current through M as a motor, and by means of the rheostat the amount of this current is regulated until it corresponds to the normal armature current of either machine. Let W = the total mechanical power in watts supplied from an external source to the motor armature, and I = the dynamo armature current which is also passed through the motor armature, the fields of both being separately excited. Neglecting the loss over the connecting leads, which can be made as small as desired, if we deduct from W the losses in the armature and brush resistances of D and M, the remainder, $W - I^2(R_{aD} + R_{aM})$, is the total loss by eddy-currents, hysteresis, and friction in the two armatures. The field of the one is stronger than that of the other, but if the E.M.F. of D is made slightly greater, and the E.M.F. of M slightly less than the normal voltage of either machine, the error will be very small when this loss is assumed to be equally divided between the two, so long as the machines are of such size, say, over 30 kW, that the efficiency of each is not less than about 90 per cent. Let

$$\frac{W - I^2(R_{aD} + R_{aM})}{2} = \frac{L}{2};$$

then the commercial efficiency of the dynamo is

$$\eta = \frac{V_1 I}{V_1 I + I^2 R_{aD} + L/2 + V_{f1} I_{f1}} \quad . \quad . \quad . \quad (223)$$

[1] A description of this method is to be found in Dr. Hopkinson's paper on "Dynamo Electric Machinery," *Phil. Trans.*, 1886, reprinted among his *Original Papers on Dynamo Machinery* (Pitman), p. 112 ff.

$V_{f1} I_{f1}$ being the watts expended in magnetizing the field of the dynamo. Where it is not necessary to obtain such great accuracy, part of the armature current of the dynamo D may also be used to excite the field-magnets of both machines; in this case, assuming the machines to be shunt-wound, if I_1 = the sum of the currents I_{aM} and i_{sM} supplied respectively to the armature and field of the motor, the efficiency of the dynamo is

$$\frac{V_1 I_1}{V_1 I_1 + I_{aD}^2 R_{aD} + L/2 + V_1 i_{sD}} \quad . \quad . \quad . \quad (224)$$

where $$L = W - I_{aD}^2 R_{aD} - I_{aM}^2 R_{aM} - V_1(i_{sD} + i_{sM})$$

It is convenient in carrying out these tests to couple the two shafts of motor and dynamo rigidly together, in one line; but it should be remarked that when this is done in the case of machines intended for belt driving the loss by friction in the bearings may not reach its normal amount, since the pull of the belt corresponds to the transmission of but a small fraction of the normal power. On the other hand, when the motor and dynamo are coupled together by belt, an extraneous loss of power in bending the belt is introduced.

A further improvement of the above method due to Dr. G. Kapp consists in the use of a third dynamo as the external source whence the waste of energy in the system is supplied. This auxiliary dynamo need be of but small size, and may be coupled either in series or preferably in parallel with the two machines which are to be tested. When the method is thus modified, all the measurements can be made electrically by one voltmeter and one ammeter, and further, as regards the efficiency of the two armatures, great accuracy in the calibration of the instruments is not of such vital importance.

Fig. 434 shows the series arrangement, in which the auxiliary dynamo A adds volts to the terminal voltage of the dynamo, and must be capable of carrying the full current of the machines to be tested. The voltage of the auxiliary machine, being approximately

$$= \frac{\text{total losses in the two machines}}{\text{armature current}}$$

will be from 20 to 40 per cent. of the voltage of either dynamo or motor according to their efficiency. The field of the dynamo D must be weakened by the rheostat r, and the two-way switch enables the observer to read in quick succession either the combined voltage on the motor (V_m) when the arm is placed on contact a, or the terminal voltage of the dynamo (V_e) when it is placed on b. If the dynamo and motor are both separately excited from a fourth dynamo, and I be the amperes passing through the system as read on the

ammeter, the power supplied to M is $V_m I$, while the power obtained from D is $V_1 I$. The voltage of the auxiliary dynamo is $V_m - V_1$, and the power added by it is $I (V_m - V_1)$. The combined efficiency of the two armatures of M and D is thus $\frac{V_1 I}{V_m I} = \frac{V_1}{V_m}$, and a close approximation to the efficiency of each separately is $\eta = \sqrt{\frac{V_1}{V_m}}$. A small percentage error in the voltmeter produces but little error in the

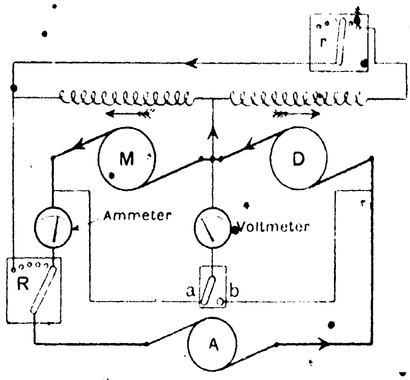

FIG. 434. Series electrical arrangement of Kapp Hopkinson efficiency test.

result, since the efficiency is only proportional to the square root of the voltage ratio. The fourth dynamo employed for separate excitation of the fields may also be dispensed with by the arrangement shown in the diagram, but in this case the system must be started by means of the switch and resistance marked R. When the arm of the switch is placed on the contact farthest to the left at starting, resistance is inserted in series with the armatures, and sufficient fall of potential is obtained to pass a small shunt current through the two fields in order to supply an initial excitation and start the motor. As the speed rises, the arm is brought over to the right until all the resistance is cut out of the armature circuit, while its presence in the shunt circuits will produce little effect. In calculating the efficiency, allowance must then be made for the fact that the dynamo armature is carrying not only its own shunt current but also that of the motor; hence, as in expression (224), if $I_1 = I + i_{sM}$, the combined efficiency of dynamo and motor is $\frac{V_1 I_1}{V_m I + i_{sM}^2 r_{sM}}$, and the square root of this gives approximately

the efficiency of either machine; or, separately, the efficiency of the dynamo

$$=\frac{V_1I_1}{\text{output of motor}}=\frac{V_1I_1}{V_mI-I^2R_{aM}-L/2}$$

and the efficiency of the motor

$$=\frac{V_mI-I^2R_{aM}-L/2}{V_mI+i_{sM}^2r_{sM}}$$

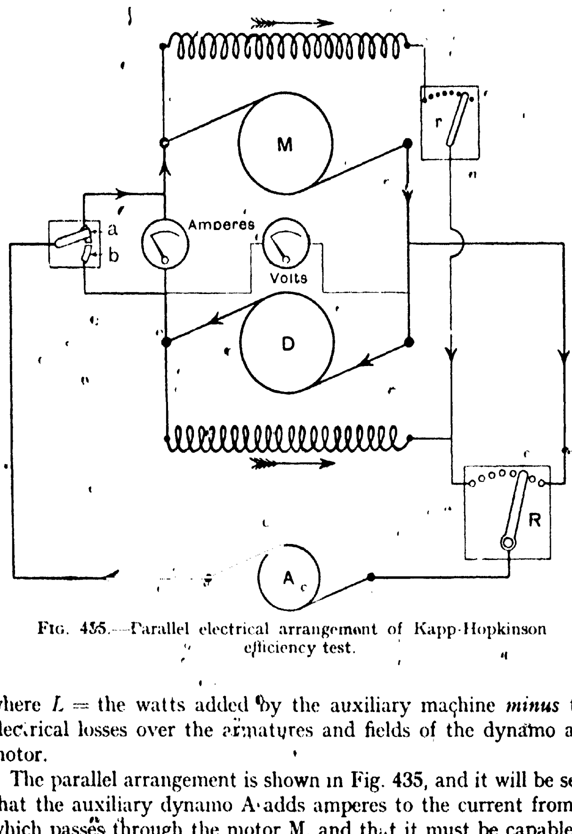

FIG. 435.—Parallel electrical arrangement of Kapp-Hopkinson efficiency test.

where L = the watts added by the auxiliary machine *minus* the electrical losses over the armatures and fields of the dynamo and motor.

The parallel arrangement is shown in Fig. 435, and it will be seen that the auxiliary dynamo A adds amperes to the current from D which passes through the motor M, and that it must be capable of giving the full voltage of the machines to be tested. It is now the field of the motor M which must be weakened by means of the rheostat r, and readings are taken of the amperes in quick succession for the two positions of the arm of the two-way switch.

When this is placed on the contact a, the dynamo current is read, exclusive of its own shunt current, and, when multiplied by the volts on the voltmeter, it gives the net output of the dynamo. When the two-way switch is placed on contact b, the ammeter reads the motor current inclusive of the shunt current of the motor, which, when multiplied by the volts, gives the total input to the motor. The combined efficiency of the two machines is then I_d/I_m, and the efficiency of each machine is very closely $\eta = \sqrt{\frac{I_d}{I_m}}$, the voltage being maintained constant during the short time necessary to take the two readings of amperes. The system is started by means of the switch and resistance R, with the arm thrown over to the left, and the same switch will also enable the voltage to be regulated to the right amount if the auxiliary dynamo gives a voltage slightly higher than that of the machines to be tested. In this arrangement, a small percentage error in the ammeter produces an almost negligible error in the result.

In both cases care must be taken to ensure the connexions to the fields being such as to cause the one machine to act as a motor and the other as a dynamo, and any change of the rheostats must be made gradually, so that its effect on the system may not be masked by the inertia of the revolving armature.[1]

§ 19. Efficiencies of continuous-current dynamos.—In Fig. 436 are given the efficiency curves of two small machines for various proportions of their full-loads. The exact shape of the curve depends upon the relative proportions of the constant and variable losses. Thus in a 1000-kilowatt traction generator, the two are nearly equal at full-load, each being about 3 per cent. of the output, and giving efficiencies—

at full-load	of	94·5	per cent.
„ $\frac{3}{4}$	„	94·1	„
„ $\frac{1}{2}$	„	93	„
„ $\frac{1}{4}$	„	87·7	„

Fig. 437 shows the efficiency at full-load that may be obtained under ordinary commercial conditions with machines of different outputs.[2] From this curve it will be seen that the efficiency increases but

[1] For a full description of these two methods, see a paper by G. Kapp on "The Determination of the Efficiency of Dynamos," *Electr. Eng.*, Vol. 9, pp. 87 and 102 (1892). *Cp.* E. Wilson, *Electr. Eng.*, Vol. 32, p. 432; and for the extension of the Hopkinson principle to the testing of a single continuous-current multipolar machine, W. Lulofs, *Journ. I.E.E.*, Vol. 43, p. 150. The limits of accuracy in input-output tests of efficiency, chiefly of synchronous motor-generators, are discussed in *Trans. Amer. I.E.E.*, Vol. 32, Part I, pp. 525, 531, 551, and pp. 626 ff.

[2] *Cp.* R. Goldschmidt, *Journ. I.E.E.*, Vol. 40, p. 455; and for detailed curves of efficiency and losses in a Siemens 1500 kW generator, see *Electr. Eng.*, Vol. 42, p. 15.

little after an output of some 75 kilowatts is reached, and that for outputs above those of the figure the curve will become nearly flat at about 94 to 95 per cent. Any such curves are, however, greatly affected by conditions of the design, and in especial by

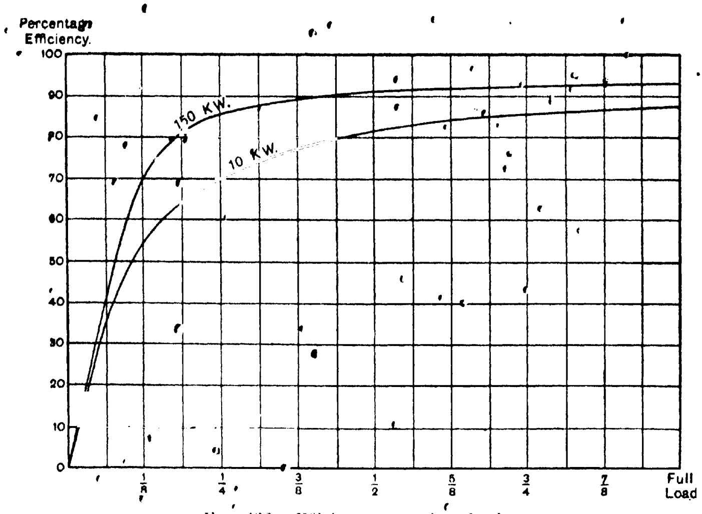

FIG. 436.—Efficiency at various loads.

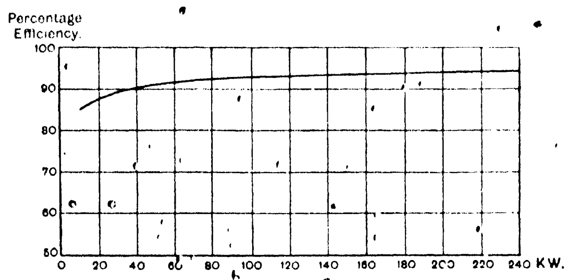

FIG. 437.—Efficiencies at full-load.

the speed, a low-speed machine being in general less efficient for the same output than one which runs at a high speed. For a given carcase, the losses in excitation and over the armature resistance remain practically the same whatever the speed. The loss from hysteresis, friction, and windage bears a nearly constant ratio to the output, and the only loss that increases faster than the speed is that from eddy-currents in the armature or pole-pieces. The

output itself rises nearly proportionally with the speed, so that, on the whole, within ordinary limits, the higher speed is favourable to the efficiency. There is, however, for a given armature with a given winding a certain speed at which the efficiency reaches a maximum, the assumption being made that the increase of the output with increasing speed is simply due to an increase in the volts; this speed is reached when the loss by eddy-currents is equal to the constant losses in the copper resistance of the armature and field-windings.[1] Such a fact is, however, of little assistance in the process of designing a machine in the first instance, since it presupposes a fixed copper loss. For a constant terminal voltage and speed, and given no-load losses, the maximum efficiency is reached when the variable losses proportional to the square of the armature current (*i.e.* practically the losses over the resistance of the armature, brushes and series windings) are equal to the no-load losses, but whether such a value for the armature current can be reached depends upon the limits set by heating and sparking. In electro-plating dynamos, owing to the large proportion of the total voltage which is lost over the contact-resistance of the brushes and other connexions, and also owing to the considerable friction loss from the numerous brushes, the efficiency is necessarily low; *e.g.* in a 10-kilowatt machine giving 4 volts and 2500 amperes, the efficiency may fall between 60 and 70 per cent. Very small dynamos are unable to excite themselves owing to the comparatively large air-gap required for mechanical reasons, and when separately excited, the excitation watts may become equal to the output; hence in very small sizes, as in models, ohmmeter generators, etc., a permanent magnet is employed.

§ 20. Weights of continuous-current dynamos.—For purposes of rough calculation and with a given type of dynamo the weight may often be taken as proportional to the two-thirds power of the watts per rev. per min., or $W = c\,(w/N)^{2/3}$ lb., and c has then such values as 200–170 in the case of multipolar machines with slotted armatures, the weights being exclusive of that of the bedplate. The proportion which the weight of the armature bears to that of the entire multipolar machine rises gradually from 20 per cent. in small to 42 in large machines, and averages about 35 per cent. Correspondingly its cost as compared with that of the whole machine is comparatively high, namely, about 45 per cent.

The above approximate formula for the weight may in the case of large machines with $D^2_a L_a > 13{,}000$ be also expressed as $25\,(D^2_a L_a)^{2/3}$, since w/N will then be $\simeq D^2_a L_a/17{\cdot}5$. But in the case of smaller machines the weight is proportional rather to the three-fourths power of D^2L. If the output kW be taken as proportional

[1] A. G. Hansard, *Electrician*, Vol. 38, p. 401.

to the fourth power of the linear dimensions, and the weight W as proportional to the third power, we should then have[1]

$$W \propto (kW)^{3/4}$$

§ 21. Homopolar design. The E.M.F. of a bar.—The design of the homopolar continuous-current dynamo, whether of the axial or radial type, is dominated entirely by the maximum peripheral speed of collector ring at which it is considered practicable to collect the current without sparking, undue heating by friction and from electrical causes, or excessive wear of the collector rings. As explained in Chapter VII, § 2, the whole of the flux of each magnetic circuit must pass once *through* the set of collector rings at one end of the armature. The total flux that can be used is therefore dependent upon the cross-sectional area of iron *within* the rings, and the flux-density at which it can be worked. In a word, the total flux is *not* dependent on the diameter and length of the armature core proper.

Let the maximum collecting speed in feet per minute that has been found to be feasible be

$$v_r' = \pi D_r' N$$

so that d_r'' or the diameter of the collecting surface in inches is $\dfrac{12v_r'}{\pi N}$. Then in the axial type of machine, taking into account the supports and fixing of the rings, the diameter of the steel core within the rings cannot be more than about 80 per cent. of the diameter of the collecting surface, or $d_c = 0{\cdot}8d_r''$. The area of the steel, assuming that there are no axial or radial ventilating holes in it, is therefore

$$\frac{\pi d_c^2}{4} = \frac{0{\cdot}64 \times 144 \times (v_r')^2}{4\pi N^2} \text{ sq. in.}$$

If B_c is the maximum permissible flux-density per sq. cm. in the steel, the possible flux through the rings at one end is

$$\frac{0{\cdot}64 \times 144 \times 6{\cdot}45 \times (v_r')^2 \times B_c}{4\pi N^2}$$

and twice this amount through the two ends of the double magnetic circuit type, which gives

$$\Phi_a = \frac{0{\cdot}64 \times 148 \times (v_r')^2 B_c}{N^2}$$

The highest possible voltage obtainable from a single bar or from a complete cylinder concentric with the shaft is thus

$$e = \Phi_a \cdot \frac{N}{60} \cdot 10^{-8} \text{ volts}$$

$$= \frac{0{\cdot}64 \times 2{\cdot}47 \times (v_r')^2 B_c}{N} \times 10^{-8} = \frac{1{\cdot}58\,(v_r')^2 B_c}{N} \times 10^{-8}$$

or less if there are any air-ducts through the steel under the rings. It will be seen that for a given speed of prime mover the possible voltage from a single bar is fixed by the flux-density B_c under the rings, and by the square of v_r'. The limiting condition for the voltage per bar is thus for a given value of B_c the peripheral collecting speed and not the peripheral speed of the armature proper.[2]

[1] *Cp.* Prof. M. Vidmar, *E.u.M.*, Vol. 36 (1918), p. 149.

[2] *Mutatis mutandis*, the same conditions also apply to the radial type. If the line-cutting length of the disc is 0·64 of the radius, which is not far from the case in practical designs, the maximum voltage from a single disc is $e = \dfrac{0{\cdot}79\,(v_r')^2 B_c}{N} \times 10^{-8}$. But in this type $B_c = B_g$ and its average value must be taken, since from the usual position of the exciting coils the density decreases somewhat as we pass from the periphery toward the shaft. Further, v_r' is also practically equal to the peripheral speed of the disc.

The number of bars and of rings at each end is of course E/e, and it is evident that a high number of revolutions per min., as when a steam turbine is the prime mover, is a positive disadvantage in so far as it increases the necessary number of rings and the length of path through the steel within them which must be worked at a high density if the machine is to give its greatest output. If B_c over a short length of path be taken as high as = 18,000, then with d_c assumed as above = $0{\cdot}8d_r''$,

$$\text{for } v_r' = 10{,}000 \text{ ft. per min.} \qquad e = \frac{28{,}400}{N} \text{ volts}$$

$$\phantom{\text{for }} v_r' = 20{,}000 \quad ,, \quad ,, \quad ,, \qquad e = \frac{114{,}000}{N} \text{ volts}$$

When the speed may be chosen at will, a decrease in the revs per min. will decrease the number of rings with their expensive outfit of brushes, but a limit is soon set to the economy in this direction owing to the iron or steel of the magnetic circuit becoming too bulky and heavy, and commercially too expensive.

The great weight of the homopolar dynamo in general has in fact been urged[1] as an insuperable objection to its use. But when comparison is made of its possibilities with those of the commutating dynamo for a given turbine speed and an output which is still within the reach of the commutating dynamo, it will be found that the advantage or otherwise of the homopolar entirely turns upon the number of rings and brushes that the designer and user are prepared to regard as a feasible proposition; for, as will be seen later, it is the number of rings that fixes in practice the value that can be assigned to the ampere-conductors per inch of rotor circumference.

§ 22. The output of homopolars.—If J = the current carried by any one bar, Z = the number of bars or of rings at one end, and ac = the ampere wires per inch of circumference of the armature, $JZ = ac \,.\, \pi D_a$, and the output of the machine is

$$e\,JZ = \frac{1{\cdot}58(v_r')^2 \,.\, B_c \,.\, ac\,\pi D_a}{N} \times 10^{-8}$$

This shows incidentally that the output is in no way determined by the length, so that a machine having a given section and length cannot be made to yield double the output by doubling its length. By this feature the continuous-current homopolar is sharply differentiated from the heteropolar machine. Further, the comparison of the values of $B_g \,.\, ac$ in the homopolar and heteropolar types is entirely fallacious. The values of B_g may be made higher in the former than in the latter machine, and so equal values might be obtained for $B_g \,.\, ac$. But the output of the homopolar is not primarily dependent upon the value of this product.

§ 23. The ampere-conductors per unit length of circumference.—The passage of the flux through the rings having once been secured, the two dimensions of the armature core have theoretically only to be so chosen that the density B_g of the main flux in the air-gap and the circular magnetization of the conducting cylinder have practicable values which are properly related to one another.

So far as concerns the proper relation between these two quantities, since the circular M.M.F. of the cylinder acts on concentric paths whose lengths are dependent upon the diameter of the circle of bars, armatures of different diameter will roughly have the same effect on the main radial flux of density B_g, if the amperes of the cylinder are also varied in proportion to its diameter, *i.e.* $\dfrac{1{\cdot}257\,JZ}{\pi D}$ or ac should be a constant. The diameter of the cylinder formed by the rotor bars can never be less than $0{\cdot}8d_r''$, but it may be greater than d_r'', if the bars as they leave the collecting rings are cranked outwards, and this could be done until the final diameter of the core reaches the safe limit of surface speed. If ac were strictly constant, this would, from the equation of the preceding section, be an advantage as increasing the permissible current or the number of bars.

[1] As by Dr. Pohl, *Journ. I.E.E.*, Vol. 40, p. 247.

§ 24. Values of ac in practice.—As a matter of fact the values of ac in machines that have been commercially built are not only much lower than in heteropolar direct-current machines but show very wide variations, ranging from 100 to 500 ampere-conductors per inch of circumference. This lack of any approach to a constant value for ac is partly due to the different number of rings that the different voltages have necessitated; with a large number of subdivisions of the inducing cylinder the length of the collector must necessarily be increased by the divisions between the rings, unless the widths of the rings and the amperes are reduced. But still more is it due to differences in the methods of collection. In one machine with a comparatively low value for v_r' the current may perhaps be collected at as many as 16 points on the circumference of each ring; in another machine with a higher value of v_r' there might be excessive wear or heating from friction unless the area of brush surface per ring and consequently the ampere-conductors per inch of circumference are very much reduced. Or if the rings are not artificially cooled and the same amperes are to be collected, their widths and consequently the length of the armature must be greatly increased. The best proportions for length and diameter of collector and the values to be selected for ac and v_r' are therefore a matter for adjustment and compromise in each particular case, the best value for ac being greatly influenced by the collecting apparatus.

§ 25. The diameter and length of armature core in homopolars.—Nothing has so far been said as to the actual diameter and length of the armature core proper. But since

$$\pi D_a L_a \times 6{\cdot}45 \times B_g = \Phi_a = \frac{0{\cdot}64 \times 148 \times (v_r')^2 B_c}{N^2}$$

their product must give

$$D_a L_a = \frac{0{\cdot}64 \times 7{\cdot}3 \times (v_r')^2}{N^2} \cdot \frac{B_c}{B_g}$$

It is, however, of practical convenience that the bars should run nearly or quite straight through from the collector rings into the slots or tunnels, and in this case the diameter of the core is usually not far from equality with the diameter of the collecting surface. D_a may then be approximately taken as equal to d_r''. Thence it results that

$$L_a = 0{\cdot}64 \times 1{\cdot}91 \frac{v_r'}{N} \cdot \frac{B_c}{B_g}$$

$$= 0{\cdot}64 \frac{d_r''}{2} \cdot \frac{B_c}{B_g}$$

and if $B_c = 18{,}000$ and $B_g = 10{,}000$,

$$L_a = 0{\cdot}575\, d_r''$$

or a little more than half the diameter of the collecting rings.

Very high densities up to $B_g = 15{,}000$ can be used without undue loss from hysteresis and eddy currents, but their practicability depends upon the air-gap being perfectly uniform and being maintained so. As soon as the bearings wear and the rotor becomes eccentric with the stator, the air-gap density becomes unequal, the core loss is increased, and the rotor suffers a strong unbalanced pull.

Lastly, still retaining D_a as approximately $= d_r''$, it is interesting to note that the watts per rev. per min. or the specific torque is

$$\frac{eJZ}{N} = 19\left(\frac{v_r'}{N}\right)^2 B_c \cdot ac \times 10^{-8}$$

$$= 0{\cdot}34 (d_r'')^2\, B_c \cdot ac \times 10^{-8}$$

where it must be observed that d_r'' varies inversely as N if the same permissible collecting speed which is the basis of the design is to be retained.

§ 26. Efficiency and uses of homopolars.—The proportions and locations of the different losses in a homopolar are entirely different from what they are in a heteropolar dynamo. The major part of the loss is in the collector rings from friction, electrical contact-resistance, and windage, while the core

loss from hysteresis and eddy-currents and the I^2R loss can be brought to a low figure. The over-all efficiency is practically the same as in a commutating dynamo.

The number of rings of the homopolar affords a ready means of obtaining a number of different voltages from one machine, so that they can be used directly on three-wire or five-wire systems as combined generators and balancers.

§ 27. The future of the homopolar.—In view of the fact that a high number of revs. per min. has been shown to be a serious disadvantage as increasing the number of rings for a given voltage, it may be asked what justification there is for the common belief that the homopolar is pre-eminently adapted to the high speeds of the modern steam turbine. The truth underlying this often-repeated statement is that when once the collection of a large current from one ring at a sufficiently high surface speed has been satisfactorily secured, there is no further difficulty in reaching very large continuous-current outputs at voltages as high as 600 and with machines of reasonable size and weight by a sufficient multiplication of the number of rings at each end. On the other hand, in the case of the commutating turbo dynamo we are met by the real physical difficulty of commuting large currents at very high speeds, which does in practice prevent them from being built

§ 28. Collecting gear at high speeds.—When the paramount importance of v_r' in homopolar dynamos has been appreciated, it is not surprising that the ingenuity of inventors has been much exercised to raise the permissible collecting speed to the highest possible value.

In the machine giving 2000 kW, 260 volts, 7700 amperes at 1200 revs. per min., which has already been mentioned in Chapter VII, § 6, with a collecting speed of 13,000 ft. per min., brushes of very thin leaf copper on copper spring-supported wearing rings were found to give the best results; to prevent the accumulation of dust at the backs of the brushes the rings were run against the brushes.[1] Sixteen brushes of the full width of the ring (3½ inches) collected the total current of 7700 amperes from each of the rings, of which there were 16 in all, and in continuous service so long as a good polish was maintained on the rings, their average wear was less than ⅜ in. per year.

Dr. J. E. Noeggerath, whose paper[2] on "Acyclic (Homopolar) Dynamos," describing a 300-kW 500-volt machine, again directed attention to the subject in 1905, has pushed the collecting speed up to as much as 20,000 ft. per min., laminated metal brushes being employed on steel rings.[3] A later machine of 2000 kW, 600 volts, had 48 inducing bars and 96 rings. The curves of brush losses of this[4] and other machines have shown that the total losses, electric and frictional, at speeds above 8000 ft. per min. do not increase much with increasing speed, while the increased ventilation is of great value in reducing the temperature rise of the rings; the increased contact resistance loss is practically balanced by a decreased friction loss. Further the friction loss decreases with higher current-density over the brush surface, and since the contact-resistance loss is not thereby increased in proportion, a fairly high current-density of 100–200 amperes per sq. inch is practicable. A high brush pressure of 3½ lb. per sq. inch is found to be advisable.

Mr. R. H. Barbour[5] has employed grooved slip rings, embraced by flexible wires made up of a stranded copper core wound over with an armouring to protect the core from wear. The pressure of the wire on the ring was maintained by adjustable tension springs, and a little lubricant was of advantage. The width of each ring to collect 500 amperes was only ½ inch.

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[1] B. G. Lamme, *Trans. Amer. I.E.E.*, Vol. 31, Part II, p. 1825 ff.

[2] *Trans. Amer. I.E.E.*, Vol. 24, p. 1.

[3] Further particulars of the practical construction of the (U.S.A.) General Electric Co.'s homopolar dynamos are given by Dr. Noeggerath in *Electr. World*, Vol. 52, p. 574.

[4] Plotted by E. W. Moss and J. Mould in *Journ. I.E.E.*, Vol. 49, p. 809 ff.; compare also P. J. Cottle and J. A. Rutherford, *Journ. I.E.E.*, Vol. 45, p. 679.

[5] *Engineering*, Vol. 92, p. 318 (1911).

In the case of a radial type of homopolar dynamo a drastic solution of the problem of collection has been sought by Dr. Boris von Ugrimoff[1] in the employment of small hatchet-shaped knives immersed in mercury. The mercury by centrifugal force is carried round the inside of an overhung annular groove on one side of the outer rim of the disc, and thus covers the edge of the knife which is arranged at the top of the disc. Water is also fed into the groove, and a layer is similarly carried round which cools the contact edges of the knives and is then drawn off after it has become heated. By this arrangement a collecting speed of 53,000 ft. per min. was reached, and from a disc of chrome nickel steel 28·6 inches diam. with a mean diameter to the centre of the field of 16⅜ inches 40 volts were obtained at about 7700 revs. per min., the mean line-cutting speed being 33,500 ft. per min.

[1] *Arbeiten aus dem Elektrotechnischen Institut zu Karlsruhe*, Vol. 2, p. 166, abstracted in *Engineering*, Vol. 92, p. 265.

CHAPTER XXIII

THE WORKING AND MANAGEMENT OF CONTINUOUS-CURRENT DYNAMOS

§ 1. The interconnexion of dynamos.—In many cases it is required to connect two or more dynamos to the same pair of mains in order to increase the amount of electrical energy that can be supplied to their common circuit. If the dynamos are coupled together in series, although the maximum current that may be passed through them is no more than the current permissible in the smaller of the two, yet the available voltage is increased. On the other hand, if two dynamos of the same voltage are coupled in parallel, the total amount of current can be increased, although the terminal voltage remains unchanged. Any such coupling together of dynamos must necessarily be so arranged that the working of one dynamo does not interfere with the proper working of the other or others; and as this requires certain precautions in their interconnexions, the more usual cases which occur in practice will here be shortly considered.

§ 2. The coupling of series-wound dynamos in series.—The simplest case is the coupling of two series-wound continuous-current machines in series. To effect this, it is only necessary to connect the + terminal of one machine with the − terminal of the other, the external circuit being then connected to the remaining terminals of the pair. Such an arrangement is not unusual in cases of transmission of power over long distances, where it is necessary to work with high pressures in order to combine economy in the first cost of the copper leads with a high efficiency of transmission. It has been already mentioned that the delicate process of commutation hardly permits of more than 4000 volts 300 amperes, or possibly 5000 volts 100 amperes, at about 250–500 revs. per min. being generated in any one continuous-current dynamo; but by the use of ten or more similar dynamos coupled in series, and each giving, say, 3000 volts, a combined E.M.F. of 30,000 or more volts is obtained on the external circuit.

The employment of such a system for the transmission of power over long distances by high-tension direct current[1] has been brought into use by M. Thury. In the St. Maurice to Lausanne transmission six turbines are employed, each driving two generators, and each of the latter giving 2250 volts and 150 amperes at 300 revs. per min.; being coupled in series, the total line pressure is 27,000 volts.

[1] *Cp.* J. S. Highfield, *Journ. I.E.E.*, Vol. 38, p. 471, and Vol. 49, p. 848; and *Elect. World and Engineer*, Vol. 48, p. 755.

In the transmission between Moutiers and Lyons, the largest turbines each drive two double machines, a complete set supplying 150 amperes and a maximum voltage of 18,250 at about 428 revs. per min. By this means a maximum line pressure of 75,000 volts has been reached. The turbine is connected to the dynamos by an insulating coupling, and each machine is separately insulated from earth, its base being bolted to a concrete block supported on stoneware insulators embedded in asphalt with pure bitumen run in to fill up the space round the beds.

§ 3. The coupling of shunt-wound dynamos in parallel.—The coupling of two or more dynamos in parallel is even more frequent. In all large installations and central electricity works, as the load increases, more dynamos have to be brought into use, without interruption to the supply from the machines already running. In such cases, in order to obtain the greatest economy, each dynamo should be worked as long and as closely as possible at its rated output: this is best attained if they are all capable of being worked in parallel, an additional machine being switched on to the same mains as soon as the load exceeds the combined output of those already at work. To connect two shunt-wound dynamos in parallel it is simply necessary to join their positive terminals to form a common + and their negative to form a common − terminal. If, as is usually the case, a second machine is to be joined in parallel with another which is already running and excited, or is to be connected to switchboard bus-bars fed by other running dynamos, this must of course not be done while it is at rest or unexcited; its armature would then form a short-circuit, and would present no E.M.F. opposing an excessive rush of current through its low resistance. Hence the in-coming machine, B, must be run up to its normal speed, and before it is thrown into parallel it must either be allowed to excite itself to approximately the same voltage as that of the bus-bars or of machine A, or B's shunt circuit must be closed on the bus-bars so as to excite B's field before its armature circuit is closed. If this is correctly done and the voltages are equal, the armature current in the machine when thrown into parallel will be and will remain zero. In order to make B take a share of the load, its excitation must be increased or its speed raised, the exact proportion in which the total current divides between the machines depending on their respective internal E.M.F.'s and armature resistances. The condition which determines this division is that, after deducting from the internal E.M.F. of a machine the volts lost by the passage of the current over its armature resistance, brushes and leads connecting it to the switchboard, the remainder or the terminal voltage at the bus-bars must be alike in all machines. Thus, if two similar machines, similarly excited and run at the same speed, be coupled in parallel, each will

take half of the total current. If the speed of one be now lowered or its field weakened, its current will gradually pass over to the other machine; when its internal E.M.F. falls to equality with the terminal voltage of the second machine it supplies no current at all, the whole of the load being thrown on to one machine, and this is the condition which should approximately be reached when a machine is to be withdrawn from service.

To take a numerical example, suppose that each machine runs normally at 1000 revolutions and is then excited with 13,000 ampere-turns giving $\Phi_a = 4{,}300{,}000$, and $E_a = 103$ volts; further, that the loss of volts over the armature at the full-load of 100 amperes reduces the terminal voltage V_b to 100 volts. If the speed of machine B falls, a larger portion of the current passes over to machine A. The increased loss over the armature resistance of A and the increased reaction of the armature current on its field combine to reduce both its internal and terminal voltages. When it takes the whole of the current, let $\Phi_a = 4{,}100{,}000$ be the number of lines that are produced by the 12,000 ampere-turns due to the terminal voltage of 92 volts; in other words, $I_a = 200$ amperes, and $V_t = 92$ volts, give a point on its characteristic curve for a constant speed of 1000 revolutions. This same terminal voltage will, however, in the case of machine B which is carrying no current, give, say, $\Phi_a = 4{,}250{,}000$, and this flux will give an internal E.M.F. of 92 volts when B is running at 900 revolutions. Thus the speed of B may be reduced by 10 per cent. before the whole of its load passes on to machine A, provided that the speed of the latter is kept strictly constant. On a further reduction in the speed or field-strength of B, its internal E.M.F. falls below the terminal voltage of A, and the latter then drives a reverse current through B's armature: the internal E.M.F. of B *plus* the volts lost over its armature resistance due to the reverse current are then equal to the terminal voltage of A. The effect of the reverse current through B's armature is to assist in turning it as a motor with the same direction of rotation as before, without mechanical damage to any part, and thereby it tends to keep up B's speed. The electrical interaction of the two machines with drooping characteristics thus exerts a considerable inherent influence, tending to equalize their speeds and loads; shunt-wound dynamos are therefore easy to work in parallel, and the share of the current which each machine takes is easily regulated by altering its speed or its field excitation. In fact, without interfering with the speed, the rheostat in the shunt circuit affords a ready means of loading or unloading the continuous-current machine to any desired extent. In practice, when, as is most often the case, the machines are driven by separate prime movers, the equalizing tendency is also assisted by the mechanical action of the prime movers themselves. Thus, when the load of

one machine increases, the speed of its engine or other prime mover falls and tends to check the increase, while, for a similar reason, when a machine fails to maintain its fair share of the load, the speed of its prime mover rises, and tends to keep up its E.M.F.

The use of shunt-wound dynamos in parallel is common in central stations for electric lighting: as soon as the load becomes too great for a single machine, a second is run up to speed, excited to an equal voltage, and switched into parallel with the first, the same process being repeated with other dynamos as often as required. When so connected, they all supply current to a common pair of "omnibus bars," whence the feeders are run to the network of mains. As a precautionary measure, a magnetic switch is frequently inserted between each dynamo and the omnibus bars: this automatically flies off and breaks contact if the armature current falls below a certain minimum; and hence, if for any reason a dynamo begins to slow down, it is cut out of circuit before a reverse current passes through it, while its load is taken up and divided among the other dynamos at work, without interruption to the general supply of current.

§ 4. The coupling of series-wound dynamos in parallel.—When series-wound dynamos are connected together in parallel after the same fashion by joining their like terminals, we are met by the difficulty that, if for any reason the E.M.F. produced by one machine B falls considerably below that of the other machine A, then the current through B is reversed; and this, since the machine is series-wound, reverses its polarity. The direction of B's E.M.F. is consequently reversed, with the result that both dynamos simply act in series round their own internal resistances, and in a short time would be damaged by the excessive current. This difficulty is, however, at once overcome by the addition of an *equalizing* wire connecting the ends of the series coils adjoining the brushes (Fig. 438).

It is especially important that this equalizing lead, $b\ b'$, should be of such large area, that its resistance is practically negligible as compared with the resistance of either of the series windings. By it the extremities of the two field-windings are joined in parallel, and the current supplied to the external circuit flows in the same direction through both and magnetizes each equally. Should the internal E.M.F. of machine B fall below the voltage at the brushes of A, a reverse current passes through B driving it as a motor, but as this motor current flows through the lead $b\ b'$, and does not pass in a reversed direction through the series winding of B, it does not reverse B's polarity. If the fall of volts over $b\ b'$ is considerable, a certain proportion of the motor current may pass through the alternative path of $b\ d\ d'\ b'$, but this is prevented by making $b\ b'$ of sufficiently low resistance, as mentioned above. If it be required

to switch B into parallel with A while the latter is running, it is necessary, as in the case of shunt machines, to excite B to approximately the same voltage: this is effected by closing the switch K while B is running, and a few seconds later switch S' may be closed.

5. The coupling of compound-wound dynamos in parallel.—Series-wound dynamos are, however, but seldom required to work in parallel, and the above remarks are merely introduced owing to their applying equally well to the series windings of compound machines when worked in parallel. The arrangement usually adopted for these is shown in Fig. 439, from which it will be seen

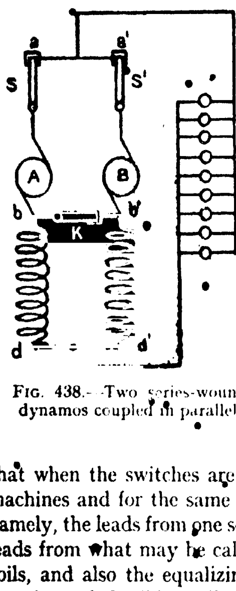

FIG. 438.—Two series-wound dynamos coupled in parallel.

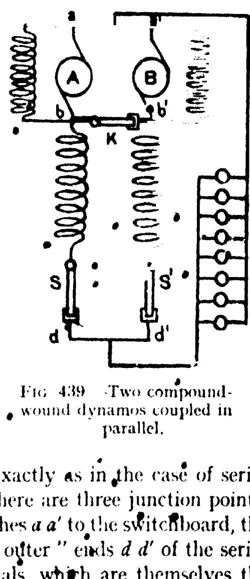

FIG. 439.—Two compound-wound dynamos coupled in parallel.

that when the switches are closed, exactly as in the case of series machines and for the same reason, there are three junction points, namely, the leads from one set of brushes a a' to the switchboard, the leads from what may be called the "outer" ends d d' of the series coils, and also the equalizing terminals, which are themselves the junctions of the "inner" ends b b' of the series coils to the other set of brushes. Again, the equalizing wire b b' must be of negligible resistance as compared with the series windings,[1] since any reverse current through the series coils of a compound-wound dynamo will partially demagnetize it and finally overpower the shunt. In order to throw the one machine, say B, into parallel with the other, A, when the latter is running, B must be run up to approximately the same voltage as A, the excitation being provided by the shunt

[1] For a more exact calculation of the possible resistance that the equalizer bar may have, see Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, p. 275.

alone; switch K is then closed, and finally switch S', while, if it be required to withdraw machine B from parallel connexion, its speed should be slightly reduced until it is supplying little or no current, after which switch S' is opened, and lastly switch K. The opening or closing of the two connexions in due order may be conveniently effected by a composite switch, by which the contact at K is made first on closing the switch, and broken last on opening it.

If the dynamo ammeters are placed in the leads to the switchboard bus bars, on the same side $d\ d'$ as the series coils, the sum of the currents gives the total amperes supplied to the external circuit, but the separate readings of the instruments in no way indicate how the load is divided between the several armatures, and one or more might be running as motors with current supplied by the other machines. It is, therefore, essential to place the ammeter of each dynamo in the lead to the bus bar which is on the side $a\ a'$ opposite to the series coils, when it will measure the current actually passing through the machine to which it is connected. Further, it is advantageous to employ moving-coil ammeters with a central zero position and a short scale on one side, so that the readings of the needle to one or the other side of the zero render the state of affairs evident at a glance. In every case, either the hand-wheel governing the speed of the engine or the shunt-regulating switch must be employed to secure equal division of the load.

§ 6. Regulation of load between compound-wound dynamos in parallel.—In the case of over-compounded dynamos, it is not easy to secure an equal division of the load between several machines owing to small differences in their characteristic curves, and, even if at full-load the division is proportional to their respective capacities, it may not retain this strict proportionality throughout a wide range. The first requirement for running compound-wound machines of different make or of different output in parallel is that the drop of volts over the series windings and the leads connecting the outer positive and negative terminals to the bus bars on the switchboard should be the same for all machines when carrying their proper share of the total current delivered to the board; in other words, the resistances of the series coils and connecting leads should vary inversely as the normal full-load current for each machine. If the junction points or the bus bars were immediately upon the machines, as in Fig. 439, and therefore added practically no resistance, it would suffice that the external characteristics of the several machines at their terminals should be alike to secure proper division of the load. But such is not the case in practice, since the switchboard is usually at some distance and also is at different distances from the machines; the connecting leads must therefore have appreciable resistances, and these may differ even when the machines are of equal size. It is then necessary to insert in the connecting

leads small resistances to carry the full-load current, and of such amounts that the drop of volts over both series winding and connecting leads again becomes in each case equal. The external characteristics at the bus bars on the switchboard are thus once again made similar, and although over-compounded each machine will take its due share of the total load, so long as it is running at the correct speed. But, whatever precautions are taken, satisfactory parallel working must in the end depend upon the mechanical

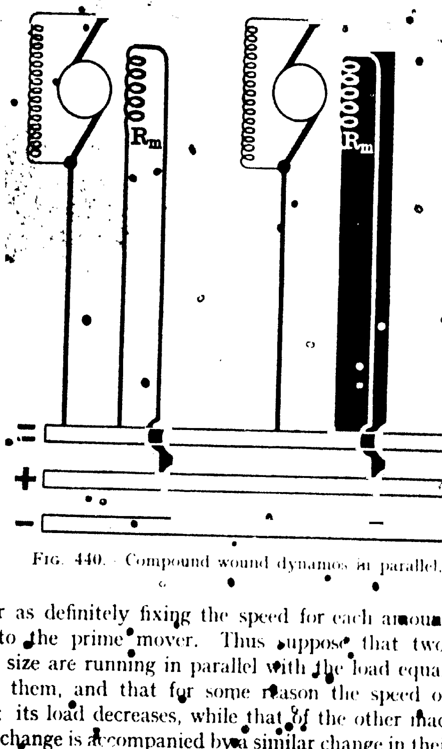

FIG. 440.—Compound wound dynamos in parallel.

governor as definitely fixing the speed for each amount of steam supply to the prime mover. Thus suppose that two dynamos of equal size are running in parallel with the load equally divided between them, and that for some reason the speed of one falls slightly: its load decreases, while that of the other machine rises, and this change is accompanied by a similar change in their voltages, so that the load tends to pass completely over to the machine which has maintained its speed, and this tendency is only held in check by the mechanical governor; no inherent electrical effect comes into action until a motor current is actually supplied from the machine of higher voltage to assist in keeping up the speed of the machine whose voltage has fallen. Perfect division of the total current between the several series coils, and also at the same time practically zero resistance between the "inner" ends of the

series coils, can be realized if the junction point between the "inner" ends is transferred to the switchboard and is effected by the equalizer bus bar (marked =) as shown in Fig. 440. The resistance of the connecting leads is then adjusted so that the drop of volts at full-load is in each case equal and the machines are on complete equality. But such an arrangement requires four leads to the switchboard from each dynamo, and a triple-break switch for each dynamo.

The amount of the compounding action of a machine is frequently regulated by a *diverting switch and resistance*, which shunts a portion of the current away from the series turns through a by-pass. When this is the case, any alteration of the diverting switch of one machine also affects the current through the series turns of the other machines. This may be corrected by the employment of a *compensated series regulator*, of which the principle is shown in Fig. 441 in connexion with a dynamo which is used as a shunt machine on a lighting load and as a compound-wound machine on a traction load. In the position of the switch shown in the diagram, the current passes immediately through the arm from a to b, and there is no compounding action at all. But when the arm is moved in a clockwise direction, a shunting resistance r_2 is inserted in parallel with the series turns R_m, and as the resistance r_2 increases step by step, more and more of the total current I passes through the series turns. At the same time, with a movement of the switch from its starting position a compensating resistance r_1 is inserted in series with the parallel branches of the series turns and shunting resistance, and this resistance r_1 is gradually cut out until with the arm on contact 6 all the current passes through the series turns, and the maximum compounding action is obtained. On any contact (except the starting position) $r_2 = \frac{I_m}{I - I_m} \cdot R_m$, and $r_1 = \frac{I - I_m}{I} \cdot R_m$, where R_m is the resistance of the series turns and of the leads therefrom to the regulating switch. By this means the resistance between the points a and b is maintained perfectly constant at a value $= R_m$, and an increase of I_m from zero to I is obtained by six equal steps without in any way affecting the other machines which are in parallel.

§ 7. **Three-wire dynamos.**—For the purpose of using a single dynamo on a three-wire system without any auxiliary balancer, it is necessary to make connexion for the third wire with a point nearly midway in potential between the positive and negative brushes. If a third set of brushes were applied at points on the commutator intermediate between the positive and negative brushes, their position would, strictly speaking, require adjustment according to the amount of load on the machine, and also according to the amount of out-of-balance current, but, apart from this, such destructive

sparking would ensue as to render the method practically inadmissible. An armature wound with two independent windings, each for half the main voltage and the two connected in series, meets the case better, as the third wire can be connected to the

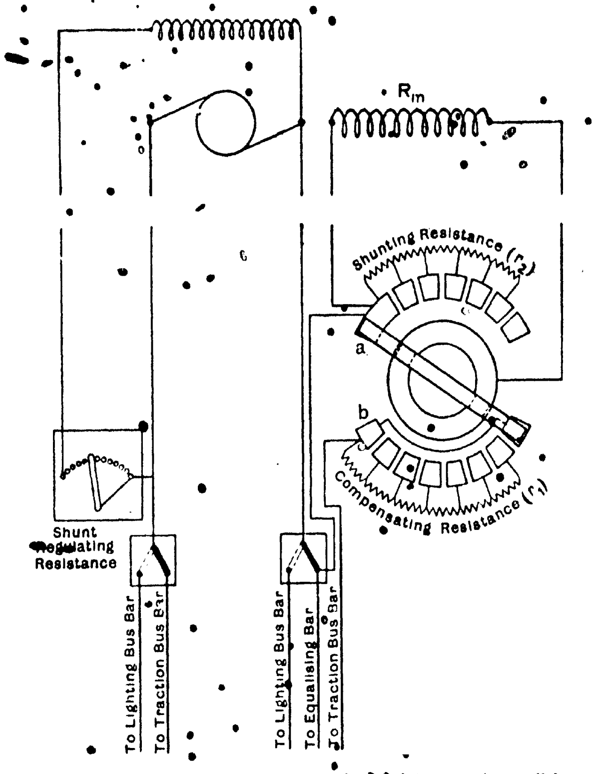

FIG. 441.—Series regulator for compound wound dynamos in parallel.

common junction uniting the positive brushes of one armature with the negative brushes of the other; but it involves two commutators, one at each end of the armature, and two sets of brush-gear, so that its cost is great, and it does not admit of the voltage of one side being regulated independently of the other save by external means.

A simpler method by which, without moving machinery and with a single armature, a central point may be found for attachment of the third wire, is due to von Dolivo Dobrowolsky, and is not infrequently employed in small generating stations, as in factories, where

some proportion of the motors or other plant is better suited by a low voltage, such as the half of the main voltage between the outers. It consists in the addition to the armature of two or more slip-rings each connected to the armature winding at a point or at points corresponding to an equivalent number of electrical degrees according as it is wave or lap, just as in a rotary converter, and in the connexion of these slip-rings to a choking coil. The latter has a laminated closed magnetic circuit and is wound for as many phases as there are slip-rings. The central point of the windings of the choking coil, which is also called the "static balancer," is then practically midway in potential between the positive and negative brushes, and furnishes a point to which the third wire may be attached.

In its simplest form two slip-rings are employed, and the choking coil is two-phase, one half representing a phase displaced 180° from the other half (although such an arrangement is usually called "single-phase") and the third wire is attached to the middle of the winding of the coil. The two slip-rings are joined to any two points 180° apart on a wave-wound armature, or on a lap-wound multipolar each is connected to p points separated by a distance corresponding to a pair of poles, and these two sets of p points are mutually displaced by a distance corresponding to the pole-pitch. The pair of slip-rings with their brush-gear may be seen in Fig. 442, which shows a 440 kW generator built by the British Thomson-Houston Co., Ltd., for 3-wire connexion. When the armature is at work, an alternating E.M.F. is set up between the slip-rings, and so long as the two sides of the network are exactly balanced or on open external circuit, this simply causes a lagging alternating current of frequency $pN/60$, which magnetizes the choking coil. As the windings of the coil should have a very low resistance, the value of the magnetizing current is practically fixed by the reactance $2\pi fL$, and the loss in watts is thereby kept small. The virtual voltage applied to the coil is dependent upon the ratio of the pole-arc to the pole-pitch, but on the assumption of a sine law distribution of the magnetic flux is with two slip-rings $1/\sqrt{2} = 0{\cdot}707$ of the voltage at the brushes on the commutator, and in general with increased numbers of slip-rings bears the same ratios to the direct-current voltage as in a rotary converter.

If a dynamo without any static balancer is connected to a three-wire network, of which the resistances R_1 and R_2 on the negative and positive sides respectively are unequal, the divergence of the potential of the third-wire from the mean between the potentials of the brushes will be

$$V_0 = \frac{V_b}{2} \times \frac{R_1 + R_m - R_2}{R_1 + R_2 + R_m} \text{ or } = \frac{V_b}{2} \times \frac{R_1 - R_2 - R_m}{R_1 + R_2 + R_m},$$

according to whether the series winding of resistance R_m is on the negative or the positive side of the network. It must be only on one or other side, since when the static balancer and third-wire are connected, the external

currents in the two sides of the network will be different, and therefore the series winding must not be halved since the voltage of the parallel circuits of the armature would then be different.[1] Hence V_0 is positive when $R_2 + R_m > R_1$ or in the second case when $R_1 > (R_2 + R_m)$, *i.e.* when the positive side has the lower resistance and is the more heavily loaded.

When the static balancer is connected to the generator, but is still left unconnected to the third-wire, the potential of the centre point of the balancer where its phase windings unite, is from considerations of symmetry always exactly midway between the potentials of the brushes. Due to the presence of the alternating current through the balancer, the terminal voltage, the

FIG. 442.—440 kW, 215 r.p.m., 3-wire generator.
(The British Thomson-Houston Co., Ltd.)

external current, and the divergence of the third-wire potential are all caused to pulsate slightly. But when the tappings to the armature winding, say, of a 2-phase balancer are passing points 90 electrical degrees from the brush positions, the instantaneous value of the pulsating initial divergence of the potential of the third-wire (still unconnected) is identical with the value V_0 given above, and this particular moment need alone be considered. Unless, therefore, the series winding of the generator happened exactly to equalize the total resistances on the two sides of the network, the potential of the third-wire differs from that of the centre point of the balancer. The consequence of connecting the two together must be the flow of such out-of-balance currents through the balancer, divisions of the armature winding, and external network as will equalize the potential of the two points that are to be joined together.

If we imagine the centre point of the balancer to be earthed, the potential of the third-wire must be lowered or raised to zero, and the numerical values of the potentials of both sets of brushes will be either raised or lowered by equal amounts; if we keep the potentials of the brushes of equal numerical value but of opposite sign, the potential of the centre of the balancer and of the third-wire

[1] To connect the series coils on the several poles alternately to the positive and negative sides would be inconvenient and undesirable with high voltages.

become alike and neither is zero. On either way of considering the effect, the new divergence of the potential of the third-wire from a value midway between that of the brushes is reduced from V_o to a lower value which at the same moment is very closely

$$v_c = \frac{I_o}{n_{ph}}\left(r_{ph} + \frac{R_a}{2}\right)$$

where n_{ph} is the number of phases, I_o is the current in the third-wire, and r_{ph} is the resistance of one limb or phase of the balancer.

The out-of-balance currents in each phase of the balancer are in the same direction relatively to the centre, *i.e.* all outwards from it or inwards to it through the windings of the balancer, according to which side of the network is the more heavily loaded; they are outwards if v_c is positive or $R_1 > R_2$, and the + side of the network is the more heavily loaded. Their values pulsate, but they are equal at the moment assumed above. Hence if i is the natural alternating current of a phase, it is approximately, though not absolutely, true to say that the current in any limb of the balancer is made up of $\frac{I_o}{n_{ph}}$ constant in direction for given resistances on the two sides of the network, combined with i which alternates in sign and value in accordance with the time.

The out-of-balance current I_o flowing in the third wire is frequently given as a datum of design, but strictly speaking its value cannot be determined until the static balancer and the generator have been designed and the resistance r_{ph} of one limb or phase is known, as well as the resistances of the armature and series winding of the dynamo. From given values simply of R_1 and R_2, the resistances of the two sides of the network, and of V_e, the terminal voltage across the outers, it is only possible to state the maximum ideal out-of-balance current if the voltage on either side of the network were the same, *i.e.* if the balancing were perfect. This ideal value of I_o is $\frac{V_e}{2} \cdot \frac{R_1 - R_2}{R_1 R_2}$, and from this must usually be deduced the values of R_1 and R_2 when these are not definitely stated.

Assuming then the values of R_1 and R_2, the real value of I_o can be more accurately stated. Let the current through the network without any balancer be $I_b = \frac{V_e}{R_1 + R_2} = \frac{V_b}{R_1 + R_2 + R_m}$ where V_b is the brush voltage. The series winding, being necessarily arranged only on one side of the network in order that the voltage of the parallel circuits of the armature may be equal, is here assumed to be on the same side as R_1, and since we are not now concerned with the sign or direction of V_o or I_o, R_1 may be simply made the higher resistance so that the conditions are at their worst.

$$\text{Then } I_o = \frac{(R_1 + R_m - R_2)\, I_b(R_1 + R_2 + R_m + R_a)}{U'}$$

where U' has the following values:—

in the 2-phase case, $U' = 2(R_1 + R_m)R_2 + r_{ph}(R_1 + R_2 + R_m + R_a) + R_a(R_1 + R_2 + R_m + \tfrac{1}{2}R_a)$

" " 3-phase " $= 2(R_1 + R_m)R_2 + \tfrac{2}{3}r_{ph}(R_1 + R_2 + R_m + R_a) + \tfrac{2}{3}R_a(R_1 + R_2 + R_m + \tfrac{1}{3}R_a)$

" " 4-phase " $= 2(R_1 + R_m)R_2 + \tfrac{1}{2}r_{ph}(R_1 + R_2 + R_m + R_a) + \tfrac{1}{2}R_a(R_1 + R_2 + R_m + \tfrac{1}{4}R_a)$

The first stage of the design must, however, be to determine r_{ph}. If y = the fraction which the excess voltage on one side of the network may be permitted to be of the half brush voltage, *i.e.*

$$y = \frac{v_c}{V_b/2} = \frac{2v_c}{V_e} \times \frac{R_1 + R_2}{R_1 + R_2 + R_m}$$

$$r_{ph} = \frac{y n_{ph}(R_1 + R_m)R_2}{R_1 + R_m - R_2 - y(R_1 + R_2 + R_m)} - \frac{R_a}{2}$$

With an increase in the number of slip-rings, the advantage is gained that the out-of-balance current is more uniformly distributed over the armature winding, and local heating is minimized.[1]

§ 8. Foundations and erection of dynamos.—It remains to add a few remarks on the fixing and working of dynamos and motors in general. In the first place, good foundations are as much an essential for successful working as a good machine, inasmuch as any vibration is most detrimental to the life of the armature, commutator, and brushes. The ground should be excavated until a sound and solid bottom is reached, on which may be built up a foundation of concrete, or of concrete and brickwork, sufficiently massive to absorb and damp the vibration of a fast-running dynamo or engine. With small motors, Lewis bolts having a tapering shank of rectangular section with jagged edges are used to hold down the bed-plate or slide-rails: and square holes required for these bolts are cut into the concrete, their position being marked off usually from a wooden template of the slide-rails, and when these latter are ready to be laid down the bolts are dropped in and fixed by lead or sulphur cement run in round them. With larger dynamos, long holding-down bolts are used, passing right through to the bottom of the concrete and terminating at their lower ends in large square plates. The holes for such bolts are formed in the concrete by inserting long tapering wooden boxes, of square section and made collapsible for the purpose of withdrawing, through which pass the bolts; when the bed-plate is in position, the cement is run in round the bolts until the holes are full, and is then allowed to set without disturbance. In all cases, the upper surface of the foundation should be carefully levelled with a straight-edge and spirit-level, or a smooth slab of York stone may be used as a seating on the top of the concrete.

Perfect steadiness of driving is of great importance, especially for direct incandescent lighting; a very slight fluctuation in the speed of the dynamo, even though it be not great enough to cause sparking, is immediately discernible as a pulsation in the light of the lamps, owing to the slight change of E.M.F. to which it gives rise. On this account some types of gas or oil engines are inadmissible for direct lighting owing to their great fluctuation of speed during each cycle, even when fitted with heavy fly-wheels. When such prime movers are used, the dynamo is usually worked in conjunction with an accumulator battery, and even then it is advisable for the dynamo to be itself fitted with a heavy disc fly-wheel. In such cases, the secondary cells should never be placed in the same room with the dynamo, since the cotton or other fibrous insulation of the dynamo

[1] For a fuller treatment, *vide* "The Theory of the Static Balancer," by C. C. Hawkins, *Journ. I.E.E.*, Vol. 45, p. 704 and *Electr.*, Vol. 67, p. 342; and H. Lorenz, *E.u.M.*, Vol. 35 (1917), p. 90.

wires is rapidly attacked and eaten away by the acid spray given off during the process of charging.

After the machine has been fixed in position, the armature should be turned round by hand to see that it revolves freely and that nothing is loose; it should then be run for some time with the brushes raised from the commutator, in order to test the alignment of the bearings and of the machine generally. The adjustment of the brushes when lowered on to the commutator requires careful attention; usually lines are cut on the collar or sectors of the commutator next to the outer bearing (these lines corresponding to the angular pitch of the poles), and to them the brushes must be set, the tips of each set of brushes carried by one arm of the brush gear being in line with one setting mark.

All screw contacts should be firmly screwed up, and any dirt or lacquer (if such there be) on the points of terminal-screws should be cleaned off. Want of contact of the brushes on the commutator or elsewhere may cause a failure of the dynamo to excite. The electrical connexions should be carefully examined and verified, in especial, the connexions to the field-magnet coils. On starting, it should be borne in mind that a shunt-wound dynamo will not excite on a very low resistance, or a series-wound dynamo on a very high resistance. If, therefore, a shunt-wound dynamo fails to excite even when the main switch is open, a short circuit in the leads is a possible cause. Any such difficulty will usually be solved by testing with an ordinary linesman's detector; or, in default of a solution by this means, trial may be made with the connexion of the brush leads to the field winding transposed, in case it is through misconnexion that the field will not excite.[1]

If a machine has been standing for long out of action, and its windings are consequently very damp, it should be run at a low voltage for some little time until it becomes thoroughly dry.

§ 9. Care of machine in working.—To bed a new set of carbon brushes, a long strip of fine emery or carborundum cloth should be strained taut against the curved surface of the commutator, and then be drawn repeatedly in the direction of rotation under the face of the brushes, which are meanwhile pressed down on to it by their pressure springs. After any such operation, the tips of the brushes must be cleaned from any adherent copper dust.

For the filling of the lubricators copper oil-cans should invariably be used, since iron cans are liable to be drawn to the magnet, and thereby perhaps cause damage by catching in the armature. All oil-pipes and waste oil-chambers require occasional attention to see that they are not clogged. If from any neglect in this respect, or from original defective construction, lubricating oil creeps from the

[1] For other causes of failure to excite and their remedies, *see* Miles Walker, *The Diagnosing of Troubles in Electrical Machines*, pp. 287–289.

bearings on to the surface of the commutator, it becomes carbonized by any sparking at the brush-tips, and forms a thin conducting film which bridges across the strips of insulation between the sectors of the commutator, the result being a loss of E.M.F. due to the local leakage which ensues between neighbouring coils; while if oil further makes its way on to the armature winding, it has a deteriorating effect on the protecting varnish, and causes adherent deposits of dirt and copper dust. Every dynamo should therefore be kept clean, and in especial its commutator requires scrupulous care on this point. A small air compressor, by means of which the dust collected by the armature connexions and winding may be blown out at intervals, is a valuable accessory to any large installation. The terminals and insulating washers of the brush gear occasionally require to be wiped to remove any dust which may have lodged on them. With carbon brushes paraffin wax may be sparingly used as a lubricant and to prevent chattering. If carbon brushes show signs of copper dust working into their bearing surface, any such coppering must be carefully scraped away. The dark burnished lustre which may be seen on the commutator of a good non-sparking dynamo is the sure evidence of a careful attendant. Occasionally, before stopping, fine emery cloth may be applied if the surface shows signs of wearing into grooves and becoming uneven. If a flat begins to develop on one or more sectors (Chapter XX, § 49), it may often be ground out by applying a hard stone with curved face to the commutator when running. But if the disease has gone too far for such remedy, the armature must be put in a lathe and the commutator surface turned up true; the tool should be sharp and fine-pointed, and the feed should be light, so as not to drag the copper over the insulating strips of mica; after turning, it should be lightly filed with a smooth file, and finally examined to see that no particles of copper are embedded in the mica, bridging adjacent sectors.' Still better is it to grind the commutator true in place by means of a small motor-driven emery wheel.

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